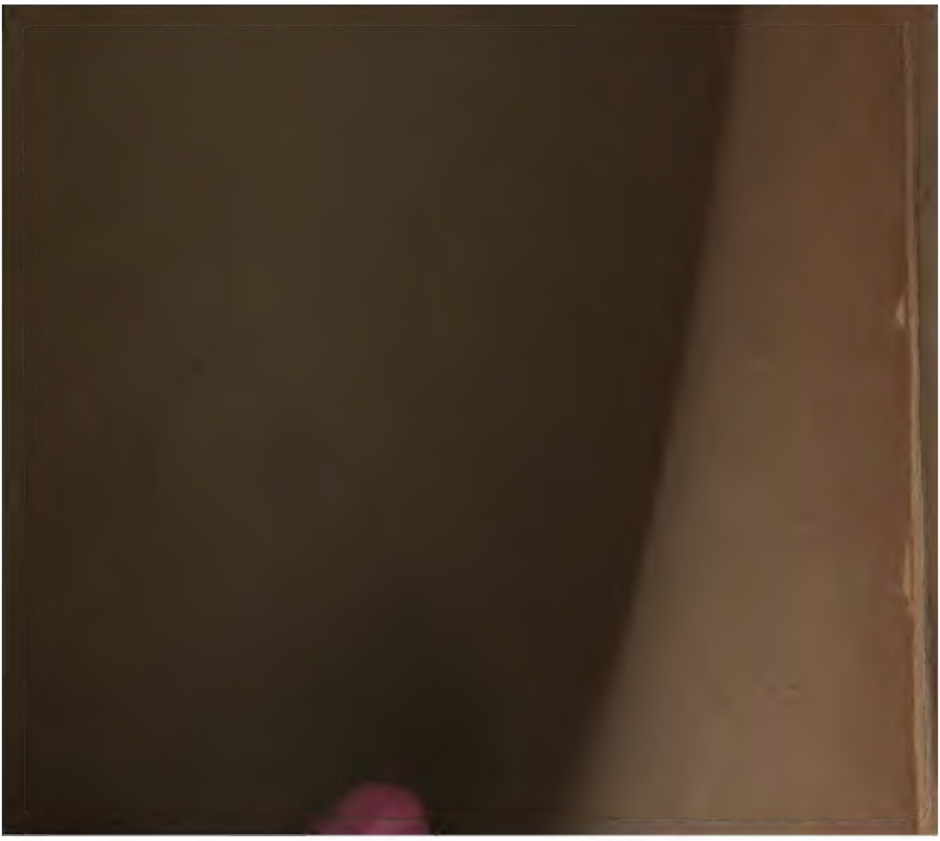

AMERICAN SCIENCE SERIES

PHYSICS

ADVANCED COURSE

BY

Frederic

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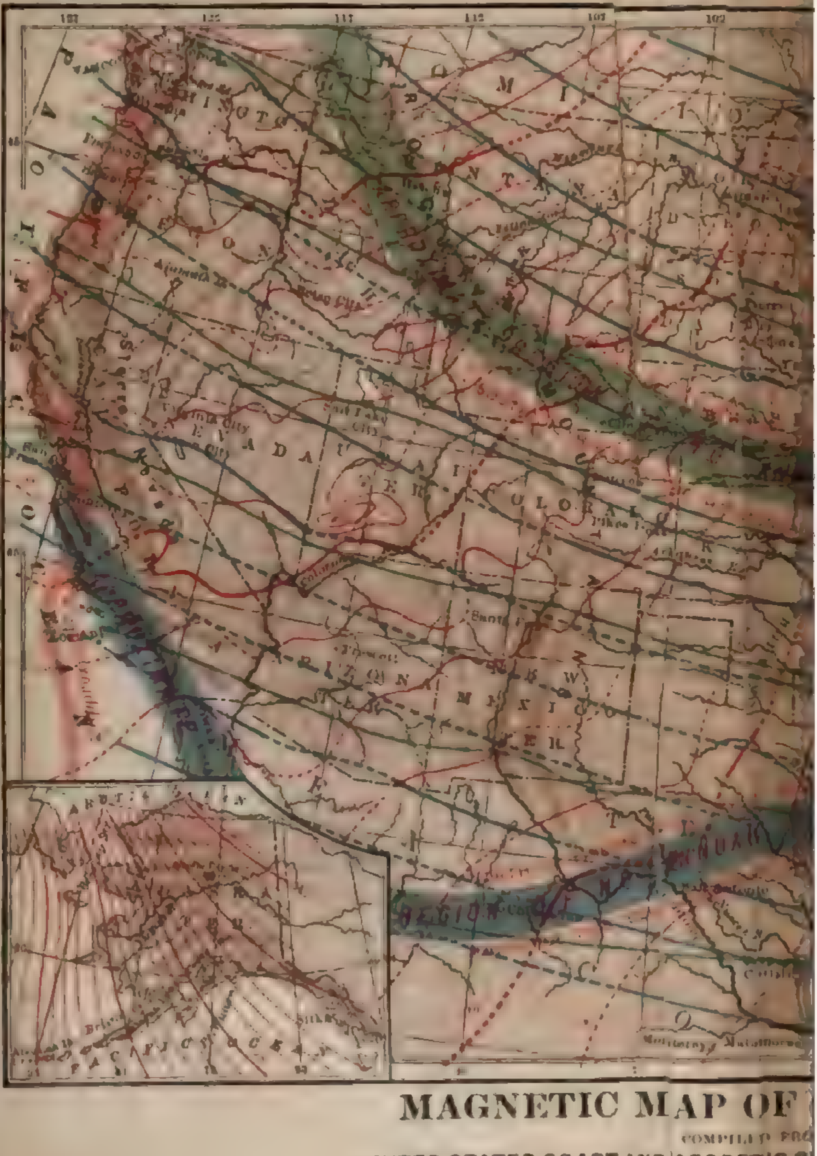
Figure 292.



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PREFACE.

WITHIN the past decade the progress which has been made in physical science has completely changed its aspect. The most striking feature of this advance, unquestionably, is the much greater importance which the phenomena of energy have assumed in all physical discussions, as compared with the phenomena of matter. The physics of to-day is distinctively the science of energy. Henceforth every physical change must be regarded as conditioned upon the transference or the transformation of energy. It is from this point of view, therefore, that any new text-book of Physics must present the subject. Hence the classification which has been adopted in the present work is based on the most recent views of energy, considered as being ultimately a phenomenon of the æther. At present, all physical phenomena seem capable of satisfactory discussion under the heads of mass-physics, molecule-physics, and æther-physics. And the fact is significant, that to the last subdivision of the subject it has been found necessary to devote more than half of the entire work.

The introductory portion of the book considers (1) physical relations in general, and (2) the laws of motion; the latter being discussed, first in the abstract, and second with reference to the action of force upon matter. Under mass-physics energy is first treated of as a mass-condition, and then work, as being done whenever energy is transferred or transformed; the subject of potential

being developed as a consequence of mass-attraction. The properties of matter are next considered, including the modern views of its structure; and then follows the subject of sound considered as a mass-vibration. Under molecular physics the phenomena of heat alone are treated; the term heat being restricted, in accordance with modern usage, to molecular kinetic energy. Under the head of æther-physics are grouped: (1) æther-vibration or radiation, (2) æther-stress or electrostatics, (3) æther-vortices or magnetism, and (4) æther-flow or electrokinetics; following the classification so well set forth by Lodge. Radiation is considered broadly, without any special reference to those wave-frequencies which excite vision and are ordinarily called light.

The author's aim has been to avoid making the book simply an encyclopedic collection of facts on the one hand, or too purely an abstract and theoretical discussion of physical theories on the other. The ground covered is that which is usually traversed by students in the more extended courses in Physics in our leading Universities, Colleges, and Technological Institutes. The mathematics required in the derivation of the formulas is only that with which students taking such courses may be presumed to be already acquainted. Obviously, however, these derivations can be omitted at the discretion of the instructor. To facilitate the use of the book in the class-room most of the illustrative and explanatory matter is printed in a smaller type.

With regard to the subject-matter of the book, the author lays no claim to any originality. He has made free use of all the sources of information at his command and has freely given credit for the material thus taken. The names of those physicists to whom the science is most deeply indebted are given in connection with the subjects on which they have worked; and in order to bring the student into more intimate contact with these great minds, the laws or principles which they formulated have frequently been given in their own

words. Especially has this been done in the case of American investigators. To make more easy a reference to these workers in science, their names have been included in the index.

The metric system has been used throughout the book; all the units employed in it being those of the C. G. S. system and their secondary derivatives. The centigrade degree has been adopted exclusively as the unit of temperature, and the water-gram-degree as the unit of heat. Illustrations have been freely introduced wherever they appeared to be desirable in order to increase the clearness of the descriptions or demonstrations. For the most part, these illustrations are diagrammatic. Two reasons for this may be given. In the first place, demonstrations, in general, require only outline diagrams. And, in the second, the actual construction of apparatus not only varies widely with different makers, but changes materially from time to time; the most approved forms in many cases soon becoming obsolete. Hence those descriptions of apparatus which are given in the text have been illustrated generally by figures showing only the principle of operation; those forms of apparatus alone being represented in detail which are typical or standard, or which mark epochs in the progress of the science. The details of construction in any actual case can easily be obtained by the student from the catalogues of the leading constructors. Most of the illustrations herein given are entirely new and were made especially for this work. A few have been borrowed from existing sources. The author would here acknowledge his indebtedness to Ducretet & Co., to J. P. Morton & Co., to Queen & Co., to P. Blakiston, Son & Co., and to Ginn & Co., for courtesies in furnishing electrotypes from their publications. His thanks are due also to Dr. Rowland for spectrum photographs, to Mr. Schott for suggestions concerning the magnetic map, and to his associate Dr. Goodspeed for assistance in reading the proofs.

To his fellow-teachers of Physics the author offers this book as the result of an earnest endeavor on his part to aid them in their work, not only by making the facts and principles of physical science more clear to the comprehension of the student, but also by assisting the instructor to present these facts and principles so as to secure a still higher grade of attainment in our more advanced institutions of learning.

PHILADELPHIA, October 1, 1892.

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PART FIRST.

INTRODUCTORY.

PHYSICS.

CHAPTER I.

THE PROVINCE OF PHYSICS.

SECTION I.—MATTER AND ENERGY.

1. External Nature.—It is through the medium of our senses alone that we derive our knowledge of the phenomena of external nature. A careful study of these phenomena reveals to us only two things, or entities, as having an actual and objective existence. These two things are **Matter** and **Energy**. So far as we can determine, therefore, these two entities, in their various forms, make up the whole of the physical universe.

2. Matter, Body, Substance.—Of the two things just mentioned, matter and energy, matter alone has dimensions. Provisionally, therefore, **matter** may be defined as that which occupies space. A limited portion of matter is called a **body**, and a definite kind of matter is spoken of as a **substance.**

EXAMPLES.—The materials which we call glass, iron, water, air, are forms of matter, since they all have dimensions and all occupy space. Each of these materials is a special kind of matter and possesses properties which distinguish it from all the others. Hence each of these kinds of matter is a distinct **substance**. Similarly, a rain-drop, a pebble, or a planet, each being a limited portion of matter, is spoken of as a **body**.

3. Mass, Volume.—Bodies, even when made of the same substance, may contain very different quantities of matter. Hence the word **mass** is used to indicate the quantity of matter contained in a body. Moreover, since

bodies may have different dimensions, the space which a body occupies is called its **volume.**

EXAMPLES.—A cannon-ball and a grape-shot, though made of the same substance, iron, have very different masses and very different volumes; the cannon-ball being the more massive in proportion as it is larger. But the mass of a volume of air equal to that of the cannon-ball would evidently be very much less.

4. Conservation of Matter.—Whatever the changes which take place in matter, its mass remains constant. So far as science can determine, not a single particle of matter has ever been brought into existence by any human agency; and no operation performed by man has ever diminished by a single atom the quantity of matter in the universe. This great fact, that matter is absolutely uncreatable and indestructible by any agency within our control, is known as the law of the **Conservation of Matter.**

EXAMPLES.—Carbon, which in its pure crystallized form we know as diamond, appears as petroleum when combined with hydrogen, as carbon dioxide gas when combined with oxygen, and as sugar when combined with both hydrogen and oxygen. But a given mass of carbon may be converted successively into these widely differing substances and yet remain absolutely invariable in amount.

5. Definition of Energy.—Energy is apparently inseparable from matter. No form of energy is known to us that is not in some way connected with matter, and there is no point in the universe, so far as we can ascertain, where matter would cease to possess energy. Energy may be provisionally defined, therefore, as a condition of matter in virtue of which any definite portion of it may be made to effect changes in other definite portions.

EXAMPLES.—The water in a flowing river possesses energy, since by its action upon a water-wheel machinery may be put in motion. It is the energy contained in gunpowder which drives the cannon-ball. It is the energy contained in the food we eat that enables us to move and act and think.

6. Conservation of Energy.—The present century has witnessed the complete establishment of the proposition that the total energy of the universe is constant.

into whatsoever forms this energy may be converted, its total amount remains absolutely unchanged. Precisely as no matter has ever been created or destroyed, so no energy has ever come into existence or has disappeared. This fundamental and far-reaching principle is known as the law of the **Conservation of Energy**.

EXAMPLES.—The energy of coal may be transformed into heat-energy, this into mechanical energy, and this again into electrical energy. But neither by these transformations, nor by any others known to science, can the minutest fraction of energy be put out of existence. Its form may change, but its total amount remains invariable.

7. Province of Physical Science.—It is the province of physical science to investigate the various transformations which matter and energy may undergo, to determine the conditions upon which these transformations depend, and to ascertain the laws according to which they are effected. Chemistry is concerned mainly with the alteration in properties which the different kinds of **matter** undergo when they act upon one another. Physics, on the other hand, has to do, chiefly, with the laws according to which the transformations of **energy** are brought about.

EXAMPLES.—When wood burns, or iron rusts, or nitro-glycerin explodes, the material changes which take place in these substances are properly the objects of chemical study. While the accompanying energy-changes, such as the production of heat, of an electric current, or of mechanical displacement, are studied by physics.

No change in matter can be effected without producing simultaneously some form of energy-change. In consequence every chemical change necessarily involves physical changes. But the converse of this is not true. Energy-changes may take place, energy may be transferred from one body to another, without the production of any corresponding matter-change.

EXAMPLES.—When hydrogen unites with oxygen, the matter-product is **water**, the energy-product is heat. When sunlight falls on the leaves of a plant, its energy is stored up in the matter-products which are formed by its agency. Indeed, the energy-changes which accompany chemical actions are always to be regarded as of quite as much importance as the matter-changes.

On the other hand, the change of energy which takes place when a stone falls to the earth, when a tuning-fork vibrates, or when an incandescence lamp emits light, is not accompanied by any corresponding matter-change, all these phenomena being due simply to energy-transference.

8. Definition of Physics.—Evidently, therefore, physics regards matter solely as the vehicle of energy. And hence from this point of view, physics may be defined as that department of science whose province it is to investigate all those phenomena of nature which depend, either upon the transference of energy from one portion of matter to another, or upon its transformation into any of the forms which it is capable of assuming. In a word, physics may be regarded as the science of energy, precisely as chemistry may be regarded as the science of matter.

CHAPTER II.

PHYSICAL QUANTITIES.

SECTION I.—PHYSICAL UNITS.

9. Physical Magnitudes.—Every physical quantity possesses a definite magnitude, the value of which may be more or less accurately determined by measurement, and which may be more or less exactly expressed numerically. This value is always stated in terms of a quantity of the same kind called **a unit.** The process of measurement consists in comparing the quantity to be measured with the unit and thus ascertaining the numerical relation between them.

EXAMPLES.—Thus, length, surface, volume, mass, time, force, and work, considered as physical quantities, are capable of measurement, each in units of its own kind. The value of a length is always expressed in units of length, that of a mass in units of mass, that of work in units of work, etc.

10. Definition of a Physical Unit.—Inasmuch as only magnitudes of the same kind can be compared with one another, the unit of measure must be a definite magnitude of the same kind as the quantity to be measured, assumed, more or less arbitrarily, for the purpose. Hence there must be as many kinds of units as there are kinds of magnitudes to be measured. A unit which has become legalized, either by statute or by common usage, is called **a standard** unit.

EXAMPLES.—The yard and the meter are standard units of length. They are both definite quantities of length which have been assumed as units for the purposes of measurement. In a similar way, the gallon has been assumed as a unit of capacity, the second as a unit of time, the pound as a unit of mass, etc.

11. Fundamental and Derived Units.—The earlier units employed in measurement were selected arbitrarily, and were in general independent of one another. An examination of the relations existing between physical magnitudes, however, has shown that by suitably selecting certain elementary units, all the other units needed in measurement may be readily derived from these, in virtue of these relations. In consequence, the former units have been called **fundamental units**, and the latter units, based upon these, **derived units**.

EXAMPLES.—Thus the fathom as a unit of length, the bushel as a unit of capacity, and the stone as a unit of mass, are purely arbitrary units having no direct relation to one another. When, however, a square yard is taken as the unit of surface, or a cubic meter as the unit of volume, there is a direct relation between these units and the corresponding units of length; since geometry teaches that surface is extension in two, and volume is extension in three, perpendicular directions. Again, a speed is defined in physics as a length divided by a time; and hence the unit of speed is expressed in terms of the unit of length and the unit of time. In these cases, the units of length and of time are fundamental units, and the units of surface, of volume, and of speed are derived units.

12. Expression of a Physical Magnitude.—Every expression representing the value of a physical magnitude consists of two parts, one of which is a numerical part, and the other a unit part. The former, which is called the **numeric**, represents the ratio of the magnitude, taken as a whole, to the particular unit selected for its measurement. Evidently the numerical part is the greater, the larger the magnitude represented, and the smaller the unit employed. Quite frequently only the numerical part is expressed, the unit part being understood

EXAMPLES.—Thus a length, regarded as a physical magnitude, may be spoken of as ten meters or as thirty-three feet; a ma[illegible] eleven pounds or as five kilograms. If l represent a definite le[illegible] and L a unit of length, the ratio l/L is the numerical value [illegible] length and denotes this value in terms of the unit employed. [illegible] the mass of a body is represented by m, this can only mean th[illegible] body contains m units of mass; and the complete express[illegible] $m[M]$, in which m is the numerical part or the numeric, and [illegible] the unit part. When a physical quantity is spoken of as havi[illegible]

17. Dimensions of Derived Units.—Every physical magnitude may be expressed in terms of the units of mass, length, and time; e.g., in C. G. S. units. The particular value of such a quantity in terms of the fundamental quantities of the system is called the **dimensions** of that quantity; and the value of the unit of this quantity, in terms of the fundamental units of the system, is called the **dimensions** of that unit.

EXAMPLES.—Since a speed is the ratio of a length to a time, the dimensions of speed are a length divided by a time; and the dimensions of a unit of speed are a unit of length divided by a unit of time; i.e., a centimeter per second, in the C. G. S. system.

18. Dimensional Equations.—Equations showing the relations existing between derived units and the corresponding fundamental units of a system are called **dimensional equations**. Although suggested originally by Fourier, these equations were first brought into use by Maxwell. They have two important functions in physics: 1st, to facilitate the conversion of physical quantities expressed in terms of the units of one absolute system into those of another; and 2d, to furnish a check upon equations of definition; since by reducing both members to fundamental units, the equation should become an identity.

EXAMPLES.—Thus, from the equation of quantities, $s[S] = l[L] \div t[T]$, we may obtain, by making the numerics s, l, and t unity, the dimensional equation $[S] = [L] \div [T]$; which asserts that the dimensions of unit speed are unit length divided by unit time.

If $[L]$, $[M]$, and $[T]$ be C. G. S. units, and $[L']$, $[M']$, and $[T']$ be F. P. S. units, the conversion from one system into the other may be easily effected. A rod whose mass is m grams may be represented as $m[M]$, or if its mass is m' pounds, by $m'[M']$. But since it is the same rod, $m[M] = m'[M']$; whence $m = m'\dfrac{[M']}{[M]}$, or the mass of the rod in grams is obtained by multiplying its mass expressed in pounds by the ratio of a pound to a gram; i.e., by 453·59, the number of grams in a pound. In the same way, a given speed may be represented as $s[S]$, or as $s'[S']$, the units varying inversely with the numerics. But $s[S] = \dfrac{l}{t} \cdot \dfrac{[L]}{[T]}$, and $s'[S'] = \dfrac{l'}{t'} \cdot \dfrac{[L']}{[T']}$;

consequently, since $s[S] = s[S']$, we must have $\frac{l}{t}\cdot\frac{[L]}{[T]} = \frac{l'}{t'}\cdot\frac{[L']}{[T']}$. But $s = l/t$ and $s' = l'/t'$; and hence $s\frac{[L]}{[T]} = s'\frac{[L']}{[T']}$; or transposing, $s = s'\frac{[L']}{[L]}\cdot\frac{[T]}{[T']}$. Since the unit of time is the second in both systems, the last factor is unity. Whence the value of the speed in centimeters per second is obtained by multiplying the speed in feet per second by the ratio of the foot to the centimeter, or 30·4797.

As an example of the second use of dimensional equations, we may apply them to test the assertion that force is time-rate of energy-change. The definition-equation is $f[F] = e[E] \div t[T]$, and the dimensional equation $[F] = [E] \div [T]$. Now the dimensions of energy are $[ML^2/T^2]$, and the dimensions of force are $[ML/T^2]$. Substituting, we have $[ML/T^2] = [ML^2/T^2] \div [T]$. Since this is not an identical equation, the fundamental assumption on which it is based is erroneous.

Every physical equation should be read in terms of the units representing the quantities involved in it, these units being either generic or specific. The equation $s = l/t$, for example, means that a body has s units of speed when it passes over l units of length in t units of time, in any absolute system; or when it describes l centimeters per second in the C. G. S. system.

SECTION II.—PHYSICAL MEASUREMENTS.

19. Measurement in General.—"All exact knowledge," says Maxwell, "is founded on the comparison of one quantity with another." Such a comparison of the quantity to be measured with a unit quantity of the same kind constitutes a physical measurement; the object being to determine the numeric, or the number of times that the unit is contained in this quantity.

20. Direct and Indirect Measurement.—Both direct and indirect methods are employed for the measurement of physical magnitudes. The **direct** method consists in applying a concrete unit of measure directly to the quantity to be measured, and expressing the result in terms of this unit. In the **indirect** method, other quantities than the one under investigation are measured, and

the value of the quantity sought is calculated from the known relations existing between these magnitudes.

EXAMPLES.—The length of a rod is ascertained directly by applying to it a rule or scale divided into centimeters; and if this length be exactly six of these divisions, it is said to be six centimeters long. But the surface of a plane rectangular figure is measured by ascertaining the lengths of its sides and multiplying these together; since the unit of surface is the unit of length squared. Surface, therefore, is measured indirectly.

Since in any absolute system a magnitude is always expressed in terms of its fundamental units, it is evident that in such a system every physical measurement involves a comparison with these units only; although in many cases intermediary or secondary units are employed whose values are known in terms of the fundamental units.

EXAMPLES.—The dimensions of work are force per unit of length; of force, are mass per unit acceleration; of acceleration, are speed per unit time, and of speed, length per unit time. Ultimately, therefore, the dimensions of work are a mass multiplied by the square of a length and divided by the square of a time. In the C. G. S. system, work in ergs may be measured either in the secondary units of force, of acceleration, or of speed, or in the fundamental units of length, mass, and time; i.e., in centimeters, grams and seconds.

But even this method of measurement may be simplified. According to Clifford "every quantity is measured by finding a **length** proportional to the quantity and then measuring this length." Since by definition the unit of mass, e.g., the gram, is the mass of unit volume, e.g., of one cubic centimeter, of water, the mass of any quantity of water is known when its volume has been ascertained. And the mass of any body whatever may be obtained in grams by noting the amount by which a given spring is stretched by it in terms of the amount by which it is stretched by one gram. Again, the fundamental unit of time may be measured as a length. If a line be traced by a body moving uniformly, equal spaces on this line represent equal intervals of time. If the line be traced on the surface of a cylinder rotating uniformly, the pencil at intervals moving parallel to the axis for a short

distance, the time-value of these intervals is determined with the greatest accuracy by measuring the length of the spaces representing them, provided the rate of rotation of the cylinder is known; i.e., the value of unit time, one second, upon the cylinder. Such an instrument is called a chronograph.

21. Measurement of Length.—Because every quantity is measured by finding a length proportional to it and then measuring this length, it is of the greatest importance to attain a high degree of accuracy in linear measurement. This is accomplished, in the first place, by decimally subdividing the linear unit, so that when the object does not cover an entire number of units, the fractional part may be read on the subdivisions. But a limit is soon reached to these subdivisions, even when a lens is used to magnify them. So that, in the second place, some additional device must be made use of to effect a farther subdivision. Two such devices are employed in practice, known as the **vernier** and the **micrometer-screw**, respectively. The vernier consists of an auxiliary rule, sliding along the main scale, ten divisions on which may correspond, for example, to nine on the scale; so that the length of one division on the vernier is one tenth of a scale-division less than that of one division on the scale. If, therefore, the quantity to be measured exceeds six divisions of the scale by eight divisions on the vernier, its length is 6·8 scale-divisions. The micrometer-screw consists of a screw of known pitch, having a graduated disk attached to its head; so that when turned through one division on the head, any point upon the screw advances through that fraction of the pitch corresponding to the ratio of this one division to the whole circumference. If, therefore, the head is divided into one hundred equal parts, and if to pass over a given length twelve turns and forty-five divisions of the head are required, the length measured is 12·45 times the value of the pitch of the screw.

EXAMPLES.—The scales used in actual measurement of length are in general multiples or sub-multiples of the meter. They are

divided into centimeters and millimeters, and these latter into fifths and sometimes tenths. For use with the microscope, scales are used divided into hundredths and even thousandths of a millimeter; i.e., into **microns**. Verniers are direct-reading when they read in the same direction as the scale, and inverse-reading when they read in the opposite direction. In the latter case the number of vernier divisions is one more than the corresponding number of scale divisions. If a length of 9 millimeters on the scale be divided on one of the verniers into ten parts and a length of 11 mm. be divided on the other into ten parts, the value of one division on the first vernier is 0·9 millimeter and on the second 1·1 millimeters; the excess in both cases, or "least count" of the vernier being 0·1 millimeter. If a length measures 6 scale-divisions and 8 vernier-divisions, its value is $6 + 0{\cdot}8$ millimeters with the former vernier and $7 - {\cdot}2$ or 6·8 millimeters on the latter, the second being read backward. If a micrometer-screw has a step or pitch of 0·1 millimeter and its head is divided into one hundred equal parts, the value of one division on the head is $0{\cdot}1 \times 0{\cdot}01$ or 0·001 millimeter.

[redacted]

PART SECOND.

MASS-PHYSICS.

CHAPTER I.

KINEMATICS.

SECTION I.—MOTION IN GENERAL.

22. Position and Motion.—The simplest change which a material particle can undergo is a change in its position. The **position** of such a particle is determined by its distance and direction from a point of reference called the origin. Any change in this position, therefore, must alter either the distance of the particle from the fixed point or its direction from the point, or both. Change of position is called **motion**. If the distance of a particle from the origin, and its direction with reference to it, continue unaltered for a given time, the particle is said to be at rest, relative to the origin, during that time. When these values are changing with the time, the particle is said to be in motion. No motion of matter is instantaneous. For a particle to pass from one position to another, it must occupy a definite time in the transit, and it must pass through, on its way, all the intermediate points between its initial and final positions.

If O (Fig. 1) represent the point of reference, or origin, and A a material particle, the line OA drawn from the origin to the particle will represent by its length and direction the position of the particle A with reference to the origin O. If B represent a second particle, the line OB will represent the position of this particle B with reference to the same point O. For, by starting from this point and passing along the line OA or the line OB, we should reach A or B.

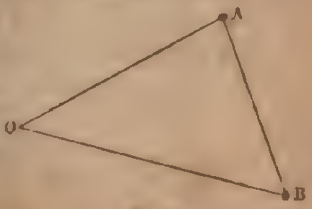


FIG. 1.

In the same way the line AB would represent the position of B with respect to A, and the line BA, the position of A with respect to B. If

a particle at the point A were to change its position so as to bring it to the point B, its direction with reference to O, as well as its distance from this point, will both have changed, and the particle is said to have moved from A to B, the motion taking place along the line AB. This motion must have required a definite time, and the particle must have occupied successively every intermediate point between A and B.

23. Definition of Kinematics. — That branch of science which studies motion in the abstract without reference to the character of the body moved, or to the antecedent or consequent conditions of its motion, is called **Kinematics**, from κίνημα, motion (Ampère.) The study of pure motion, then, uncomplicated by collateral phenomena, constitutes an appropriate introduction to the consideration of mass-physics.

24. Classification of Motions.—By material particle is meant a body so small that the distances between its parts may be neglected in any given case without sensible error. The motion of such a particle causes a change either in its distance from the origin or in its direction from it, or in both. If the change is one of distance alone, the particle describes a straight line; if it is one of direction alone, it describes a curved line. This motion along a line, which is the only motion a particle can have, is called a **motion of translation**. A body of sensible magnitude is said to have a motion of translation when all the particles contained in it move in parallel and equal lines; so that the motion of the body is completely defined by the motion of any one of its particles. Again, the particles in a body may move in circles whose centers are on a common line or axis. Such a motion is called a **motion of rotation**. While in a motion of translation the direction of every straight line drawn within the body remains fixed and the line moves always parallel to itself, in a motion of rotation the direction of such a straight line in the body is continually changing. Both translatory and rotatory motion may be continuous or intermittent, uniform or variable. In both, the direction of the motion may be constant or alternate, the body in the latter case moving to and fro

to equal distances on each side of its normal position when at rest. Such a motion is called a **motion of oscillation.**

EXAMPLES.—The motion of a sail-boat on water or ice when rectilinear may be taken as an example of motion of translation, and the motion of the fly-wheel of a stationary steam-engine as an example of motion of rotation. In general, however, these two motions coexist; as in the case of a rifled projectile, of a screw entering wood, of the driving-wheel of a locomotive, or of a planet in its orbit. The motion of a pendulum, and of the balance-wheel of a watch, are motions of oscillation.

Hitherto the body moving has been assumed to be **rigid**, so that no change in its size or shape takes place during the motion. Bodies which are not rigid are said to be **elastic**; and the relative motion of the parts of an elastic body produces a change either in its size or in its shape, or in both. Such changes in the size or shape of an elastic body are called **strains.**

25. Province of Kinematics.—It is the object of Kinematics to teach us "how to describe motion accurately and how to compound different motions together" (Clifford). For purposes of study, Kinematics is conveniently subdivided into three sections based upon the classification above given. These sections are as follows:

1. Motion of particles. Translatory motion.
2. Motion of rigid bodies. Rotatory motion.
3. Motion of elastic bodies. Strains.

SECTION II.—MOTION OF TRANSLATION.

26. Rectilinear and Curvilinear Motion.—In simple translatory motion the position of a particle with respect to the origin may vary (1) in magnitude alone, as when the motion is along the line connecting the particle with the origin, the motion being rectilinear; (2) in direction alone, as when the motion is constantly perpendicular to this line, the motion being curvilinear; or (3) in both magnitude and direction, as when the motion takes place

along an oblique line, the motion being either rectilinear or curvilinear.

27. Velocity, Speed.—Whenever a particle moves continuously its motion is measured not only by the distance through which it moves, but also by the time occupied in traversing this distance; the motion being greater for a given time the greater the distance, and for a given distance the smaller the time. The rapidity with which a particle moves, i.e., its rate of motion, or rate of change of position, is called its **velocity**. Velocity, therefore, may vary in both magnitude and direction. When regarded as so varying it is evidently a directed quantity, or a vector. If the magnitude only of the change in position be considered, the rate of motion is usually expressed by the term **speed**.

28. Uniform and Variable Velocity.—The velocity of a particle may be uniform or variable. It is **uniform** when the change of position in the second, third, or any subsequent unit of time is the same as that described in the first unit. In uniform motion, equal changes of position take place in equal times, and unequal changes of position take place in times which are proportional to these changes. If the times are not proportional to the changes of position, then the velocity of the particle is said to be **variable**.

EXAMPLES.—The speed (or magnitude-velocity) of a particle moving over 10 centimeters in each successive second, or over 20 centimeters in 2 seconds, 50 centimeters in 5 seconds, or 100 centimeters in 10 seconds, is said to be uniform. But if it move over 5 centimeters in 1 second, or 12 centimeters in 2 seconds, and over 21 centimeters in 3 seconds, its speed is said to be variable. The direction-velocity of a particle moving in a circle so as [illegible] the an arc of ten degrees each successive second is uniform [illegible] the locus of the particle's motion be the spiral of Archimedes [illegible] ity will be uniform both in magnitude and direction.

29. Dimensions of Speed.—Evidently w[illegible] the speed is uniform the entire space described by [illegible] ing particle is obtained by multiplying the [illegible] passed over in one unit of time by the number [illegible] ts of time during which the motion continues. [illegible] if

the entire distance be l units of length, the space described in unit of time be s units of length, and the time occupied be t units of time, the characteristic equation of uniform motion will be

$$l = st. \qquad [1]$$

Since by transposition $s = l/t$, the speed of the particle, or the rate of its motion, is measured in terms of the space over which it passes in one unit of time. If in this equation l and t both be made unity, s will also become unity; so that the unit of speed is at once defined as that rate of motion in which a unit of length is described in a unit of time. Hence in the C. G. S. system the unit speed is a centimeter per second. It is a derived unit and its dimensions are $[L/T]$ or $[LT^{-1}]$. It has received no special name.

EXAMPLES.—If a particle, moving uniformly, passes over 100 centimeters in 20 seconds, or over 180 centimeters in 36 seconds, it obviously moves over five centimeters in each second. Its rate of change of position along the line of its motion, therefore, i.e., its speed, would be five centimeters per second.

30. Acceleration.—If the linear space described by a moving particle in each successive unit of time be greater or less than that described in the first unit, its speed is variable, and the rate of motion of the particle is said to be **accelerated**; positively if the variation be an increase of speed, negatively if it be a decrease. The acceleration itself is said to be uniform when in each successive unit of time the increase or decrease of the speed has the same value. The measure of acceleration is simply the ratio of the total change of speed to the time in which it is effected. So that acceleration may be defined as **rate of change of speed**, precisely as speed is defined as **rate of change of position**. Since the total change of speed is obtained by multiplying the change per unit of time by the number of units of time required, the fundamental equation of acceleration will be

$$s = at, \qquad [2]$$

in which s represents the total increase or decrease of speed during the time t, and a the increase or decrease of speed per unit of time. Evidently since $a = s/t$, the acceleration of a moving particle is unity when a unit of speed is gained or lost in each unit of time. If acceleration be defined as the time-rate at which speed is lost or gained, the unit of acceleration will be that acceleration in which a unit of speed is lost or gained per unit of time. In the C. G. S. system the unit of speed is a centimeter per second; and hence the unit of acceleration is an acceleration of a centimeter per second per second. The dimensions of the unit acceleration are $[LT^{-2}]$.

EXAMPLES.—A particle starting from rest and moving over five centimeters the first second, ten centimeters the second second, and fifteen centimeters the third second, obviously gains five centimeters each second; i.e., it has a positive acceleration of five C. G. S. units. If its motion continue for 25 seconds, it is evident that the total positive acceleration will be 5×25 or 125 units, and the particle will have a final speed of 125 centimeters per second. So a particle moving with a speed of 12 units the first second, 10 the second, 8 the third, and so on, has a negative acceleration of 2 units; so that if this condition of things continue for 6 seconds, the particle will come to rest.

31. Space described in Accelerated Motion.—The space described by a moving particle in a given time is always the product of the speed by the time. In uniform motion, however, the speed is constant; while in accelerated motion it is variable. In the latter case, therefore, it is the **mean** or **average speed** of the particle by which the time must be multiplied in order to obtain the distance traversed. This mean or average speed is defined as the speed which a uniformly moving particle must have in order to describe the same total space in the same time, and therefore is the quotient of the entire space by the entire time taken to describe it. It will represent the actual speed at any given instant the more exactly in proportion as the interval of time is smaller. If the acceleration be uniform, the mean or average speed is one half the sum of the initial and final

speeds; or if the particle start from rest, one half of the final speed. If the speed of a particle be s_0 at the beginning of the time considered, and s_1 at the end of this time, the total change of speed during the interval is clearly $s_1 - s_0$; and the rate of change of speed or the acceleration is $(s_1 - s_0)/t$. This is positive or negative, obviously, according as the final speed s_1 or the initial speed s_0 is the greater. The mean speed during the interval is $\frac{1}{2}(s_1 - s_0)$. In case the particle start from rest, s_0 is zero; whence the acceleration is s_1/t and the mean or average speed is $\frac{1}{2}s_1$. The space traversed, being the product of the mean speed by the time, is $\frac{1}{2}s_1 t$. Hence the second fundamental equation of uniformly accelerated motion may be written:

$$l = \tfrac{1}{2}s_1 t. \qquad [3]$$

It expresses the space described in terms of the mean speed and the time. Evidently this space is the same as that described by a particle moving uniformly for the same time t with the speed $\frac{1}{2}s_1$.

32. Graphic Method of Illustration.—This subject may be illustrated graphically. Since a line is measured in units of length, it is evident that any physical quantity whatever may be represented by a line, upon the convention that a unit length of the line shall be taken to represent a unit of the quantity. Thus a line five centimeters long may be taken to represent five units of time, five units of speed, or five units of acceleration, etc. Moreover, since the product of two lines is geometrically an area, it is clear that if we lay off on the axis of abscissas a length equal to the time, and on the axis of ordinates a length equal to the speed in uniform motion, the space described, being the product of the speed by the time, will be represented by the area of the rectangle having these co-ordinates as sides; as is illustrated in Figure 2. So in uniformly accelerated motion, if the initial speed be zero and the final speed s_1, the space described in the time t will be represented by the

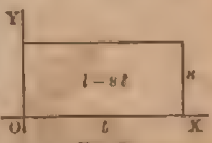


Fig. 2.

area of a right-angled triangle as in Figure 3; i.e., by the product $\frac{1}{2}s_1t$. Hence the linear space described by a particle moving uniformly for the time t with the speed s is twice that described by a particle which, starting from rest, acquires this speed at the end of the time t. Moreover,

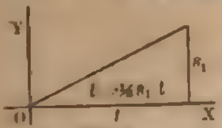


FIG. 3.

the area of this triangle is the same as that of a rectangle of half the height, as is shown by Figure 4. Hence the space described by a given particle moving for t units of time with an acceleration of a units is the same as that described by a second particle moving uniformly for the same time with a speed one half as great as that finally attained by the first.

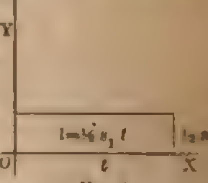


FIG. 4.

33. Other Equations of Uniformly Accelerated Motion.—We have now obtained two equations applicable to uniform acceleration, one of which, $s = at$, gives the total speed acquired in terms of the acceleration and the time, and the other, $l = \frac{1}{2}st$, the space described in terms of the final speed and the time. If we substitute in the second equation the value of s from the first, we obtain the new equation $l = \frac{1}{2}at^2$, which gives the space described in terms of the acceleration and the time. And further, if t be eliminated between the last two equations, still another equation is obtained, $s^2 = 2al$, which gives the speed acquired in terms of the acceleration and of the space described during the motion. The four equations of uniform acceleration are, therefore,

$$s = at; \qquad [2]$$

$$l = \frac{1}{2}st; \qquad [3]$$

$$l = \frac{1}{2}at^2; \qquad [4]$$

$$s^2 = 2al. \qquad [5]$$

If instead of starting from rest, as supposed, the particle has an initial speed s_0, its final speed is simply the

sum of this initial speed and the acquired speed. And the above equations become

$$s_1 = s_0 + at; \tag{6}$$

$$l = \tfrac{1}{2}(s_1 + s_0)t; \tag{7}$$

$$l = s_0 t + \tfrac{1}{2}at^2; \tag{8}$$

$$s_1^2 = s_0^2 + 2al. \tag{9}$$

Equation [7] may be graphically represented as follows (Fig. 5). Here it will be seen that the total area which represents the space described is the sum of the areas representing the space described uniformly and the space described with accelerated motion. Hence the equation may be written

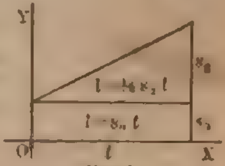


FIG. 5.

$$l = s_0 t + \tfrac{1}{2}s_2 t,$$

in which $s_2 = s_1 - s_0$.

In these cases the acceleration has been supposed positive. If it be negative, it will diminish the initial speed and finally reduce it to zero, thus bringing the particle to rest. Then if it continue, an increasing speed will be generated in the opposite direction. The same equations are applicable in this case, the sign of the acceleration only being changed.

34. Curvilinear Motion.—Hitherto the velocity has been supposed to vary only in magnitude. But it may vary also in direction. In case it varies only in direction the distance of the particle from the origin remains unaltered but its direction continually changes; so that the particle describes a circular path about the origin as a center. In general, however, the velocity of a moving particle changes continually both in magnitude and direction, its path being a curvilinear one. The rate of change of direction of a curvilinear path per unit length

of the path is called its **curvature**; and this curvature may be constant, as in the circle, or variable, as in the parabola. A straight line has zero curvature.

If the path described by a moving particle be a circle, and tangents be drawn to the circle at two of its points, the angle α between these tangents measures the change of direction of the path. The ratio of this angle to the length of the arc a between the points, i.e., α/a, measures the space-rate of change of direction or the curvature. But since in the circle the angle between the tangents is the angle between the radii drawn to the same points, the arc a expressed in circular measure is the product of this angle into the radius, or $r\alpha$. Whence, substituting this value for the arc a itself in the above ratio, we have $\alpha/r\alpha$ or $1/r$ as the measure of the curvature. Hence the curvature of a circle is numerically equal to the reciprocal of its radius. In the case of any other curve, a circular arc may be found which, for any particular point of it, coincides with the curve. The curvature at this point is the reciprocal of the radius of this osculating circle.

35. Direction - acceleration. — Hodograph.—If, at successive instants, lines be drawn from a given point as origin so as to represent in magnitude and direction the actual velocity of a moving particle at these instants, the extremities of these lines will lie on a curve which is called the **hodograph** of the motion of the particle. If two points be taken on this hodograph, the portion of the curve lying between them will represent the change of velocity in the interval, or the mean acceleration. The actual acceleration at any point is the limiting value of the hodographic velocity, as the distance between the points is continually diminished. Moreover, the direction of the tangent to the hodograph at any point represents the direction of the acceleration of the moving particle at the corresponding point of its path; and the speed in the hodograph at this point represents the amount of the acceleration of the moving particle. (Thomson and Tait.)

EXAMPLES.—If the velocity of a moving particle be uniform, the magnitude and direction of its motion are both constant and the hodograph is evidently a point. If the magnitude of the velocity be uniformly accelerated while its direction remains constant, the hodograph of the motion is a straight line along which the speed is uniform. If, on the other hand, the magnitude of the velocity be constant while its direction changes uniformly, as when the particle describes a circle with uniform motion, the hodograph is also a circle along which the speed is uniform. Hence the acceleration of such a moving particle is constant in magnitude; and since the tangent to the hodograph at any point is parallel to the corresponding radius of the circle in which the particle is actually moving, the direction of this constant acceleration is towards the center of the circle.

To determine the magnitude of this constant acceleration, we may proceed thus (Fig. 6): Suppose the particle to move in the

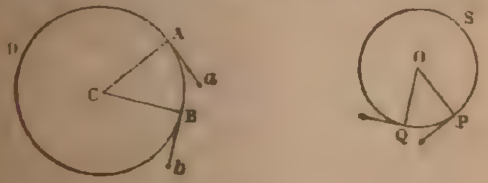


FIG. 6.

circle ABD, of radius r with the uniform speed s. The magnitude and direction of the velocity at A are represented by the tangent Aa, and at B by the equal tangent Bb. To construct the hodograph, assume any fixed point O as origin, and draw the lines OP, OQ equal in magnitude to these tangents and parallel to them in direction. These two lines will represent the velocity of the moving particle at these two points A and B in both magnitude and direction, and the curve connecting the outer extremities of these lines is the hodograph. Since the speed is uniform, all the lines drawn from O will be equal in length; and hence the hodograph will be a circle. Moreover, the tangent at P is parallel to AC, and the tangent at Q is parallel to CB. Hence, since the tangent to a hodographic curve represents the direction of the acceleration, the acceleration of a particle at the point A is along AC, and at B along BC; the acceleration being always directed toward the center of the circle in which the particle moves. Further, the point P describes the circle PQS in the same time that the particle itself moves over ABD; and hence their speeds are proportional to the radii of these circles, or as AC to OP. Since AC is r and OP is s, we have, the speed in the hodograph : the speed of the particle : : the radius

of the hodograph : the radius of the path described by the particle; or

$$V : s :: s : r.$$

From which we have $V = s^2/r$. But the speed of the point P in the hodograph represents the acceleration of the particle :—

Consequently, when a particle is moving uniformly in a circle it experiences a constant acceleration toward the center of this circle, the magnitude of which is directly proportional to the product of the square of its speed by the curvature of the circle. It will be noticed that although the motion of the particle is constantly accelerated, its speed remains uniform.

SECTION III.—COMPOSITION AND RESOLUTION OF MOTIONS, VELOCITIES, AND ACCELERATIONS.

36. Composition of Uniform Motions.—Since motions are definite quantities, they may be added, subtracted, or otherwise combined with each other. Experiment shows that the result of the concurrent action of two or more motions is a single motion; and that if these original motions are constant in speed and direction, the result is a motion in a straight line, also constant in speed and direction. This single motion is called the **Resultant** motion, and the motions producing it are called **Component** motions. If the component motions take place along the same straight line, each will produce its own effect, and the resulting motion will be either the sum or the difference of the two, according as they are similarly or oppositely directed. In other words, the resultant motion will be the algebraic sum of the component motions.

EXAMPLES.—If a person walk upon the deck of a moving ship, his resultant motion will depend upon the direction in which he himself moves. If he walk toward the bow, the component motions are both positive or both negative, according to the convention adopted, and the resultant motion is their arithmetical sum. If toward the stern, one component is negative and the other positive and the resultant is the arithmetical difference of the two. If in

the latter case the two motions are equal in magnitude, the resultant is zero, and the position of the person relative to an outside point remains unchanged. In every case the resultant of two component motions along the same straight line is their algebraic sum.

If, however, the two component motions are not along the same straight line, their directions will form an angle with each other; and then their resultant may be found by the aid of a geometrical construction. In the first place, let us assume that these motions originate at the same point. Then if from this point two lines be drawn in the component directions whose lengths represent the magnitudes of these components respectively, and if upon these lines, as sides, a parallelogram be constructed, it is found that the diagonal of the parallelogram will represent the resultant motion also in direction and magnitude. Thus by construction and measurement the magnitude and direction of such a resultant may be ascertained.

To determine the resultant magnitude and direction by calculation, we may suppose as a first case that the two directions form a right angle with each other. The parallelogram constructed as above will then be a rectangle (Fig. 7). Since its opposite sides are equal, $BC = OA$; and a particle starting from O may reach C, either by moving along OB and then BC, or by moving first along OA and then AC. But since the same result would also be attained by moving along OC, this diagonal represents a single motion which is the equivalent of the other two. By geometry, OBC is a right-angled triangle, and the square of the hypothenuse is equal to the sum of the squares of the sides; i.e.,

Fig. 7.

$$OC^2 = OB^2 + BC^2.$$

In magnitude, then, the resultant motion is equal to the square root of the sum of the squares of the component motions. In direction, trigonometry teaches us that the

ratio of BC to OB is the tangent of the angle . . This gives the angle made by the resultant with . . the components. That made with the other is its complement and hence is obtained simply by subtracting the former angle from 90°.

As a second case suppose that the two directions are not rectangular; then the diagonal of the parallelogram representing the resultant may be calculated by means of the known trigonometrical relation between the sides of an oblique triangle. Calling the three sides P, Q, R, respectively, (Fig. 8,)

$$R^2 = P^2 + Q^2 - 2PQ \cos \alpha.$$

But in this case the angle α is the obtuse angle which the direction of P makes with the direction of Q; i.e., the angle OBC. Whereas since the motions originate at O, the acute angle AOB or β is the angle between their directions.

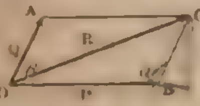


Fig. 8.

But this angle is the supplement of OBC; hence while its cosine has the same value it has an opposite sign; and the formula becomes

$$R^2 = P^2 + Q^2 + 2PQ \cos \beta. \qquad [10]$$

From which the value of R can be readily calculated. Moreover, in the triangle OBC, $Q : R :: \sin COB : \sin \alpha$; whence $\sin COB = Q \sin \alpha / R$. This gives the angle made by the resultant R with the force P.

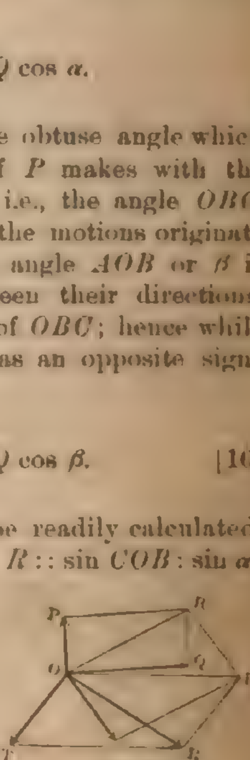


Fig. 9.

The resultant of several simultaneous motions may be obtained by finding first the resultant of two of the motions, say P and Q (Fig. 9), in the way above described. Then, by combining this resultant R' with the third motion S, we may obtain a second resultant R''. This resultant, com-

bined with the fourth component T, gives a third resultant R, and so on. The last resultant obtained is the resultant of all the motions.

37. Triangle and Polygon of Motions.—In the cases above given the components and the resultant are represented by the three sides of a triangle, which may be called the triangle of motions. Either side may represent the resultant, the other two representing the components. Hence if from a given point a line be drawn representing in magnitude and direction one of the motions, and if from the end of this line a second one be drawn representing similarly the second component motion; then a third line drawn from the fixed point to the end of the second line will represent the resultant motion also in magnitude and direction. In the same way, the resultant of several component motions may be found by the polygon of motions. From a (Fig. 10)

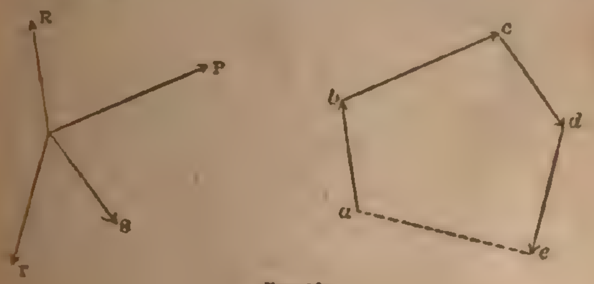


FIG. 10.

draw ab, representing the first component motion R in magnitude and direction. From b draw bc, representing the second component P; and so on. Connect the point e, where the last component line terminates, with the initial point a. The line ae will represent in magnitude and direction the resultant of all the motions R, P, S, and T. It will be observed that the direction of the resultant motion ae which closes the figure is opposite to that of the component motions round the polygon. Hence the resultant of motions which are repre-

sented by all the sides of a polygon but one, taken in order, is represented by that one taken in the opposite direction.

38. Composition of Velocities and Accelerations.—Velocities and accelerations, being directed quantities like motions, may be similarly compounded. The principles which have been discussed with reference to motion are equally true of velocity and acceleration. We may have, therefore, the parallelogram of velocities and of accelerations, and also the triangle and the polygon of velocities and of accelerations; all constructed in the same way as in the case of motions.

39. Resolution of Uniform Motions.—Not only may several given component motions be compounded into a single resultant, but a given single motion may be resolved into two or more components. If a line of definite length and direction be taken to represent a given motion, it may be made the diagonal of a parallelogram, the sides of which will constitute the components. But inasmuch as an indefinite number of parallelograms may be constructed upon a given line as a diagonal, the problem is indeterminate unless the direction and magnitude of one of the components is also given. In the most frequent cases, the resolution takes place so that one component is parallel to, and the other is perpendicular to, a given direction. The component in the given direction is called the **resolute** in that direction.

40. Resolution along Co-ordinate Axes.—Another method which is frequently employed to obtain the resultant of two or more component motions, velocities, or accelerations consists in resolving each of the components along rectangular coordinate axes, according to the methods of analytical geometry. In case the motions are all in one plane they

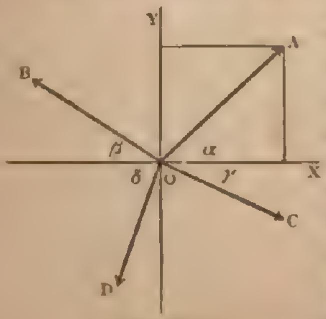


FIG. 11.

may be resolved along the axes OX and OY (Fig. 11). Calling α the angle which the motion A, represented by the line OA, makes with the axis of X, the two components thus resolved are $A \cos \alpha$ along X, and $A \sin \alpha$ along Y, respectively. The components of B are $B \sin \beta$ along Y, and $-B \cos \beta$ along X'; of C, $-C \sin \gamma$ along Y', and $C \cos \gamma$ along X; and of D, $-D \sin \delta$ along Y', and $-D \cos \delta$ along X'. Adding together the components along each of these axes, regard being had to their signs, we have for the algebraic sum of the components along X

$$A \cos \alpha - B \cos \beta + C \cos \gamma - D \cos \delta;$$

and for the sum of those along Y

$$A \sin \alpha + B \sin \beta - C \sin \gamma - D \sin \delta.$$

Calling X and Y these total values, we have now but two components, and these at right angles to each other, from which to find a resultant. Hence $X^2 + Y^2 = R^2$, which gives the magnitude, and $Y/X = \tan \theta$, which gives the direction, of this resultant; i.e., the angle θ which it makes with the axis of abscissas.

If the motions to be compounded do not lie all in one plane, then each of them may be resolved along three mutually perpendicular axes, X, Y, and Z, and the values of the sum of the components along each of these three axes may be obtained in the same way; the component of each motion along each axis being the product of the line representing it into the cosine of the angle which the line makes with that axis. Then the final resultant $R = \sqrt{X^2 + Y^2 + Z^2}$; which gives the magnitude of this resultant. The expressions X/R, Y/R, and Z/R represent respectively the cosines of the angles formed by this resultant with the axes of X, Y, and Z.

It is obvious, therefore, that the principle of the parallelogram of motions, of velocities, and of accelera-

tions may be extended so as to include lines representing these quantities drawn in any direction whatever. And hence, since the diagonal in this case would be that of a solid, that we may speak of the parallelopipedon of motions, of velocities, and of accelerations. Thus in Fig. 12, R is the resultant of P, S, and Q, and $R^2 = P^2 + S^2 + Q^2$. Moreover, the principle of the polygon of motions applies equally well when the motions, etc., are not restricted to one plane. If such a skew-polygon be constructed, in a way analogous to the plane polygon above mentioned, the missing side will always represent the resultant. If the polygon be closed, there is no resultant; i.e., either side is a resultant to all the others.

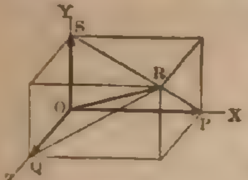


FIG. 12.

41. Composition of Uniform with Variable Motion.—Thus far it has been assumed that the motions, etc., to be compounded are uniform. But plainly uniform motion may also be compounded with variable motion. The most important case is where a uniform motion is compounded with a uniformly accelerated motion. The path of the particle in this case is curvilinear, being in fact a parabola, the path of a projectile.

Thus, if a particle be made to move uniformly in a horizontal direction, and at the same time vertically with a motion uniformly accelerated, each motion will produce its separate effect. If the particle move horizontally with a speed s, the space thus described in t seconds will be st units of length. The vertical space described in t seconds, the acceleration being a, will of course be $\frac{1}{2}at^2$ units of length. Calling the first space l and the second l', and eliminating t between the two equations, we have $l' = (a/2s^2)l^2$. If l be made the abscissa and l' the ordinate of a curve, then, since a and s are constant and the ordinate varies as the square of the abscissa, the locus of this equation is a parabola. This result may be represented graphically as follows (Fig. 13): From the origin O lay off on a horizontal line a number of equidistant points, 1, 2, 3, 4, 5, etc., to represent the equal distances passed over in equal times in the uniform motion. On a vertical line drawn from O lay off distances 1,

1, 9, 16, etc., on the same or on a different scale, proportional to the squares of the times and, therefore, characteristic of accelerated motion. At the end of the first unit of time the particle will be at the intersection 1, 1; at the end of the second, 4, 2; of the third, 9, 3; of the fourth, 16, 4; of the fifth, 25, 5; and so on. Drawing a smooth curve through these points, it is readily seen to be a parabola.

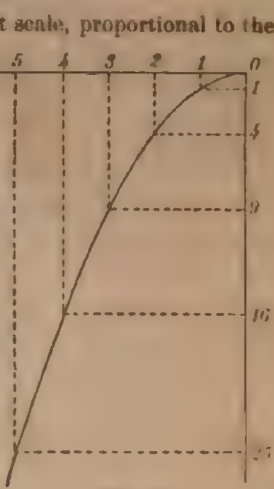


FIG. 13.

The same result follows whatever the direction of the one motion with reference to the other. A particle moving uniformly from A to B (Fig. 14) and having simultaneously a uniformly accelerated motion from A to D will be found at C; the path described from A to C being a parabolic one as before. In the above equation $l^2/l' = 2s^2/a$. In the parabola, $CD^2/AD = 4FA$. But $CD = l$ and $AD = l'$; and hence $4FA = 2s^2/a$. If, therefore, a parabola be drawn through A whose axis is vertical and whose vertex is upward, and in which $2s^2/a$ is equal to four times the distance from the focus F to the point A, the point moving from A to C will describe this parabola.

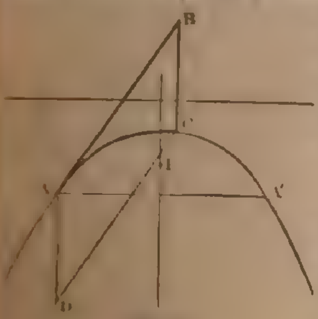


FIG. 14.

It should be borne in mind that a curved path is in general a resultant path; and that the components are a uniform motion in some given direction, and an accelerated motion making some given angle with this direction.

42. Degrees of Freedom.—When a particle is free to move in any direction in space, it is said to have three degrees of freedom; since, as above shown, every possible direction of motion may be resolved along three mutually perpendicular co-ordinate axes. If the conditions of its motion be restricted, however, it may have less freedom. Thus if the particle be required to move in a plane surface, it can evidently have no motion perpendicular to that surface, and its degrees of freedom are limited to two. If, further, the condition is that the

particle shall move in two plane surfaces simultaneously, it can fulfil the condition only by moving along the intersecting line of those surfaces; in which case it has only one degree of freedom. Or lastly, if the condition is that it shall be in three surfaces simultaneously, it can obviously do this only by being at the point in which they intersect. In this case its position is fixed and it has no degree of freedom whatever.

Section IV.—Motion of Rotation.

43. Angular Velocity.—A material particle can have a motion of translation only. A material body, however, made up of particles rigidly connected, may move (1) so that a line joining any two of its particles remains always parallel to itself; in which case its motion is one of translation. Or it may move (2) so that such a line drawn within it changes its direction continually; in which case the motion is said to be one of rotation. Motion of pure rotation takes place about some fixed direction, either within or without the body itself, which is called the **axis of rotation.** All the particles in the body describe circles about this axis, those which are nearest to it moving through the smallest circumferences. Inasmuch as all these circles are described in the same time, it is evident that the linear distance traversed by a given particle is the greater, the greater its distance from the axis. Every such particle, therefore, has a distinct speed of its own, depending upon this distance. We may speak of the rotational velocity of the entire body, however, meaning by this the number of complete rotations which it makes in a second. In general, the velocity of rotation is measured in terms of the linear velocity of those points which are at unit distance from the axis; this velocity being called **angular velocity.** This angular velocity in motion of rotation is the correlate of linear velocity in motion of translation. It is usually denoted by ω; so that if the linear velocity of a particle

at unit distance from the axis is ω, that of a particle at the distance r is $r\omega$. Angular velocity, then, represents the angle through which the rotating body turns per second, and is of course the same for every portion of the body. Since, in circular measure, an angle is measured by the ratio of the arc to the radius, it is evident that the linear velocity, which corresponds to the arc, is measured by the product of radius and angle as above stated.

Thus, calling the circumference of a circle $2\pi r$, where r is the radius and represents the distance of the moving particle from the axis, and supposing the body to make one rotation in T seconds, it is obvious that the linear velocity of the particle, or the distance described per second, is $2\pi r/T$; and further, that if the particle be at unit distance from the axis, $r = 1$ and the linear velocity will be $2\pi/T$; which, since it is the linear velocity of a particle at unit distance from the axis, is the angular velocity, or ω.

If, in the equation $\omega =$ arc/radius, the value of the arc and of the radius be both made unity, the angle will also be unity. In other words, the unit of angular velocity is described by a particle, which in a circle of unit radius moves over a path of unit length in unit time. A rotating body has unit angular velocity when it traverses unit angle in unit time. This unit angle is an angle whose arc is equal in length to the radius; it is called a **radian**, and its value in angular measure is obtained by dividing the semi-circumference 180° by π, or 3·1416. The quotient is 57°·29578, or 57° 17′ 44″·8, nearly; which is therefore the value of unit angle in degrees.

Motion of rotation is said to be **negative** when its direction is the same as that of the hands of a watch; and to be **positive** when in the opposite direction.

Angular velocity, like linear velocity, may be uniform or variable. In the latter case the rate of change of angular velocity is called **angular acceleration**.

Thus as in translatory motion $a = s/t$, so in rotatory motion $\alpha = \omega/t$; or in other words, angular acceleration is the quotient of

change of angular velocity divided by time; i.e., it represents the rate of change of the angular velocity. Since $\omega = s/r$, $\alpha = v/rt = a/r$. Hence linear acceleration is represented by the product of the angular acceleration multiplied by radius.

44. Composition and Resolution of Rotations.—Rotations, angular velocities, and angular accelerations may all be compounded and resolved in precisely the same way that translatory motions, velocities, and accelerations may be. If two or three simultaneous rotations about axes passing through a fixed point be impressed upon a body, the resultant motion is a rotation about a single axis whose direction and magnitude may be found by a construction similar to the parallelogram or parallelopipedon of motions; the sides being drawn parallel to the directions of the axes, and being in length equal in magnitude to the angular velocities. The diagonal will then represent in direction the resultant axis of rotation, and in magnitude its angular velocity. So a motion of rotation may be compounded with one of translation, as in the case of a screw entering its nut.

45. Degrees of Freedom of Rotation.—A rigid body free to rotate in any direction has three degrees of freedom, since it may rotate about three axes perpendicular to one another. Combining these with the three degrees of freedom of translation, we see that in all a rigid body has six degrees of freedom. If a point within it be fixed, however, the body has no motion of translation; but it may rotate about any axis passing through the point, and then retains three degrees of freedom. If a line (i.e., two points) be fixed within it, there can be no translation, nor any rotation except about this line. Hence the body has now only one degree of freedom. If a surface (i.e., three points) be so fixed, the body can have neither rotatory nor translatory motion; it has no degree of freedom. Again, if a point in the body be limited so as to move only along a given line, it may rotate about any axis, and so has four degrees of freedom. If a line in the body is restricted to coincide

with a line in space, the only rotation possible is about this line, and the body has only two degrees of freedom. The restriction of a point to a given surface gives two degrees of freedom of translation and three of rotation; that of a line to a given surface gives four degrees of freedom, two of rotation and two of translation. When three points in a body are restricted to a surface, there is only one rotation possible, with two translations along the surface; and hence there are three degrees of freedom.

46. Moments of Motion.—The moment of a directed quantity is defined as the magnitude of the quantity itself multiplied by a length taken perpendicular to its direction. Since motions, velocities, and accelerations are all directed quantities, we may speak of the moment of a motion, of a velocity, or of an acceleration. In the case of motion of rotation, all lines drawn from the moving particle perpendicular to its direction of motion at successive instants are evidently radii of the circle which it describes. So that if s be the uniform speed with which it moves along the circumference, and r be the radius of the circle, the product rs will represent the moment of the speed considered with reference to the point about which the rotation takes place. Moreover, moments may be compounded and resolved; and the moment of the resultant motion about any given point in the plane of the components is in magnitude the algebraic sum of the moments of the components, these moments being taken with their proper signs.

Section V.—Strains.

47. Definition of Strain.—The bodies hitherto considered have been supposed rigid. Hence no change has taken place in the relative positions of their parts, and the bodies themselves have undergone no change in volume or figure. Bodies which are not rigid are called **elastic**; and elastic bodies may suffer changes in their

size or shape. Any definite alteration in the form or dimensions of an elastic body is called a **strain** (24).

EXAMPLES.—"Thus a rod which becomes longer or shorter is strained. Water when compressed is strained. A stone, beam, or mass of metal in a building or in a piece of framework, if condensed or dilated in any direction, or bent, twisted, or distorted in any way, is said to experience a strain. A ship is said to 'strain' if, in launching or when working in a heavy sea, the different parts of it experience relative motions." (Thomson and Tait.)

48. Homogeneous Strain.—The simplest strain is a linear one, as when an elastic cord is stretched. This strain is called homogeneous when every portion of the cord has its length changed in the same ratio, so that the ratio of the initial to the final length of each part of it is the same as this ratio for the whole. The ratio of the final to the initial length is called **the ratio of the strain**; it represents evidently the quantity by which the initial length must be multiplied to obtain the final length. The **elongation** is the ratio of the change in length to the initial length. A negative elongation is called a **compression.**

Thus if a length ab be changed to a length $a'b'$ either by extension or contraction, the ratio of the strain thus produced is $a'b'/ab$, and the elongation is $(a'b' - ab)/ab$. If c be a point between a and b, the distance ac becomes $a'c'$ after the strain; and the strain is homogeneous when $ac : a'c' :: ab : a'b'$. If the elongation be e, the initial length l, and the final length l', then the change of length is $l' - l$, and the ratio of this change of length to the initial length is $(l' - l)/l$; and this is the elongation or e. Whence $l' - l = el$ and $l' = l(1 + e)$; so that $1 + e$ represents the ratio of the strain.

When all lines in a body parallel to a certain direction are changed in the same ratio, and no lines perpendicular to these are changed either in length or direction, the body suffers a strain of simple elongation. If, however, a second set of lines at right angles to these also suffer such a change, then there is elongation in two perpendicular directions; and if these lines are all

in the same plane, the strain is a **surface strain**. A square elastic sheet, if the elongation be e in a direction parallel to one edge, and e' parallel to another, will be converted by the strain into a rectangular sheet, the sides of which are proportional to the strain-ratios. Evidently two equal and parallel lines drawn on the square will remain equal and parallel after the change in form; and the strain will be **homogeneous**. If the elastic sheet be circular, the strain will change the circle into an ellipse; the two perpendicular directions which remain perpendicular after the strain becoming the axes of the ellipse. If these lines remain parallel to their original directions, the elongations take place along them and the strain is called a **pure strain**. If not, the strain is compounded of a pure strain and a rotation.

49. Shear.—The principal axes of a strain are the principal axes of the ellipse into which the strain converts a circle. If the increase of length along one such principal axis is equal to the decrease of length along the other principal axis, the strain under these conditions is called a **shear**. Evidently in a shear the area of the plane itself, or of any figure in it, remains unaltered. Thus if (Fig. 15) the length Oa be increased by aA, and Ob equally decreased, the rhomb $aba'b'$ will become the rhomb $ABA'B'$, of the same area, the strain being pure. The same result may be attained in another way. Suppose that by means of a motion of translation and one of rotation, and not therefore involving strain, the line ab is made to coincide with AB. Then it is evident that the figure $aba'b'$ may be converted into the figure $ABA'B'$, simply by sliding $a'b'$ in its own direction until it coincides with $A'B'$ (Fig. 16). So in general any plane figure may be converted into a strained figure, i.e., the shearing strain may be produced, simply by fixing one of its sides, and moving all lines parallel to this

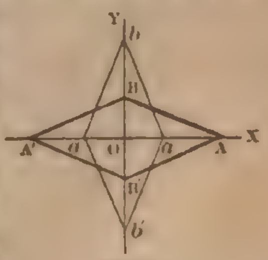


FIG. 15.

fixed side in their own directions, through spaces which are proportional to their distances from this fixed line.

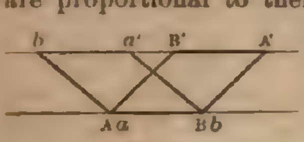


FIG. 16.

The amount of this sliding motion which takes place between lines which are unit distance apart is called the **amount of the shear**, and the ratio $OA : Oa$, **the ratio of the shear.** If one principal axis is elongated in the ratio $1 : \alpha$ and the other contracted in the ratio $\alpha : 1$, the ratio of the shear is α; and the amount of the shear is equal to the excess of the ratio of the shear above its reciprocal; i.e., to $\alpha - \alpha^{-1}$.

50. Strain-ellipsoid.—When a solid body undergoes a strain, a change may take place in its dimensions in one or more of three perpendicular directions. If the strain is such that all parallel lines within it are altered in length in the same ratio, the strain is called a uniform or **homogeneous strain.** Thus for example, a sphere when subjected to strain is converted into an ellipsoid; a solid every plane section of which is an ellipse. This ellipsoid is called the **strain-ellipsoid.** In any homogeneous strain of a solid body, there are three directions at right angles to one another, which remain perpendicular after the strain. These directions are those of the three principal axes of the strain-ellipsoid. Along one of these directions the elongation is greater, and along another less, than along any other directions in the body. Along the remaining one, the elongation is intermediate. The principal axes of a strain are the principal axes of the ellipsoid into which it converts a sphere. The principal elongations of a strain are the elongations in the directions of its principal axes. "Any strain whatever may be viewed as compounded of a uniform dilatation in all directions, superimposed on a simple elongation in the direction of one principal axis, superimposed on a simple shear in the plane of the other two principal axes." (Thomson and Tait.)

Section VI. — Motion of Oscillation.

51. Definition.—The motions of translation and of rotation which have now been considered were assumed to be continuous in direction. There are other translatory and rotatory motions which are of great importance in physical science, in which the motion is alternate in direction. These motions are called **oscillatory** or **vibratory** motions. They may be observed in the motions of a swinging pendulum or of a sounding tuning-fork.

52. Projection of Circular Motion.—If a particle be made to revolve uniformly in a circle, and the eye be placed in the plane of this circle, the particle will appear to oscillate along a diameter of the circle. Since the motion repeats itself at regular intervals of time, it is said to be **periodic.** Periodic motion of oscillation, then, is simply the projection of uniform circular motion upon a diameter. The law according to which it takes place may be easily deduced from the following considerations.

Let ABC (Fig. 17) be the circle in which the particle is moving—and which is called the circle of reference or the auxiliary circle—and let AB be the diameter on which the uniform circular motion is projected. Let the direction of motion of the particle be indicated by the arrow. When the particle is at B, it is moving in the line of sight and there is no resolute along the diameter; hence it appears to be at rest. When it reaches a, it will appear projected on the diameter at t. As it passes over the equal arcs ab, bc, cd, de, and eC, it will appear to describe the continually increasing intervals ts, sr, rq, qp, and pO; its apparent speed constantly increasing and reaching a maximum at O. Here its apparent motion is the same as its actual motion in the auxiliary circle. After passing the point O, its apparent

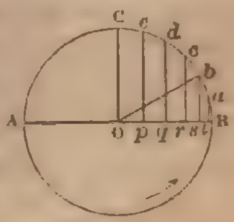


Fig. 17.

motion decreases again and becomes zero when it reaches A.

Draw now the radius Ob of length r. Then, since the triangle Osb is right-angled at s, the line Os, which represents the displacement of the oscillating particle from its mean position, has the value $r \cos \alpha$; in which α is the angle bos. Moreover, since the motion in the circle is uniform, equal arcs are described in equal times. So that if ω be the arc described in unit time, and t be the time of describing the arc Bb, $\alpha = \omega t$. Calling d the displacement Os, a the amplitude OB (which is equal to Ob or r), and substituting for α its value, the equation representing the displacement becomes

$$d = a \cos \omega t. \qquad [11]$$

Evidently when ωt or α is zero or is 2π, $\cos \alpha = 1$ and $d = a$; the displacement then reaching its positive maximum and the particle being at B. When $\alpha = \pi/2$ or $3\pi/2$, $\cos \alpha = 0$ and $d = 0$; there being no displacement, the particle is at the center O. When $\alpha = \pi$, $\cos \alpha = -1$, $d = -a$, and the particle is at A, its negative maximum. Since, as the angle is increased still further, the values of d repeat themselves indefinitely in the same order, d is said to be a periodic function of α.

Again, suppose the particle to be at d (Fig. 18) in the auxiliary circle. Draw the tangent dm to represent its speed s, and resolve it into the component dn parallel to AB, and into nm perpendicular to it. The resolute dn represents the component of the speed parallel to the diameter. Calling this s', we have $s' = s \cos mdn$. But the angle mdn is equal to Ods, since their sides are perpendicular; and $\cos Ods = \sin dOs$ or α. Whence replacing, we have $s' = s \sin \alpha$, and

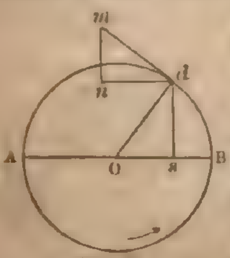


FIG. 18.

the speed at any point on the diameter is proportional

to the sine of the corresponding arc of the auxiliary circle. Hence in Fig. 17, the speeds at the points O, p, q, r, s, t are represented by the lines OC, pe, qd, rc, sb, and ta; these being the sines of the corresponding angles to unit radius. Since the maximum speed is OC, which is equal to OB or a, and since this is the speed in the auxiliary circle, $s = a$, and the speed-equation becomes

$$s' = a \sin \omega t. \tag{12}$$

The function in this equation also is a periodic one. As ωt or α becomes successively 0, $\pi/2$, π, and $3\pi/2$, the value of s' becomes 0, a, 0, and $-a$; corresponding to the positions B, O, A, O, respectively; the values repeating themselves for every 2π of rotation.

It will be observed that the displacement-equation and the speed-equation are complementary; the one representing the same condition of things as the other, but one quarter of a rotation later. Hence when the displacement is a maximum the speed is zero; and when the speed is a maximum the displacement is zero. If two particles oscillate across perpendicular diameters of a circle so that when one reaches the end of its path the other is at its middle point, the displacement-equation of the one particle corresponds to the speed-equation of the other.

53. Simple Harmonic Motion.—The motion of oscillation now described, since it is a motion characteristic of vibrating tuning-forks, stretched strings, and other bodies emitting musical sounds, is called simple harmonic motion. The extent of the excursion of the vibrating body on either side of its middle point, or point of rest, is called its **amplitude**. It corresponds to the radius of the auxiliary circle. The interval of time between two successive passages of the vibrating body through a given point of its path in the same direction is called its **period**. The fraction of the period which

has elapsed since the body last passed through the extreme point of its path in the positive direction is called the **phase**. When the body is at the extremity of its path on the positive side, it is said to be in the position of **maximum positive elongation**; and when at the opposite end, in the position of **maximum negative elongation.** Time may be reckoned from the point of maximum positive elongation or from some other point. When reckoned from some other point, the interval of time, estimated from this point of reckoning until the vibrating body next comes to its position of maximum positive elongation, is called the **epoch.**

Since angular velocity is the angular space described in unit time, $\omega = 2\pi/T$; and the angle BOd or α, which as above is equal to ωt, is also equal to $2\pi t/T$; the time in both cases being reckoned from B, the point of maximum positive elongation. If, however, the time be reckoned from X or Y (Fig. 19), calling XOd or YOd, α, and BOX or BOY, ϵ, we have $BOd = \alpha + \epsilon$ in the former case, and $\alpha - \epsilon$ in the latter. The angle ϵ or its equivalent in time, ϵ/ω or $\epsilon T/2\pi$, is called the **epoch**, and BOd the **phase**.

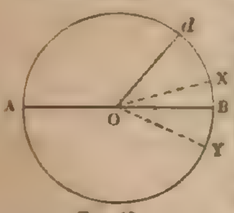


FIG. 19.

54. Acceleration Proportional to Displacement.— In order that a body may vibrate with simple harmonic motion it is necessary that the acceleration toward the middle point of its path should be directly proportional to the displacement from this mean position. In Fig. 17, for example, when the vibrating body is at s its displacement from the mean position is Os; hence its acceleration should be sO. It has already been proved (35) that when a body is moving uniformly in a circle of radius r with a speed s, it has an acceleration toward [illegible]ter of that circle equal in magnitude to s^2/r. The [illegible] of this acceleration along the diameter, the body [illegible]posed at b, will be $- \cos \alpha(s^2/r)$, the accelerat[illegible]ng from right to left. But $\cos \alpha = Os/Ob = O$[illegible] and therefore the acceleration $a = - Os(s^2/r^2)$. Sin[illegible] and

r are constant, this acceleration is proportional to $-Os$ or to sO; or in other words, the acceleration is proportional to the displacement.

55. Isochronism.—Since angular velocity is s/r (43), the value s^2/r^2 in the above equation is evidently the square of the angular velocity in the auxiliary circle, or ω^2; and the equation becomes $a = -d\omega^2$; since $Os = d$, the displacement. Whence $\omega = (-a/d)^{\frac{1}{2}}$; or the angular velocity in the auxiliary circle is equal to the ratio of the square root of the acceleration to the square root of the displacement. Evidently if this ratio be constant, the angular velocity will be constant. And since $\omega = 2\pi/T$, if ω is constant, T will be so also. Consequently, inasmuch as T represents the time of one complete oscillation,—i.e., the period,—the period will be constant. Clearly, if the acceleration and displacement vary together, their ratio will be constant and independent of their absolute values. Hence the period of simple harmonic motion is independent of the amplitude of vibration. In other words, if the acceleration of a vibrating particle toward its mean position bears a fixed proportion to its displacement from that position, the particle will execute a simple harmonic motion whose period is independent of the amplitude of the vibration. This property of oscillating in equal times whatever the amplitude of the swing is called **isochronism**, and a particle thus oscillating is said to vibrate **isochronously**.

56. Composition of Simple Harmonic Motions.—Harmonic Curve.—Motions of vibration may be compounded and resolved, like motions of translation and rotation. When a simple harmonic motion along a given line is compounded with a uniform motion of translation in a direction perpendicular to this line, the resultant is a locus called the **harmonic curve**. Such a curve is seen in Figure 20. It may be traced as follows:

On a horizontal line AB lay off a series of equal distances to represent the uniform translatory motion. Draw the circle $ACDE$ to represent the circle of reference of the simple harmonic motion, this motion itself

taking place along the diameter CE. On this circle lay off equal arcs Ea, ab, bc, etc., to represent the uniform motion in the auxiliary circle. The projections of these arcs upon the diameter EC will represent their simple harmonic resolutes; i.e., the spaces passed over in equal times along the diameter. Let the vibrating particle start from C to move downward, and from 0 to move to the right; and represent its position by 0′, the intersection of $C0'$ and $00'$. At the end of the first time-interval it would be vertically at t and horizontally at 1; and therefore will be actually at 1′, the intersection of $t1'$ and 1, 1′. At the end of the second interval its vertical motion will have brought it to s, and its horizontal motion to 2;

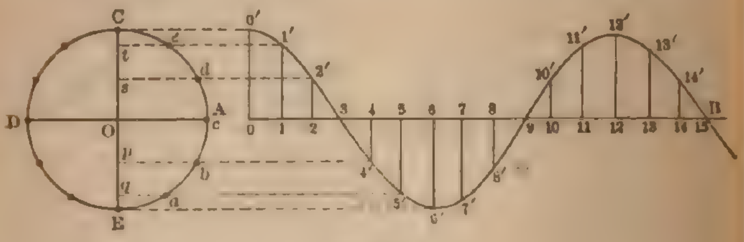


FIG. 20.

so that it will be found at 2′. So at the end of the third interval it will be at 3; having reached the middle point of its swing. Here the acceleration changes sign, and the space described begins to diminish; and at the end of the fourth, fifth, and sixth intervals the particle will be at 4′, 5′, and 6′, respectively. Having now reached the end of its swing, it stops and retraces its course, passing through the points named in the inverse order and occupying the positions 7′, 8′, 9, 10′, 11′, and 12′. It has now completed one vibration, and has returned to its initial position, the time occupied being the same as that required to describe the auxiliary circle uniformly. In twelve time-intervals the particle has moved over twelve equal horizontal spaces, and simultaneously over twelve simple harmonic spaces. If now a regular curve

be drawn through the resultant points of these two motions, this curve will be the **harmonic curve.**

57. Equation of the Harmonic Curve.—An inspection of this curve shows that it is a periodic or repeating curve. In order to obtain its equation, it is necessary only to combine the equation of uniform rectilinear motion $l = st$ with the equation of simple harmonic motion $d = a \cos \omega t$; which gives for the equation of the harmonic curve

$$d = a \cos \omega l/s. \qquad [13]$$

This equation shows that d, the ordinate to the curve, which represents the displacement, varies directly as the amplitude of the vibration, and as the cosine of an angle which itself varies directly as the abscissa and inversely as the speed of the rectilinear motion, and which represents the phase. Since $\omega = 2\pi/T$, the equation becomes $d = a \,.\, \cos 2\pi l/sT$. But sT is evidently the rectilinear distance travelled during a complete period; and this is called the wave-length. Representing it by λ, the complete equation of the harmonic curve is

$$d = a \,.\, \cos 2\pi l/\lambda. \qquad [14]$$

Evidently when l is zero, $2\pi l/\lambda$ is also zero; and since $\cos 0 = 1$, $d = a$, or the displacement is a maximum and is equal to the amplitude. When $l = \frac{1}{4}\lambda$, $\cos 2\pi l/\lambda = 0$ and $d = 0$; the moving particle being at the center of its swing. When $l = \frac{1}{2}\lambda$, $\cos 2\pi l/\lambda = -1$, and $d = -a$; or the particle is at the point of maximum negative elongation. If l be increased another quarter wave-length, or to $\frac{3}{4}\lambda$, the cosine and the displacement again become zero, and the particle is again at the center, but moving in the opposite direction. So that finally when $l = \lambda$, $\cos 2\pi l/\lambda = \cos 2\pi = 1$, and $d = a$, as at first; the particle having completed an entire vibration. It appears, therefore, that whenever l is increased by λ, $2\pi l/\lambda$ is increased by 2π; and the value of d is unaltered.

If, in like manner, the speed-equation of simple harmonic motion be combined with the equation of uniform rectilinear motion, the resulting equation will be

$$s' = a \,.\, \sin 2\pi l/\lambda. \tag{15}$$

The resulting harmonic curve is shown in Figure 21. If the particle oscillate along the diameter AD, its speed at A, q, p, o, etc., will be proportional to the ordinates drawn from these points to the circle of reference; i.e., to the simple harmonic resolutes perpendicular to the line of oscillation. Laying off as before equal spaces on the line AB to represent the uniform translatory motion, and erecting ordinates at each of these points to represent

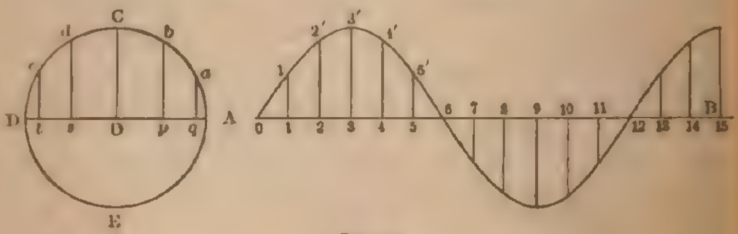


FIG. 21.

these resolutes, the line joining the ends of the ordinates will be the harmonic curve. If both motions originate at A, the speeds at this point are both zero, and both the abscissa and ordinate are zero; hence the curve passes through the origin. At the end of the first time-interval, the particle is at q and its speed is aq. Drawing an ordinate at 1 equal to aq, the end 1′ of this ordinate is also a point on the curve; and so are the points 2′ and 3′, the ends of ordinates corresponding to bp and CO. Here the speed is a maximum, and it decreases according to the same law. Evidently during the time required for the particle to oscillate from A to D and back again, its rectilinear motion has carried it from 0 to 12; and hence the distance 0 to 12, since it represents the rectilinear space traversed during one complete vibration, is the wave-length.

An inspection of the figure shows that the speed-curve differs from the displacement-curve by one quarter of the period. So that if the one be slipped along the translation axis by one quarter of a wave-length, it will coincide with the other. The same result follows from a comparison of the equations to these curves. In the speed-equation, whenever l is equal to 0 or to an even number of quarter wave-lengths, the harmonic speed of the particle is zero; and whenever l is equal to an uneven number of quarter wave-lengths, the harmonic speed of the particle is a maximum and is numerically equal to the amplitude. From its form the harmonic curve is ordinarily known as the **curve of sines**, or the sinusoidal curve. The slope of the curve is obviously a function both of the amplitude and of the wave-length.

EXPERIMENT.—The vibrations of a pendulum are approximately simple harmonic when the amplitude is small. Construct a pendulum with a hollow bob, so that when it oscillates, the sand with which this bob is filled shall be delivered in a fine stream; and notice that the sand-trace is thickest at the ends of the swing, where the motion is slowest. Place a sheet of paper beneath the moving pendulum and draw it uniformly along in a direction perpendicular to that of the oscillation. The falling sand will trace out the harmonic curve approximately. If a little dry aniline color be mixed with the sand, and alcohol be sprayed upon it after it has been distributed upon the paper, the path described will be left as a permanent stain.

38. Composition of Simple Harmonic Motions.—I. Of the same Period.—The problem of compounding two simple harmonic motions with each other may be readily solved either by construction or by calculation. Two cases are to be considered: 1st, that in which the two motions take place along the same line; and 2d, that in which the motions are perpendicular to each other. In the former case the resultant will also be a simple harmonic motion, whose amplitude is the algebraic sum of the component amplitudes. If the motions compounded are equal and are in the same phase, the resultant will be a simple harmonic motion of double amplitude. If they are equal but in opposite phases, the resultant amplitude will be zero.

In the latter case, when the motions compounded are perpendicular to each other, there will also be two different results, according to the relative phases of the components. If these components are equal in amplitude and are either alike or opposite in phase, their resultant will be a simple harmonic motion along a line bisecting the angle between them. If both components are in the same phase, this resultant diagonal will lie in the first and third quadrants; if they are in opposite phases, the resultant will lie in the second and fourth quadrants and be perpendicular to the former. If, however, the difference of phase between the components be one quarter of the period, then evidently one of the particles will be at the middle of its swing and moving fastest, when the other is at the end of its swing and moving slowest; the consequence of which is that the resulting path will be a circle, the circle of reference in fact, described with a uniform motion. If the phase-difference be three quarters of a period, i.e., one quarter of a period in the opposite direction, the resultant will still be a circle, but the moving particle will describe it in the opposite sense.

ILLUSTRATION.—To illustrate this by construction, let AB and CD (Fig. 22) be the two rectangular simple harmonic motions to be compounded. Since the amplitudes are equal, the speeds at corresponding points will be equal. Suppose the motions to be both in the same phase, let them both be positive, and both start at O. At a subsequent instant, when by the horizontal motion alone the moving particle would be at 1, by the vertical motion alone it would be at c. Hence its actual position will be x at the end of the diagonal of the parallelogram whose sides are $O1$ and Oc. At the end of the second interval the particle will be at y; and so on, reaching P at the end of the fourth interval and continuing to oscillate along the line PP', whose direction is inclined 45° to the direction of the component

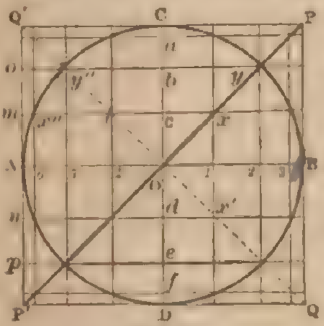


FIG. 22.

motions. If, however, the two motions be in opposite phases, then, although these motions may simultaneously originate in O as before, one of them will have a negative direction. Suppose it to be the vertical one. Then when by the horizontal motion the particle would be carried to 1, by the vertical motion it would be carried to d, and thus, under the influence of both these motions, the particle will oscillate along the diagonal QQ', perpendicular to PP'. Since the projection of a simple harmonic motion is also a simple harmonic motion, the particle in each of the above cases will describe simple harmonic motion along a diagonal.

Suppose now there be a difference of phase between the two motions of one quarter of a period, and let the origin of the motions be at A. A particle at this point will have its maximum speed vertically and will have zero speed horizontally, since it is at the extremity of its swing. During the first time-interval, if both motions be positive, the particle would move by the vertical motion alone through the distance Am, and by the horizontal motion alone through the distance $A6$. Both these motions together will bring it to x''. In the same way, at the end of the second interval it will be at y'': until at the end of the fourth interval it will be at C, having described the circular arc $Ax''y''C$. Hence the particle will move uniformly in the circle $ACBD$ in the same direction as that in which the hands of a watch move; i.e., in the negative direction. If, however, the horizontal motion be negative, and if the particle begin its motion at B, the difference of phase will now be three quarters of a period, and the path of the particle will again be the circle of reference; but now this path will be described in the positive direction.

Evidently, therefore, uniform motion in a circle may be considered as compounded of two simple harmonic motions taking place along two perpendicular diameters of this circle. Moreover, it will be observed that when the difference of phase between these two component motions is zero, the resultant is a straight line; that when this difference is one quarter of the period, the resultant is a circle; that when it is one half of the period, the two motions are in opposite phases and the resultant is again a straight line; that when it is three quarters of the period, the resultant is again a circle; and that when the difference is four quarters, the components are again in the same phase. Hence the circle and the straight line are the limiting values of the resultant. Any intermediate difference of phase between these

limits will give an intermediate resultant; i.e., an ellipse; as may easily be shown by constructing such a resultant with an eighth period difference of phase. Obviously if the amplitudes of the component motions be not equal, the resultant, if it be a straight line, will not be inclined at 45° to the components; and with a difference of phase of one quarter period the resultant will be an ellipse whose axes are parallel to the direction of the component motions.

59. Composition of Simple Harmonic Motions.—II. Of Different Periods.—If the simple harmonic vibrations are perpendicular and are not of the same period, the resultant is a special curve, the form of which is a function of the two component periods. The simplest period ratio next to that of 1 : 1 is that of 1 : 2. Two harmonic motions whose periods are as 1 : 2 compound into a resultant which, when the difference of phase is equal to ¼ the shorter period, is a parabola; and when the difference of phase is zero or is ½ the shorter period, is the figure-of-eight curve known as the lemniscate. The following figures (Figs. 23 and 24) show the changes in the resultant curve produced by changes of phase alone:

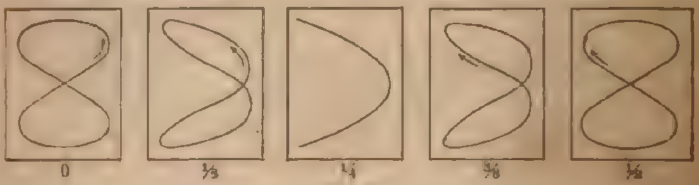


FIG. 23.

In case the periods are as 2 : 3, the curves are as follows:

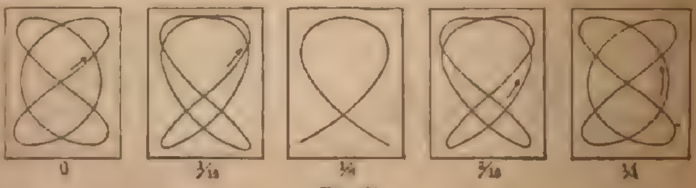


FIG. 24.

If the period of either of the two motions be only approximately exact, i.e., if the ratio between them be not represented by simple whole numbers, then one of the motions gains more or less rapidly on the other, the difference of phase continually changes, and the resultant curve passes in succession through all the forms which are characteristic of the period-ratio. The time required to pass through its cycle of changes will be the greater, the larger the numbers representing the ratio. When this is 1 : 2, the phase is constant. When it is 10 : 21, the phase slowly changes, completing the cycle only after 20 complete vibrations of the slower motion. When it is 1000 : 2001, it will take 2000 vibrations of the slower to complete the cycle.

60. Analytical Determination of the Resultant.—The magnitude and direction of the resultant in these cases may also be obtained analytically. The equation of simple harmonic motion is $y = r \sin \omega t$ for one of the components and $x = r \cos \omega t$ for the other. If the two motions originate at the middle point, the speed of each particle, supposing the period and amplitude to be the same for both, will be the same at this point; i.e., will be a maximum and equal to r, the radius of the auxiliary circle. Since the resultant of two equal rectilinear motions r at right angles is $\sqrt{2r^2}$ in magnitude, and since the angle made by the resultant with either of them is the angle whose tangent is the ratio of the other motion to that one, the resultant in this case is obviously the diagonal of the parallelogram constructed on these motions as sides, and equally inclined to the two; for $r/r = \tan \alpha = 1$ and $\alpha = 45°$. In general, since $R^2 = x^2 + y^2$, i.e., the resultant of two rectangular motions is the square root of the sum of their squares, we have, by squaring the above equations of harmonic motion,

$$x^2 = r^2 \cos^2 \alpha \qquad \text{and} \qquad y^2 = r^2 \sin^2 \alpha ;$$

whence

$$x^2 + y^2 = r^2(\cos^2 \alpha + \sin^2 \alpha) = r^2.$$

But this is the equation of a circle of radius r with the origin at the center. If the epoch of one harmonic motion be zero, while that of the other is ϵ, then the two equations above become

$$x = r \cos \alpha \qquad \text{and} \qquad y = r \cos (\alpha - \epsilon).$$

Combining these, we have

$$y = x \cos \epsilon + (r^2 - x^2)^{\frac{1}{2}} \sin \epsilon,$$

which readily reduces to

$$y^2 - 2xy \cos \epsilon + x^2 = r^2 \sin^2 \epsilon. \qquad [16]$$

If in this equation the epoch-angle ϵ be $0°$, we have $x = y$ or the equation of a straight line making an angle of 45° with the axis of abscissas along which the harmonic motion x takes place. If $\epsilon = 90°$, then the equation becomes $y^2 + x^2 = r^2$, the equation of a circle. For intermediate values, the expression becomes the equation of an ellipse, referred to the diagonals (Fig. 22) as axes.

If the two component motions have different amplitudes, their equations are

$$x = r \cos \alpha \qquad \text{and} \qquad y = r' \cos (\alpha - \epsilon).$$

Combining these two equations, and putting the epoch $\epsilon = \frac{1}{2}\pi$, the resultant equation reduces to

$$\frac{x^2}{r^2} + \frac{y^2}{r'^2} = 1. \qquad [17]$$

which is the equation of an ellipse referred to its center and axes. If the period of the two oscillations be different and be commensurable, so that $\alpha' = n\alpha$ or $(\omega t)' = n(\omega t)$, then the resultant equations represent the curves above described, which are characteristic of the period-ratio. For example, if $\alpha' = 2\alpha$, the equations become

$$x = r \cos 2\alpha \qquad \text{and} \qquad y = r \cos (\alpha - \epsilon);$$

which, if $\epsilon = 0$, becomes $y = r \cos \alpha$. The resultant of these two equations is

$$x = \frac{2}{r} y^2 - r, \qquad [18]$$

which is the equation of a parabola, the abscissa varying as the square of the ordinate. If ϵ be made equal to π, then the equation is that of the curve known as the lemniscate.

61. Composition of Harmonic Curves.—The best mode of compounding harmonic vibrations along the same line is to combine with each of these vibrations a uniform motion of translation perpendicular to it, thus forming a harmonic curve; and then to compound these curves. Since the resultant of two or more motions along the same line is always the algebraic sum of these motions, it is evident that the ordinates of the compound curve will be simply the algebraic sum of the ordinates of the simple curve. Thus, for example, if the two component curves have the same amplitude, their ordinates will be equal; and if the epoch be zero, the compound curve will be a curve of twice the amplitude. If the epoch be π, then the component ordinates will have opposite signs, and being equal their resultant will be zero; i.e., the two motions will mutually destroy each other.

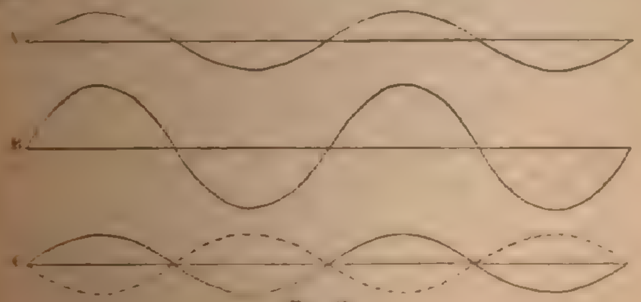
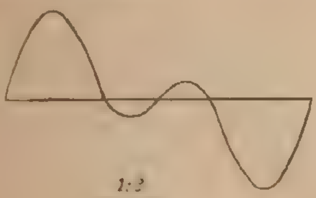


FIG. 25.

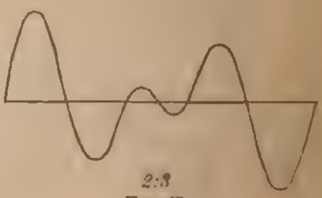
Thus if A represent a harmonic curve of amplitude one and this be combined with the full curve of C, a precisely similar harmonic

curve, the resultant curve will be represented by B, of twice the amplitude; the ordinates of the latter curve, at all corresponding points, being the sum of the component ordinates. But if A be combined with the dotted curve of C, i.e., if we combine the two curves shown in C, the resultant will be a straight line; since the equal ordinates of the two curves, being of opposite signs, will destroy each other mutually.

If the harmonic curves have not the same period, the method of compounding them is still the same; the ordinates of the resultant being always the algebraic sum of the ordinates of the component curves. The following figures show the compound curves obtained by combining two harmonic curves of different periods. In the first the periods are as one to two, and in the second as two to three.



1:2
FIG. 26.



2:3
FIG. 27.

62. Phenomena of Wave-motion.—A wave in its simplest form consists of a series of particles all vibrating according to the simple harmonic law and having between them a uniform difference of phase. Thus, for example, if a row of particles, forming a straight line when at rest, be made to vibrate with simple harmonic motion along the vertical lines a, b, c, d, etc., in the figure (Fig. 28), and if the particle at a be at a given

FIG. 28.

instant at its middle point, while the particles at b, c, d, etc., are successively in advance of this position by $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, etc., of an entire period, then it is evident that the

complete row of particles thus vibrating will have the form of a simple harmonic wave. Moreover, as the difference of phase between them is constant, the wave-form is preserved constant also. If the particle at *a* be moving upward, then at the next succeeding time-interval all the particles will have changed their positions, the highest part of the wave will have moved on to *c* and the lowest to *i*, the wave as a whole having moved from right to left. Evidently during a complete period, the wave would move from *m* to *a*; and this distance, since it represents the distance travelled during a complete oscillation of one of the particles, is a **wave-length.** Hence the length of a wave may be defined as the distance from a given particle on the wave to the next particle which is in the same relative position and is moving in the same direction. The form of this wave resembles that of the simple harmonic curve already mentioned. But it should be borne in mind that the harmonic curve represents the successive positions of a single particle, while the wave-form represents the simultaneous positions of a row of particles. In the former case the abscissas represent times; in the latter they represent distances.

63. Speed of Propagation.—With reference to the speed with which the wave is propagated, this depends in general upon the medium which it traverses. If, however, the time of vibration of the particles in one case be half as great as in another, the difference of phase remaining the same, then evidently while the wave-form will remain the same, since it depends only upon the relative positions of the particles, and these are unchanged, the wave itself will move twice as rapidly. So, on the other hand, if the phase-difference be independent of the rapidity of vibration of the particles, the rate at which the wave is propagated will be constant, the time in which the particles vibrate affecting only the wave-length. Obviously the shorter the wave the greater the number which pass a given point in a given time, other things being equal.

64. Classification of Waves.—Waves are commonly divided into three classes according to the relation existing between the direction of propagation of the wave itself and the direction of vibration of its particles. These are: (1) **Transversal** waves, in which the particles oscillate along lines perpendicular to the line of propagation, as in the figure above given. (2) **Longitudinal** waves, in which the oscillation of the particles takes place along the line of propagation. And (3) **Circular** waves, in which the particles move in circles whose planes are in the direction of propagation, and of which the first two may be regarded as limiting cases. The motion in light-waves may be taken as an example of transversal vibrations, in waves of sound of longitudinal vibrations, and in water-waves as an example of circular vibrations, in the moving particles. In the first and third of these classes, the wave consists of a portion raised above the normal level and called the **crest**, and of a portion depressed below it called the **trough**. In the second class the particles approach and recede from each other, forming a **condensed** portion over one half the wave and a **rarefied** portion over the other.

65. Wave-front.—Suppose a number of parallel rows of particles to be all occupied in transmitting the same wave. It is evident that a plane may be drawn through the particles in the different rows which are in the same phase; and that this plane will be normal to the direction of propagation. Such a plane is spoken of as the **wave-front**. It is the surface in which a single wave-form, or portion of a wave-form, is conceived of as lying. Thus if waves radiate from a point in an isotropic medium (i.e., one in which the speed of propagation is the same in all directions), the successive wave-fronts will be enveloping spheres; of progressively increasing radii and therefore decreasing curvature as time goes on. Evidently these wave-fronts are closer together in proportion as the waves themselves are shorter; i.e., as the wave-frequency is greater. When such spherical waves impinge upon the plane surface of

a second and denser medium, one portion suffers reflection, the reflected waves appearing to come from a center as much behind the surface as the actual center is in front of it. At the same time, another portion enters the medium, producing a second set of spherical waves, whose speed of propagation, and therefore whose wave-length, is diminished. If, however, the wave impinges upon the surface of a less dense medium, not only is there a wave of greater amplitude and of increased length propagated into this rarer medium, but the wave which is reflected back into the denser medium, while unchanged in length, is exactly reversed in phase. So that the crest of a wave when reflected becomes a trough; and a compression becomes a rarefaction. In other words, there is a loss of half a wave-length whenever reflection takes places at the surface of a rarer medium.

66. Interference of Waves.—According to Fourier, any periodic curve whatever can be produced by compounding simple harmonic curves having the same axis, whose wave lengths are aliquot parts of its own. And conversely, all such compound curves may be resolved into these simple harmonic curves. Since all waves have the forms of these sine-curves approximately, they may be compounded, i.e., be made to interfere, in the same way and with the same results. As an illustration, suppose two waves of the same length and amplitude but in opposite phases be made to interfere. Since they are equal in magnitude and are opposite in sign they will mutually destroy each other. Thus, for example, if two such waves be made to move continuously over a stretched wire in opposite directions, they will meet at certain points of the wire in the same phase and at other points in opposite phases; producing **nodes** or points of rest at the latter, and **antinodes** or points of maximum motion at the former. Intermediately a harmonic gradation will result, so that the wire will vibrate in segments, producing what are known as **stationary waves.** The distance from a node to the next antinode will evidently be one quarter of the wave length.

Suppose two waves, one slightly longer than the other, to be started together over the same course. The longer one will gain upon the other until at a certain distance it will be half a wave-length in advance of it and the two will be opposite in phase. At twice this distance the longer will have gained an entire wave upon the shorter and the two will again be in accord. If, for example, one vibrating body makes 100 vibrations per second and the other 101, then in half a second, after the first has made 50 vibrations the second will have made 50½ and the two will oppose each other. At the end of a whole second the former will have made one complete vibration more than the latter and they will again be in accord. The resultant curve varies, therefore, in amplitude between zero and the sum of the component amplitudes. In the case of sound, the effect is an alternate increase and decrease in its intensity, giving rise to the phenomenon known in music as **beating**.

When a wave-front meets a screen, near its edge and perpendicular to its plane, it causes a series of secondary waves having this edge as their common center. These secondary waves, since they move in all directions, pass into the space behind the screen or are inflected; thus disturbing the exactness of outline of the geometrical shadow. If the wave-length be not small compared with the size of the obstacle, the inflection is very considerable and there is practically no shadow. Thus in the case of sound-waves, which are in general of considerable length in comparison with ordinary objects, shadows are rare and ill defined; while in the case of light, the waves of which are very short, shadows are sharp and well defined. This bending of waves around an obstacle is called **diffraction**.

CHAPTER II.

DYNAMICS.

SECTION I.—CLASSIFICATION.

67. Definition of Dynamics.—In the last chapter motion was discussed solely from the geometrical standpoint, without reference to the amount of matter moved or to the antecedent or consequent phenomena of the motion. It is customary, however, probably in consequence of the experience of our own muscular sense, to assume the existence of an entity called **force** as the cause of motion. That department of physics which considers the action of force in producing motion or pressure is called **Dynamics** (from *δύναμις*, force).

68. Subdivisions of Dynamics.—Observation teaches us that force may act upon matter in one of two ways: 1st, so as to produce motion in it; or, if the motion is already existing, so as to change the direction of this motion; or, 2d, so as to prevent motion, i.e., to compel rest. Hence Dynamics is conveniently divided into two branches: **Kinetics**, or that branch of Dynamics which investigates the action of force in producing the motion of matter or in modifying its direction; and **Statics**, or that branch of Dynamics which investigates the action of force in preventing motion. Kinetics therefore considers matter only when in motion under the action of force, while Statics treats of it only when in a state of equilibrium under this action.

SECTION II.—KINETICS.

69. Quantity of Motion.—Momentum.—In discussing motion kinematically only the speed and direction of the motion were considered, without reference to the mass of the moving body. In Kinetics, however, the amount of motion produced by the action of a force upon a body depends upon the mass of that body, as well as upon its speed, being directly proportional to both these quantities. If m be the mass of the moving body and s its speed, the product ms will represent the amount of motion in it. This product is a quantity of great importance in Physics and is called **momentum**. Since $s=l/t$ we have $ms=ml/t$. So that momentum is the rate of mass-displacement in the same sense that speed is the rate of linear displacement. Evidently if the mass and the speed both be unity, the momentum will also be unity. And the unit of momentum may be defined as the momentum of a unit mass moving with unit speed. Since momentum is a directed quantity, momenta may be compounded and resolved like motions and velocities. The dimensions of unit momentum are $[MLT^{-1}]$.

70. Mass, Volume, Density.—**Mass** has already been defined as the quantity of matter in a body expressed in units of mass; and **volume** as the space which a body occupies expressed in units of volume. **Density** is sometimes defined as the ratio of mass to volume. But if the whole amount of matter in a body, stated in units of mass, be divided by its volume given in units of volume, the quotient will obviously be the number of units of mass contained in one unit of volume. Hence the density of a substance is better defined as the mass contained in unit volume of that substance.

EXAMPLES.—In the C. G. S. system, the unit mass is a gram, and the unit volume is a cubic centimeter. But by legal definition, a gram is the mass of a cubic centimeter of water. Hence the density of water is unity. If the density of iron is given as 7·8, this is simply equivalent to the statement that the mass of one cubic centimeter of iron is 7·8 grams. Moreover, just as density is the quo-

tient of mass by volume, or $\delta = m/v$, so volume is the quotient of mass by density, or $v = m/\delta$, and mass is the product of volume by density, or $m = v\delta$. The mass of a cubic decimeter of iron, for example, is $1000 \times 7{\cdot}8$ or 7800 grams. If the substance be homogeneous, the density obtained as above is its actual density. If not, the density is the mean or average density.

71. Moment of Momentum.—The moment of a directed quantity has already been defined (46) as the product of the quantity into a length perpendicular to its direction; and the moment of a particle moving with a speed s in a circle of radius r has been given as rs. In motion of rotation, it is evident that, in addition to mass and speed, distance from the axis of rotation is an important factor in the expression for the amount of motion. And since the moment of a physical quantity is the numerical measure of its importance, the moment of the momentum, or msr, measures the importance of this momentum in maintaining rotation. Each of the particles composing a rotating body, it is evident, has its own moment of momentum msr. The sum of these products, extended to all the particles in the body, is $\Sigma(msr)$. This represents the moment of momentum of the entire mass.

EXAMPLES.—That the effectiveness of a particle for maintaining rotation is directly proportional, not only to its mass and to its speed, but also to its distance from the axis of rotation, is well illustrated in the fly-wheel of a steam-engine. This is a large and heavy wheel fastened to the shaft and driven with it, which is employed as an equalizer of the motion. It is massive in size and material, and it is driven at a high speed; and hence its momentum is great. But in addition its mass is concentrated in its rim; so that, being as far as possible from the axis, it shall have as large a moment of momentum as possible, and shall produce the maximum turning effect.

Evidently, if in the expression msr the mass be unity, the value sr will represent the moment of the speed; and if s be unity, the expression mr will represent the moment of the mass; i.e., the moment of momentum of a mass m moving with unit speed. So, similarly, the product ωr represents moment of angular velocity, and αr moment of angular acceleration.

72. Moment of Inertia.—If, in the expression for moment of momentum msr, we replace s by its value ωr (43), the moment of angular velocity, we have $mr^2\omega$; an expression for the moment of momentum in terms of the angular velocity. If we call the product of a directed quantity by the square of a distance at right angles to itself the second moment of that quantity, then the above moment of momentum $mr^2\omega$ is simply the second moment of mass multiplied by the angular velocity. This second moment of mass mr^2 is called the **moment of inertia.** It is generally represented by I. Evidently if m be the mass of a single particle, and r its distance from the axis, the sum of these second moments $\Sigma(mr^2)$ or $M\Sigma(r^2)$ will be the second moment of the entire mass M; and the moment of momentum of the entire mass will be $\Sigma(mr^2)\omega$ or $M\omega\Sigma(r^2)$. If this entire mass were collected at a distance k from the axis, the moment of inertia remaining the same, then the expression $\Sigma(r^2)$ would equal k^2, the moment of inertia would be Mk^2, and the moment of momentum $Mk^2\omega$. The value k is called the **radius of gyration.**

EXAMPLE.—In the case of the fly-wheel above mentioned, since its motion is rotatory and the angular velocity is the same for all its particles, the moment of its momentum, i.e., the numerical measure of the importance of this momentum in maintaining the motion, is simply the product of its angular velocity by the sum of the second moments of mass of all the particles which it contains.

73. Definition of Force. — Force may be defined provisionally as that which produces motion or pressure. So far as we at present know, "the change of motion of any body depends partly on the position of distant bodies and partly on the strain of contiguous bodies" (Clifford). So that, as we shall see subsequently, what we are accustomed to call force is simply the space-rate at which energy is transferred from one body to another. Hence force is defined by Tait as "the measure of the tendency of energy to transform itself."

74. The Unit of Force.—Force is measured by the

amount of motion which it produces in unit of time. But amount of motion is momentum; and hence

$$f = ms/t, \tag{19}$$

or force is the rate at which momentum changes with time. Since, however, acceleration is the rate at which speed changes with time, or $a = s/t$, the expression for force may be written

$$f = ma, \tag{20}$$

the form in which it is usually given. This is the fundamental equation of dynamics, and states that the measure of a force is the product of the mass moved into the acceleration produced in it. If the mass be unity, the force is proportional to the acceleration it generates in unit mass; and if the acceleration be unity, to the mass to which it communicates unit acceleration. Since, when m and a are both unity, f is also unity, the unit of force is defined as that force which generates continually unit acceleration in unit mass. The dimensions of unit force, therefore, are $[MLT^{-2}]$.

EXAMPLES.—In the C. G. S. system a gram is the unit mass, and a centimeter per second per second is the unit acceleration. Hence the C. G. S. unit of force is that force which, acting upon a gram-mass for one second, will generate in it a speed of one centimeter per second. This unit of force is called a **dyne**.

75. Impulse of a Force.—The impulse of a force is the product of the force multiplied by the time during which it acts; and hence is represented by ft. Evidently to produce a finite acceleration in a given mass requires that the force should act for a finite interval. Moreover, the action of a comparatively small force for a long interval may obviously produce the same effect as the action of a proportionately greater force through a smaller interval. Again, since $ft = ms$, it follows that an infinite variety of forces may generate a constant momentum, provided that the times during which they act

are inversely proportional to the magnitudes of the forces. The unit impulse may be called a **dyne-second.**

76. Newton's Laws of Motion.—We may now enunciate the three laws in which Newton expressed the effect of forces in altering the motions of bodies. "The two centuries which have nearly elapsed since he first gave them have not shown a necessity for any addition or modification." (Thomson.)

First Law of Motion.—"Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it may be compelled by force to change that state."

The truth of this law is made evident by the well-known fact that whenever any alteration takes place in the state of motion of a body, this alteration can be traced to some action between the body itself and some other body; i.e., to the action of an external force. The fundamental principle which underlies this law is the principle of the **inertia** of matter. It states that matter of itself has no power to change its condition either of rest or of motion; and hence that when at rest it must continue at rest, or when in motion it must continue in motion, unless some external force intervenes to change its condition. Both motion and rest are equally normal conditions of matter. But both are relative. Bodies at rest with reference to certain points are in motion with reference to others. While to alter the condition of a body, therefore, requires the action of force, none is required to maintain its condition constant.

EXAMPLES.—Common experience does not seem to accord with this law. The normal condition of matter seems to be that of rest; and while force seems to be needed to put a body in motion, none seems to be required to destroy this motion. A ball rolled on the ground or thrown into the air is brought speedily to rest; and a railway train or a steamboat requires the continuous action of the steam to maintain its motion. But this is simply because, under the conditions in which we live, the resistances to motion are numerous; and friction, the air-resistance, etc., soon destroy it. In proportion as these are removed, however, the motion persists. A ball rolled on a smooth floor or on ice moves over a greater distance before it

stops; a smooth and well-lubricated bearing allows the wheel a longer run. So that we cannot doubt that the law is true, and that if we could remove all impediments to motion, it would continue uniform.

Second Law of Motion.—"Change of motion is proportional to force applied and takes place in the direction of the straight line in which the force acts." Or, as stated by Maxwell in more modern phraseology: "The change of momentum of a body is numerically equal to the impulse which produces it and is in the same direction."

The scope of this law appears clearly from the following paraphrase of Newton's own comments upon it: "If any force generates motion, a double force will generate double motion; and so on whether simultaneously or successively, instantaneously or gradually applied. And this motion, if the body was moving beforehand, is either added to the previous motion if directly conspiring with it; or is subtracted if directly opposed; or is geometrically compounded with it, according to the kinematical principles already explained, if the line of previous motion and the direction of the force are inclined to each other at an angle." (Thomson and Tait.)

Third Law of Motion.—"To every action there is always an equal and contrary reaction; or, the mutual actions of any two bodies are alalways equal and oppositely directed."

This law may be called the law of stress. It teaches us that all force is of the nature of a stress; i.e., that "force is always due to the mutual action of two bodies or systems of bodies; that every force in fact is one of a pair of equal opposite ones—one component, that is, of a stress—either like the stress exerted by a piece of stretched elastic, which pulls the two things to which it is attached with equal force in opposite directions, and which is called a **tension**; or like the stress of compressed railway buffers or of a piece of squeezed india-

rubber, which exerts an equal push each way and is called a **pressure**." (Lodge.)

In the light of Newton's own comments upon it, the third law may be read as follows: "If the activity of an agent be measured by its amount and its velocity conjointly; and if similarly the counter-activity of the resistance be measured by the velocities of its several parts and their several amounts conjointly, whether these arise from friction, cohesion, weight, or acceleration;—activity and counter-activity, in all combinations of machines, will be equal and opposite." (Thomson and Tait.)

EXAMPLES.—Illustrations of this law are abundant. If a stone is pressed with the finger, the finger is pressed in the opposite direction and with the same force by the stone. A horse drawing a canal-boat or a tramcar uniformly, or a locomotive thus drawing a train, is pulled backward by a force precisely equal to that which it exerts forward. When a bullet is fired from a gun, the amount of motion, or momentum, of the bullet forward, is exactly equal to the momentum of the gun backward. The same holds true when there is no visible connection between the acting bodies. The force exerted by the sun upon the earth is precisely the same as that exerted by the earth upon the sun. The force with which a magnet attracts a piece of iron is exactly equal to that with which the piece of iron attracts the magnet; as Newton himself showed by floating upon water, on the same light board, a magnet and a piece of iron, and observing that there was no resultant force in either direction.

According to the second law, when no force acts upon a body, no change is produced in its motion. Hence the body if in motion will continue in motion, and if at rest will continue at rest; which is the first law. Again, by the third law, the forces acting between two different parts of the same body are equal and opposite. If this were not so, there would be an excess of one or the other; and this resulting force would produce motion of the body. The possibility of this, however, the first law denies; since a body cannot move except under the action of an external force. Inasmuch, therefore, as the third law is readily deducible from the first, and the first in like manner from the second, it is evi-

dent that the second law might with propriety be called *the* law of motion.

77. Measurement of Mass.—The second law gives us the means of measuring both masses and forces. It asserts in fact the truth of the formula $ft = ms$, which we have already used in defining force. Suppose that two equal impulses (or forces, if the time be unity) act upon unequal masses. By the second law the momentum, which is always equal to the impulse which produces it, will be the same in both cases. Hence we shall have $ms = m's'$; or $m : m' :: s' : s$. That is, the masses are inversely proportional to the speeds generated by equal impulses. Again, suppose the masses and the times equal; then dividing the equation for the first force $ft = ms$ by that for the second $f't = ms'$, we have $f : f' :: s : s'$; or the forces are directly proportional to the speeds which they generate in equal times in equal masses. Again, if we suppose the speeds and the times constant, we have in the first case $ft = ms$ and in the second $f't = m's$; whence $f : f' :: m : m'$. Hence we see that to communicate equal speeds to different masses, the forces acting must vary directly as the masses. And conversely, if under the action of forces unequal masses acquire equal speeds, these forces must be exactly proportional to the masses.

EXAMPLES.—It is a matter of every-day experience that the muscular force required to move a body is greater the larger the quantity of matter in the body. When made of the same substance the masses of bodies are proportional to their volumes, and hence the force is the greater the larger the body. The amount of liquid in a cask is roughly ascertained ordinarily by giving the cask a kick. The same force which would move it through a considerable distance if empty would scarcely stir it if full. Since the mass of a body is determined by its density as well as by its volume, a small body of denser material may have a greater mass than a large body made of a less dense substance. The masses of several bodies of exactly the same size may be ascertained easily by measuring the forces required to give them the same amount of motion in the same time. Thus, suppose four spheres of cork, wood, iron, and gold, respectively, suspended from four strings, after the fashion of pendulums. Give each of them a sudden knock of about the same strength. Clearly

the cork sphere will be driven farthest, the others going over distances inversely proportional to their masses. Or the knock may be so adjusted as to drive each sphere to the same distance from its position of rest. In which case the masses would be proportional to the forces. This is the dynamical method of measuring mass.

78. Elements of a Force.—In order that a force may be completely determined it is necessary that the following three elements of the force be known: 1st, its place of application; 2d, its direction; and 3d, its magnitude.

1*st. Its Place of Application.*—Since a material particle occupies space, the place of application of any force which acts upon it must be either its surface or its solid content. It is often found convenient to assume a certain point on or in the body considered, such that a certain force acting at that point would produce sensibly the same effect as that actually produced.

2*d. Its Direction.*—The second element of a force is its direction. The direction of a force is the line along which it acts. If the force be applied at a point, a line drawn through that point in the direction in which the force tends to move the body is the direction of the force.

3*d. Its Magnitude.*—The third element of a force is its magnitude. The magnitude of a force is the number of units of force contained in it. In the C. G. S. system the magnitude of a force is measured in dynes.

79. Composition and Resolution of Forces.—Since a force is a directed quantity or a vector, it may be completely represented by a line; the extremity of the line being the point of application of the force, the direction of the line the direction of the force, and the length of the line the magnitude of the force. Hence forces may be compounded and resolved in the same way and upon the same principles as the vector quantities already considered; namely, motions, velocities, and accelerations.

This conclusion is readily deduced from the second law of motion. "Since forces are measured by the changes of motion they produce, and their direction as-

signed by the direction in which these changes are produced; and since the changes of motion of one and the same body are in the directions of, and proportional to, the changes of velocity,—a single force measured by the resultant change of velocity, and in its direction, will be the equivalent of any number of simultaneously acting forces." (Thomson and Tait.)

The resultant of two forces acting upon a point may therefore be found from the parallelogram or triangle of forces; and that of several forces so acting, by the polygon of forces, in precisely the same manner, either by construction or calculation, as the resultant of two or more velocities under the same conditions was obtained in the chapter on Kinematics (36).

EXAMPLES.—Thus, to obtain the resultant of several forces in one plane by construction: "Draw a set of lines, one after the other, without taking the pen off, parallel to and in the same sense as the successive forces acting on the body, and proportional to them in magnitude; then the line required to complete the polygon taken in the reverse sense, i.e., drawn *from* the starting-point, will be the resultant in magnitude and direction." (Lodge.)

Analytically the resultant of two forces may be found from the equation $R^2 = P^2 + Q^2 + 2PQ \cos \alpha$, where R represents the resultant and P and Q the component forces, α being the angle between their directions. From this equation the magnitude R is directly obtained. In the triangle (Fig. 29), whose sides are P, Q, and R, the angle α, supposed given, is equal to $\beta + \gamma$; and $\delta = 180° - (\beta + \gamma) = 180° - \alpha$. Hence, since the sides of a triangle are proportional to the sines of the opposite angles, we have $\sin \beta : \sin \delta :: Q : R$. This gives the angle β which the resultant R makes with the force P.

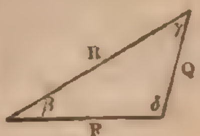


FIG. 29.

Or the forces may be resolved along co-ordinate axes, and the resultant of two or more forces found from these components in the manner described for the compounding of motions and velocities (40).

It should be noted that the formula $R^2 = P^2 + Q^2 + 2PQ \cos \alpha$ is absolutely general. For example, if the two forces P and Q are equal, the formula becomes $R^2 = 2P^2 + 2P^2 \cos \alpha$ or $2P^2(1 + \cos \alpha) = 4P^2 \cos^2 \frac{1}{2}\alpha$; whence $R = 2P \cos \frac{1}{2}\alpha$. If $\alpha = 0$, the forces act along the same line, $\cos \frac{1}{2}\alpha = 1$, and they are both positive. Hence $R = 2P$, or the resultant is the arithmetical sum of the components.

If $\alpha = 180°$, $\cos \frac{1}{2}\alpha = 0$, and $R = 0$; or the resultant is the difference of the components. If the forces are not equal, then in the first case $R = P + Q$, and in the second $R = P - Q$. If, in the general formula, $\alpha = 90°$, $\cos \alpha = 0$, and the components being at right angles, $R^2 = P^2 + Q^2$. If now the component forces be equal, the resultant is $P\sqrt{2}$ in magnitude and is inclined 45° to each component. Consequently whenever the forces are equal the resultant bisects the angle between them.

80. Action of Force in producing Rotation.—Moment of a Force.—When a force acts on a particle, it produces motion of translation only. When a force acts upon a rigid body, however, it may produce both translation and rotation, according to the number of degrees of freedom of the body (45). If two points be fixed within it, the body has only one degree of freedom, and can only rotate about a line drawn through these points as an axis. The effect of a force in producing rotation depends not alone upon the impulse of the force, i.e., the product of its magnitude by the time during which it acts, but also upon the distance of its point of application from the axis of rotation. Hence, calling this distance r, the effect of the force is measured by ftr; which may be called the moment of the impulse (46). If the angular velocity produced be ω, the angular momentum or the moment of momentum generated will be $\Sigma(mr^2)\omega$ or $\Sigma(mrs)$ (72). But this moment of momentum must be equal to the moment of the generating impulse. Hence we have

$$frt = \Sigma(mr^2)\omega\,;$$

or, calling $\Sigma(mr^2)$, the moment of inertia, I,

$$frt = I\,\omega. \qquad [21]$$

Since ω, the angular velocity, is equal to αt, the product of the angular acceleration and the time, we may write,

$$\text{Angular acceleration} = \alpha = \frac{fr}{I} = \frac{\text{moment of force}}{\text{moment of inertia}}.$$

analogous to the equation for translatory acceleration,

$$\text{acceleration} = a = \frac{f}{m} = \frac{\text{force}}{\text{inertia}}.$$

81. Composition and Resolution of Parallel Forces.—When several forces act on a rigid body, they may have a common point of application, or each of them may have a separate point of application. Since, however, forces in one plane whose directions are not parallel must intersect if produced, and since each of the forces may be transferred along its line of action to the point of intersection, non-parallel forces acting upon different points of a body may be compounded from the point of intersection according to the methods already described (79). There remains then only the case of parallel forces. The resultant of a number of parallel forces acting on different points of a rigid body is in magnitude equal to the algebraic sum of the forces, and has the direction of the greater. As to its point of application, this may be found when the two forces are like or have the same sense, i.e., act in the same direction, by the following construction:

Compound the two parallel forces P and Q (Fig. 30) acting at A and B with the two equal and opposite forces AM and BM acting on the same points. The resultant of P and AM will be AC; that of Q and BM will be BD. Let E be their point of intersection. Now at E resolve BD and AC into components parallel to P and Q and to AM and BM. Then at E we shall have (*a*) two equal and opposite components parallel to AB which may be removed, since their resultant is zero; and (*b*) two components parallel to P and Q and each equal to one of these forces in magnitude. The resultant of these latter components is evidently their sum, and this resultant is applied at the point E. But since a force may be transferred to any point on its line of action, the place of application of the resultant of P and Q is the point F, where a line drawn through E, parallel to the directions of P and Q, cuts the line AB. FR is therefore the resultant.

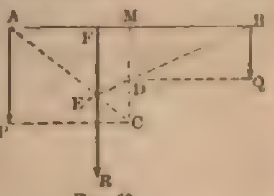


FIG. 30.

A similar construction may be employed when the forces are unlike; i.e., are of contrary sense, as P and Q in the figure (Fig. 31). Compounding Q with BM we have BD; and from P and AM we have AC. Producing these resultant directions, they intersect at E; from which drawing ER parallel to AP, we have FR as the resultant.

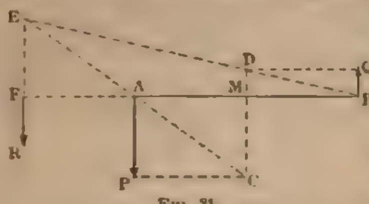


FIG. 31.

It will be observed (1) that the point of application of the resultant lies between the points of application of the components when the forces have the same sense; (2) that it is outside these points on the line joining them when these forces are of contrary sense; and (3) that in both cases it is nearer to the greater force. In general the point of application of the resultant divides the line joining the forces, or this line produced, into segments which are inversely proportional to the magnitude of the forces. In all cases $P : Q :: AF : BF$. Hence the resultant is completely determined.

82. Equal and Unlike Parallel Forces.—Couples.—One case of great importance remains to be provided for. If the two unlike forces are equal, then, in the construction given above, the diagonals AC and BD of the parallelograms will be parallel and therefore can intersect only at infinity. Hence such a pair of parallel forces can have no resultant. Taken together they constitute a **couple**, and their only effect is to produce rotation. The efficiency of a given couple in producing rotation is proportional to the magnitude of either of the forces multiplied by the perpendicular distance between the forces; this distance being called the **arm** of the couple, and the product the **moment** of the couple. Obviously the moment of a couple about any point is independent of the position of that point. The **axis** of a couple is a line perpendicular to the plane of the couple whose magnitude and direction represent respectively the magnitude of the moment and the direction of rota-

tion. If the plane of the couple be horizontal, and the direction of rotation is counter-clockwise or positive, the axis of the couple rises above the plane; falling below it if the rotation is negative. Since a couple is completely represented by its axis, couples may be compounded and resolved with reference to their axes in the same manner as forces and velocities may be with reference to the lines which directly represent them.

SECTION III.—STATICS.

83. Equilibrium.—As already stated (68) **Statics** is that branch of Dynamics which investigates the action of force in maintaining bodies in equilibrium. If but one force act upon a rigid body, the body must move and this in the direction in which the force acts. But if several forces so act, these forces may be so distributed as not to affect the condition of the body in any way whatever. Since both motion and rest are equally normal conditions of matter, matter is said to be in equilibrium when no change is going on in its condition. Matter in motion is in equilibrium when its acceleration is zero. Matter at rest is in equilibrium when the resultant of all the forces acting upon it is zero. The conditions of equilibrium then are simply the conditions under which acceleration is impossible; or those, in other words, under which rest is possible.

EXAMPLES.—Thus a book lying upon a table is in a state of static equilibrium, since the forces acting upon it are balanced and their resultant is zero. A bucket descending into a well with constant speed is in a state of kinetic equilibrium, because since its speed is constant, its acceleration is zero.

84. Conditions of Equilibrium of a Particle.—The condition of equilibrium for two forces which act upon a particle is only that their resultant shall be zero. For this to be true, it is necessary: 1st, that the two forces be equal; 2d, that they act along the same straight line; and 3d, that their directions be opposite. But this proposition may be readily extended to any number of forces, since in order for them to have a zero resultant,

it is necessary only that the resultant of certain of the forces be equal and opposite to that of all the rest. If the forces be three in number, for example, one of them must evidently be equal and opposite to the resultant of the other two. Hence two of the forces must meet in a point, through which their resultant passes. The third must evidently lie in the same plane with the others, must act on the same point, and must be equal in magnitude and opposite in direction to this resultant. But this is equivalent to saying that for three forces to be in equilibrium, it is necessary only that the three forces represent in magnitude and direction the sides of a closed triangle, taken in order. On the same principle any number of forces acting on a particle will keep it in equilibrium if these forces can be represented by the sides of a closed polygon, parallel to the several forces and taken in order. We have already seen (79) that in an unclosed polygon, whose sides are parallel (or perpendicular) to and proportional to a number of forces, the open side drawn from the initial point represents the resultant of all the forces. Consequently if the open side be drawn to the initial point, it represents a force equal and opposite to the resultant. Since no line is required to complete a closed polygon, the system of forces it represents has no resultant.

EXAMPLES.—Thus the two equal forces OS and OR (Fig. 32) acting on the particle O in opposite directions, will maintain it in equilibrium. And the three forces OP, OQ, and OS will also have a zero resultant if OS be opposite and equal to OR, the resultant of OP and OQ. The above conditions of equilibrium are here fulfilled since the forces may be represented in magnitude and direction by OP, PR, and RO, the three sides of the triangle OPR, taken in order.

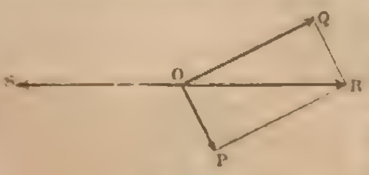


FIG. 32.

Another criterion of equilibrium may also be mentioned. If several forces act upon a particle and these

forces be resolved along any three straight lines not in one plane, the particle will be in equilibrium if the sum of the component portions along these lines is equal to zero.

85. Conditions of Equilibrium of a Rigid Body.—Under the conditions named above no motion of translation is possible; but if the forces act upon a rigid body there may still be motion of rotation. In order, therefore, that there should be no rotatory motion additional conditions are required. The most general condition for no rotation is that the moments of all the forces about every possible point or axis of rotation shall have zero for their algebraic sum. If, however, we limit the motion of the body to one plane, and if, at the same time, this be the plane in which the forces act, it is obviously quite sufficient in order to maintain the body in equilibrium (1) that there be no resultant force along any two intersecting straight lines drawn in this plane, and (2) that the algebraic sum of the moments round any point in the plane be equal to zero.

SECTION IV.—FRICTION.

86. Reaction of Surfaces in Contact.—Whenever the plane surfaces of two bodies are in contact, these surfaces being inclined to the horizontal, a stress exists between them, and the upper surface presses on the lower with a force which is proportional to the weight of the upper body. At the same time the lower surface reacts against this pressure; so that when in equilibrium the downward action due to the weight of the body is exactly equal to the upward reaction of the lower surface, the direction of these two opposite forces making an angle with this surface. Resolve now the reaction of the lower surface into two components, one perpendicular to this surface and the other parallel to it. The former component is called the **pressure** and the latter the **friction** between the surfaces. If the last-named component be absent, the surface is said to be **smooth**:

a smooth surface being defined as a surface capable of exerting pressure upon another surface only in a normal direction. Ordinarily, however, both components are present and the surface is said to be **rough**; a rough surface being one capable of exerting pressure in other directions besides the normal one. Evidently, a smooth surface offers no resistance to motion along it. But such a surface is unattainable in practice; and hence every actual surface, even when most highly polished, must be considered a rough surface, since it exerts pressure in other directions than the normal one.

87. Statical Friction.—A body at rest upon a plane surface is so, obviously, because the reaction of the plane upon it is equal in magnitude and opposite in direction to the resultant of all the other forces acting on it. If the plane be inclined, the component of its weight acting down the plane is balanced by the component of the reaction of the plane itself, parallel to the plane, and acting up the plane; i.e., by the friction. So that in general, whenever force is applied to a body to slide it along a surface, a resistance is developed acting in the contrary direction and tending to prevent the motion. This resistance is called **friction**. So long as the body does not move, the friction developed is exactly equal to the acting force. But as the latter increases in value, it finally overcomes the friction, which, having reached a maximum, can increase no more. The resistance to be overcome in starting a body from rest is called **statical friction**.

88. Coefficient of Friction.—Before motion actually takes place, the statical friction actually developed is only that required to balance the acting force and to prevent the motion. At first, as the force increases, the friction increases with it; but as there is a limit to the friction which can be thus called out, this limiting value is finally reached and the body slides along the surface.

EXAMPLES.—Place a wooden block on a planed board. When the board is horizontal (Fig. 83) observe that the reaction R, which is wholly vertical, is therefore perpendicular to the surface. If now

the board be inclined, the reaction is no longer perpendicular to the surface, and a component F of the reaction is developed (Fig. 34)

Fig. 33.

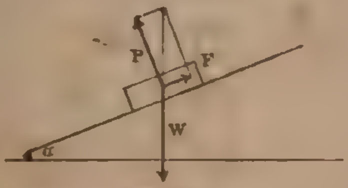


Fig. 34.

parallel to the surface and acting to prevent sliding. This is the friction-component. As the angle of the plane increases, the friction-component F increases and the pressure-component P decreases (Fig. 35). Hence $F = W \sin \alpha$ and $P = W \cos \alpha$, and $F/P = \tan \alpha$. Now experiment shows that for every two substances in contact, one will begin to slide upon the other whenever the angle α, and therefore the ratio F/P, attains a certain value: the weight-component down the plane overbalancing finally the friction-component of the reaction.

Fig. 35.

This limiting value of the ratio of the friction-component along the plane to the pressure-component perpendicular to it is called the **coefficient of friction** and is generally represented by μ. It evidently depends upon the nature of the substances whose surfaces are in contact, and it is found to be independent of the size and shape of these surfaces.

89. Angle of Repose.—Whenever a body is placed on a plane surface and the surface is gradually inclined, an angle of inclination is finally reached, as we have seen, when the body begins to slide; this angle depending on the coefficient of friction μ. Since $\mu = F/P$, we have evidently $F = \mu P$; so that the limiting value of F, or the friction, is the product of the coefficient and the pressure. Since μ is constant, while F increases and P decreases with the angle of inclination, this limiting

value is readily reached. Inasmuch, however, as $\mu = F/P = \tan \alpha$, the limiting value of $F = P \tan \alpha$, and the angle at which sliding is just about to take place is the angle whose tangent is F/P. But the angle of inclination of the plane is equal to the angle which the reaction of the plane makes with its normal. Hence when the tangent of this latter angle is equal to the coefficient of friction, sliding is just about to begin. In consequence this angle is called the limiting angle of friction, or the **angle of repose.**

COEFFICIENTS OF FRICTION (MORIN).

	Coef. (Statical).	Angle.
Steel on Glass (polished)	0·11	6° 17′
Marble on Marble (polished)	0·16	9° 6′
Wrought-iron on Brass	0·17	9° 39′
Steel on Cast-iron	0·20	11° 19′
Marble on Birch	0·44	23° 45′
Oak on Oak (fibres parallel)	0·48	25° 38′
Wrought-iron on Oak (fibres parallel)	0·62	31° 47′
Birch on Birch	0·64	32° 38′

90. Angle of Resistance.—Let a body of weight W be placed on a horizontal surface (Fig. 36), and let traction be applied to it by means of a cord passing over a pulley and supporting a weight F. In this case we have two forces at right angles acting on the body: its weight producing the pressure P, and the traction F; and by the parallelogram of forces $F/P = \tan \alpha = \mu$.

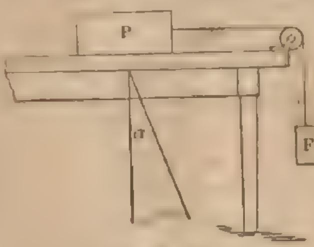


FIG. 36.

Hence the line of action of the resultant, and of course the direction of the actual reaction also, will be the diagonal of a parallelogram of which F and P are the sides. Calling T this reaction, we have from the figure

$P = T \cos \alpha$ and $F = T \sin \alpha$. Evidently, therefore, the body will not slide unless the tangent of the angle α between the resultant and the perpendicular to the surface be greater than the ratio F/P or μ. When $\tan \alpha = F/P$ or μ, the angle α has its maximum or limiting value; and it is then called the **limiting angle of resistance.**

In a similar way, if a rod AB be made to exert force upon the top of the block C represented in the figure (Fig. 37), and this rod be gradually inclined to the vertical, a component of the force will be developed, acting to move C along the plane D; this component being antagonized by the friction between the surfaces up to a certain point. When this point is reached, the component of the force acting along the plane to move the block is exactly equal to the friction; and the angle made by the rod with the vertical, which is also that made by the direction of the reaction with the vertical, is the angle whose tangent is the ratio of the horizontal component of the force, or its equal the friction F, to the vertical component, or its equal the pressure P; i.e., is α when $\tan \alpha = F/P$ or μ. In this case also, α is the limiting angle of resistance. If the force T exerted by the rod be resolved along the surface and perpendicular to it, $T \sin \alpha$ will represent the sliding component, and $T \cos \alpha$ the pressure-component. If the ratio of these components be less than μ or be equal to it, i.e., if the sliding-component be equal to or less than μ times the pressure-component, there will be no sliding.

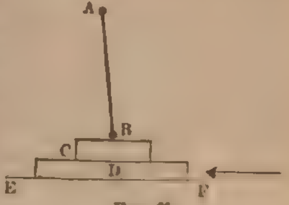


FIG. 37.

If, under the conditions shown in the figure, force be exerted in the direction of the arrow tending to slide D upon the surface FE, it is evident that the block C, and with it the end of the rod B, will be pushed in the same direction; and further, that if the rod be fixed at A, a great pressure will be produced on C tending to prevent

any motion. Such a device as this is called a **friction-grip**.

91. Kinetical Friction.—Experiment shows that the force required to maintain a body in motion is less than that required to start it originally; and hence that the coefficient of **kinetical friction**, κ, or the ratio of the resistance to the pressure experienced by a body actually in motion, is less than the coefficient of statical or starting friction, μ. Moreover, surfaces in motion relatively, may slide upon or roll over each other; whence kinetical friction may be of two sorts, **sliding** friction and **rolling** friction, the latter being much the smaller. The value of κ in general has been found to be for metal upon metal about 0·18, for wood upon wood about 0·36, and for wood upon metal about 0·42. By the use of lubricating materials the friction is greatly reduced; these coefficients being diminished by the use of tallow to the values 0·09, 0·07, and 0·08, respectively.

92. Laws of Friction.—The laws of friction have been obtained solely by experiment. As usually given, they are three in number, as follows:

1st. The friction is directly proportional to the normal pressure between the surfaces in contact; and is equal to this pressure multiplied by the friction-coefficient.

2d. The friction depends only on the nature of the surfaces in contact, and is independent of the size or shape of these surfaces. Evidently if P be the total pressure, and a the area of the surfaces in contact, P/a will be the pressure per unit of area. But to this, by the first law, the friction per unit of area is proportional. Hence the friction over the entire area, a units, which is a times the friction over unit area, is aP/a, or P; and the friction is independent of a.

3d. Kinetical friction is in general less than statical friction. But when the motion is once established, the friction is found to be independent of the speed.

EXAMPLES.—Since friction always appears as a resistance to motion, it is in some cases of the greatest utility. Without friction

rch would not stand, a nail or a screw would be useless, and a ray train could not leave the station. The transmission of r by belting would be impossible were it not for the friction of elt upon the pulley. On the other hand, when motion is to be tained between two surfaces, friction becomes a serious disad- age. To diminish it the surfaces are made as smooth as possi- nd then lubricated ; and wherever possible, sliding is converted rolling friction.

CHAPTER III.

WORK AND ENERGY.

SECTION I.—WORK AND ITS MEASUREMENT.

93. Definition of Work.—Whenever a body is made to move through a certain distance against resistance, work is said to be done; and the amount of work so done is proportional jointly to the resistance overcome and to the distance through which it is overcome. If we call force that which produces the motion, then, since resistance is measured by the force required to overcome it, the work done is represented by the product of the force into the distance through which it acts. The dimensions of work are therefore ML^2T^{-2}.

EXAMPLES.—To lift a mass of iron from the floor to a table requires that a resistance due to the weight of the iron be overcome through a distance equal to the height of the table. So, on the same principle, work is done in raising coal from a mine, in lifting stone to the top of a building, and in pumping water into a reservoir. As illustrations of other resistances than weight to be overcome, the work done in drawing a loaded train, in compressing a spring, in condensing air into a hollow cylinder, in separating a piece of iron from a magnet, may be mentioned.

94. Measurement of Work.—Work, like every other physical quantity, is measured in terms of a definite amount of its own kind, arbitrarily chosen. In the C. G. S. system this quantity is called an **erg** (from ἔργον, work). Since work is proportional to the product of the force acting and the distance through which it acts, we may write as the equation of work $w = fl$. Evidently if f and l be both unity, w will also be unity;

in other words, a unit of work is done by a unit force acting through a unit distance. Inasmuch as in the C. G. S. system the unit of force is a dyne and the unit of distance is a centimeter, their product, a dyne-centimeter, is the amount of work called an erg. Since this unit is so small as to require the use of excessively large numbers in expressing ordinary quantities of work, multiples of it by 1000, by 1,000,000, by 10,000,000, and by 10,000,000,000 are in use. These larger units are called respectively a **kilerg**, a **megerg**, a **joule**, and an **erg-ten.**

Since the weight of a gram-mass, as will be subsequently shown, represents about 980 C. G. S. units of force, the work done in raising the weight of a gram through one centimeter, which is called a gram-centimeter, is equal to 980 ergs. To raise a kilogram through one centimeter, a kilogram-centimeter of work must be done, equal to 980,000 ergs; or nearly a megerg. To raise a kilogram through one meter requires one kilogram-meter of work, equivalent to 98,000,000 ergs or to 9·8 joules.

95. Graphic Representation of Work.—Since work is a product of two factors, it may evidently be represented by an area whenever the two factors are suitably represented by lines. Thus when a uniform force f acts through a distance l, the work done, which is fl, may be represented by the area of the rectangle (Fig. 38), the product of the ordinate representing f, and of the abscissa representing l. If the force be uniformly variable, i.e., if it increase uniformly from zero to the value f, then the work done, being the product of the mean or average value of the force into the distance, will be $\frac{1}{2}fl$; since $\frac{1}{2}(0+f)$, or half the sum of the initial and final values, equals $\frac{1}{2}f$. But the area of the triangle (Fig. 39), which is half the product of the base by the height, is also $\frac{1}{2}fl$.

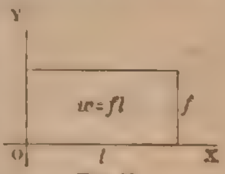


FIG. 38.

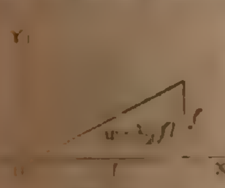


FIG. 39.

In case the force does not vary uniformly, the work

done, which is always represented by the product of the mean force into the distance, is still the area of the figure enclosed between the curve-line representing the variation of the force, the axis of abscissas, and the two ordinates; the distance between these ordinates representing the space through which the force acts. Thus suppose air be compressed in a cylinder. Let Oa (Fig. 40) be the volume and Aa the pressure at the beginning of the operation. During the compression let the volume diminish to Ob and the pressure increase to Bb. The force will not vary uniformly, and hence A and B will be connected by a curve-line ACB drawn from A to B. The area $aABb$, however, will still represent the work done upon the air during the compression. If

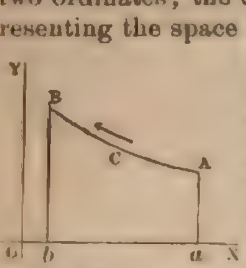


FIG. 40.

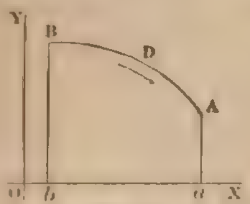


FIG. 41.

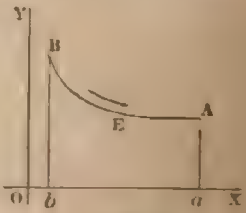


FIG. 42.

now the air be made to expand again to its original volume in such a way as to do more work than was originally expended upon it, then the new curve BDA drawn from B to A, representing the variation of pressure and volume, will be above the compression-curve (Fig. 41). If less work is done in the expansion than in the compression, then the new curve BEA, which is drawn also from B to A, is below the compression-curve, as shown in Figure 42. The work done in expansion in both these cases is represented by the areas $aABb$ in the two diagrams. Combining now both operations in one, so as to represent a complete cycle and to return the air to its initial volume and pressure, we have two condi-

tions according as the curve of compression is united with the one or the other expansion-curve. In the former case (Fig. 43) the work done by the gas is greater than that done upon it; and this by the area $ACBD$; hence the machine acts as a motor. In the latter the reverse is the case (Fig. 44) and the machine acts as a compressor; the difference being represented by the area $ACBE$. It will be observed that in the former case the operations of the cycle, as shown by the arrows, are in

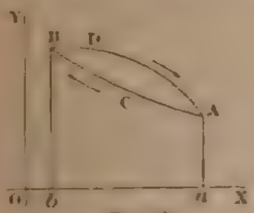


FIG. 43.

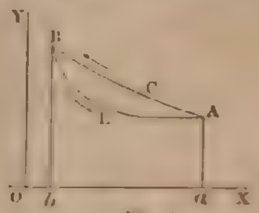


FIG. 44.

the direction in which the hands of a watch move, or negative; while they are in the opposite or positive direction in the latter.

Such diagrams as those now discussed are called diagrams of work. Knowing the enclosed area and the change of volume, ab, simple division gives their quotient or the mean pressure acting. This diagram was first suggested by James Watt to measure the work of the steam-engine; but Clapeyron and Rankine have greatly extended its usefulness by making its application general.

96. Rate of Work.—Activity.—The rate at which work is done by any agent is called the **activity** of that agent; and hence the activity is obtained by dividing the work done by the time required to perform it. The unit of activity is therefore reached when a unit of work is done in a unit of time; or in the C. G. S. system, when the **rate** of working is an erg per second. The word **power** is sometimes used to indicate rate of work.

EXAMPLES.—If a kilogram-meter of work be done by a machine in a minute, the rate at which the machine is working, i.e., its activity, is 9.8×10^7 divided by 60; or somewhat more than 1.6×10^6

erg-seconds. The activity called a horse-power is $7 \cdot 45 \times 10^9$ erg-seconds. For commercial purposes, a larger unit of activity is employed called a **watt**. The watt is defined as 10^7 erg-seconds; or, as 10^7 ergs is a joule, as a joule-second. Hence a horse-power is 745 watts, and a kilowatt is 1·34 horse-powers.

Since activity as above defined is fl/t, and since $l/t = s$, it is evident that activity may also be represented as fs, or as the product of force by speed.

97. Work done in overcoming Friction.—Since friction is a resistance, force is required to overcome it. And the product of this force into the distance through which it overcomes the friction measures the work done. If on applying a dynamometer to a tramcar, for example, it is found that 25 megadynes are required to keep it in motion, then the work done in moving the car over a kilometer will be $2 \cdot 5 \times 10^{12}$ ergs or 250 erg-tens. If this distance is traversed in seven minutes, the activity or rate of work will be about 0·6 erg-ten per second.

Section II.—Energy and its Transformations.

98. Definition of Energy.—In all the cases above mentioned, the raised or separated masses, in virtue of their position, i.e., of their condition with reference to other masses, have acquired an advantage which can be utilized to reproduce, at least in part, the work which has been expended upon them. Bodies upon which work has thus been done and which in consequence are themselves in a condition to do work, are said to possess energy. Hence energy may be defined as *a condition of matter in virtue of which the matter may be made to perform work* (5). Energy, like work, is measured in ergs.

Examples.—When water is raised to a definite height above the sea, either by natural or by artificial means, every one knows that it may be made to do work by allowing it to act upon a suitable water-wheel. This water consequently possesses energy. We commonly speak of storing **water** in reservoirs. But **energy** is also stored at the same time, the amount of which is exactly equal to the work done in raising the water. When a watch is wound up, work is done

in coiling the spring; and the energy thus stored up in this spring is gradually expended for the purpose of keeping the watch-train in motion.

99. Potential Energy.—The energy which a body possesses in virtue of its position has been called by Rankine **Potential Energy**. The work which is done upon a body in order to store up potential energy in it or in the system of which it forms a part, is work which is expended in separating attracting masses or material particles; as will be more fully discussed in the next chapter. And it is the force of attraction between these separated masses which brings them again together and reconverts the potential energy therein stored up into work.

EXAMPLES.—When the weight of a kilogram is raised through a meter, a kilogram-meter of work is done against the earth's attraction, and a kilogram-meter of potential energy is stored up. When an elastic cord is stretched, the force applied acts to overcome the molecular attraction of the material; and this force multiplied by the distance through which it acts represents the potential energy stored up in the extended cord. When carbon and oxygen are separated, it is chemical attraction which is overcome, and the potential energy of the carbon is a measure of the work done in effecting the separation.

100. Kinetic Energy.—When, however, force is applied to a mass of matter, under circumstances where no external resistance is to be overcome, the entire force is expended in generating acceleration in the body. Conversely, when, at the end of a given time, the body has acquired a definite amount of motion, it is found that the force necessary to bring it to rest in the same time is exactly equal to that which was required to generate the motion originally. Hence it is evident that energy may be stored in a body by communicating motion to it as truly as by moving it into a position of advantage against attraction; and therefore that motion is one of the conditions of matter above mentioned in virtue of which this matter may do work. The energy which a body possesses in virtue of its motion has been very appropriately called by Thomson **Kinetic Energy**.

EXAMPLES.—Thus, moving water may be employed to do work just as really as raised water. Windmills perform work in consequence of the kinetic energy which is contained in the air in motion. The work which a projectile can do in penetrating the target is a function of its speed ; as is also the work which is done by a hammer in driving a nail.

101. Measurement of Kinetic Energy.—Since in all cases the energy of a body or system of bodies is measured by the amount of work it can do, it is evident that the kinetic energy in a moving body may be determined in any case simply by ascertaining the height to which the body would itself be raised against the earth's attraction, by the expenditure of its own energy.

Thus let a body of weight w be moving vertically upward with a speed s. From the kinematical principles already discussed (33) we see that $l = s^2/2a$. Or in other words, that if a body be moving with a speed s, the height to which it will rise, and therefore the work which it will do in raising itself, is proportional to the square of s. If now the above equation be converted into a dynamical one by introducing into it the weight of the body, we shall have

$$wl = ws^2/2a. \qquad [22]$$

But wl represents the work done against the earth's attraction in raising the weight w to the height l ; and therefore also represents the potential energy stored in the body when at this height. To attain this height, however, the body has been projected upward with a speed s ; and in its course it has continuously lost kinetic energy and acquired potential energy. At its highest point it is at rest, and all its kinetic energy has become potential, represented in the equation by wl, the first member. But by the equation, the potential energy wl is equal to $ws^2/2a$. This therefore must represent the kinetic energy of the body originally when moving with the speed s. If m be substituted for its value w/a, in this expression, it becomes $\frac{1}{2}ms^2$. Whence it follows that the kinetic energy of a mass m moving with a speed s is proportional to half the product of the mass by the square of the speed. If the mass be expressed in grams and the speed in centimeters per second, the kinetic energy will be obtained in ergs.

102. Relation between Kinetic and Potential Energy.—The relation existing between the kinetic and the potential forms of energy may be illustrated in various ways. If, for example, the kinematical equation

$s^2 = 2al$ be combined with the kinetical equation $f = ma$, we obtain

$$\tfrac{1}{2}ms^2 = mal \quad \text{and} \quad \tfrac{1}{2}ms^2 = fl; \qquad [23]$$

this last equation expressing the fact that the kinetic energy of a mass m moving with a speed s is equal to the potential energy of the same mass under the action of a force f which would give it this speed in passing over a distance l. Hence the work which a body of mass m moving with a speed s will do is $\frac{1}{2}ms^2$. And the work which the same body will do in falling from a height l is mal. If, however, the body fall from the height l to the height l', how much work will it do? If it fell to the ground, its speed on striking would be in the first case $s^2 = 2al$, and in the second $s'^2 = 2al'$. Subtracting and dividing by two, $\frac{1}{2}(s^2 - s'^2) = a(l - l')$. But $l - l'$ is the distance through which the body falls. Calling it l'', and multiplying both sides by m, we obtain $\frac{1}{2}ms^2 - \frac{1}{2}ms'^2 = mal''$. From which it appears that the amount of work done by the body in its fall is equal to the increase of its kinetic energy during the interval.

103. Transference of Energy.—Let a body in motion strike another body at rest. A part of the motion possessed by the first body is transferred to the second; that of the first body being proportionally diminished. But the motions of the two bodies represent their kinetic energies. So that in this experiment, kinetic energy has been simply transferred from one body to another. In common language, the first body has exerted force upon the second. Moreover, other things being equal, the amount of force exerted is obviously the greater, the greater the space over which the bodies move while in contact. Hence force may be considered only as the rate at which energy is transferred with space.

But not only may kinetic energy be transferred from one body to another. Potential energy is equally capable of a similar transference. When water-power is employed to raise coal from the pit or ore from the mine, for

example, the potential energy of the water is transferred to the coal or the ore. The water falls through a certain height and loses potential energy proportional to the product of the weight of water and the height of fall. The coal or ore gains energy equal to the product of its weight and the height of rise. Since no transfer of energy is perfect, the energy lost by the water is always greater than that gained by the substance raised.

104. Transformations of Energy.—Again, the two forms of energy above mentioned are themselves capable of transformation, the one into the other. Whenever the fall of water from a reservoir moves a water-wheel and with it the machinery of a cotton-mill, for example, the potential energy of the water disappears in consequence of its fall, and the kinetic energy of the water-wheel and the shuttle appears in its place; and we say that the former is transformed into the latter. So whenever compressed air is employed to drive an engine, or a coiled spring to drive a train of wheel-work, energy of position is transformed into energy of motion. A ball suspended by a string and allowed freely to oscillate after the manner of a pendulum, is an excellent example of the continuous transformation of energy, and in both directions. When at the highest point of its swing, it is for an instant at rest, and all its energy is energy of position, or potential energy. But now it begins to fall, and as it does so it acquires the energy of motion; so that when it reaches its lowest point all its potential energy is lost and its kinetic energy is a maximum. As it rises on the other side, its motion diminishes and its elevation increases until it reaches the end of its swing in the opposite direction, when all its energy is again the energy of position. The total kinetic energy which it can acquire can never exceed its potential energy; and this, as we have already seen, is always proportional to the product of its mass into the vertical height through which it has been raised. Owing to the resistance of the air, some loss is experienced in each transformation; and so the oscillations finally cease.

We have already intimated that energy of position depends upon the presence of other bodies in the vicinity of the body in question, in virtue of an attraction taking place between them which is proportional to the masses of matter concerned. It may be said, perhaps more probably, to depend upon a condition of strain in the medium between the attracted masses, the two aspects of which are called stresses in the two directions, or, more commonly, force. Force, therefore, as viewed from this standpoint, is simply one aspect of a stress. This condition of things may be well illustrated by an elastic band stretched between two lead-pencils. There is a stress exerted between the pencils which is due to the strain in the rubber band. When one of these pencils only is considered, then we speak of the force acting upon it alone. But from the third law of motion, action and reaction, though oppositely directed, are equal. Hence the complete phenomenon can be studied only by taking both aspects of the stress into the account.

In general, therefore, whenever two attracting bodies are separated, work is done upon them, and energy is stored up, either in them or in the medium between them, to an extent precisely equal to the work done. Since this principle is independent of the size of the bodies separated, molecules or atoms may have potential energy, as well as visible masses. One gram of carbon possesses $3 \cdot 36 \times 10^{11}$ ergs of energy stored within it; one gram of hydrogen, $1 \cdot 43 \times 10^{12}$ ergs; one gram of zinc, $5 \cdot 46 \times 10^{10}$ ergs; one gram of alcohol, $2 \cdot 9 \times 10^{11}$ ergs. The attraction in these cases is atomic; the stored energy representing in all cases the work done in separating the atoms of these substances from the oxygen with which they were in combination. Conversely, when the atoms of these substances unite again with oxygen this potential energy is for the most part converted into kinetic energy. Coal, wood, gas, petroleum, and the like, are therefore to be looked upon simply as substances in which energy is stored; and this in a readily transportable form. It is this fact which gives them their commercial value. A kilogram of coal contains sufficient potential energy stored within it to raise a weight of 3000 kilograms to the height of 1000 meters. The energy of a kilogram of zinc would raise a similar weight to the height of 400 meters.

Moreover the same principles apply equally to the articles which we consume as food. One gram of beef contains 664 kilogram-meters of potential energy, one gram of bread 945, one gram of sugar 1418, and one gram of butter 3077 kilogram-meters. It is by the constant change of this potential energy of food into other forms of energy that the various phenomena of the animal organism are constantly maintained.

105. Energy of Rotation.—In the above discussion relating to kinetic energy, the motion has been supposed

to be one of translation only. But the same principles apply to motion of rotation. The kinetic energy of a material particle of mass m moving with a speed s in a curvilinear path is $\frac{1}{2}ms^2$, as we have already proved (101). If the path be a circular one, then, as stated in the chapter on Kinematics (43), the linear speed along the arc will vary precisely as the arc itself varies; i.e., will vary directly as the radius of the circle and as the angular velocity; hence $s \propto r\omega$. Consequently the energy of a rotating particle is represented by $\frac{1}{2}mr^2\omega^2$; an expression obtained simply by replacing s in the ordinary kinetic-energy equation by its value in terms of the angular velocity and the radius. For the entire body we shall have $\frac{1}{2}\Sigma mr^2\omega^2$; and since $\Sigma mr^2 = Mk^2 = I$, as stated (72) in Kinetics, the kinetic energy of a rotating body is represented by $\frac{1}{2}Mk^2\omega^2$, or by $\frac{1}{2}I\omega^2$; that is to say, by the product of half its moment of inertia and the square of its angular velocity.

106. Varieties of Energy. — From what has now been said, it will be evident (1) that all bodies which are in motion possess kinetic energy, and (2) that all bodies which are in a position of advantage with reference to the attraction of other bodies possess potential energy. But a somewhat more special classification of the different varieties of energy may be made. These are as follows:

I. Potential Energy.
   1. Strain, whether extension, compression, or distortion.
   2. Gravitative separation.
   3. Chemical separation.
   4. Electrical separation.
   5. Magnetic separation.

II. Kinetic Energy.
   1. Translatory or Rotatory motion.
   2. Vibration, including sound.
   3. Radiation, including light.
   4. Heat, both latent and sensible.
   5. Electricity in the form of current.

It therefore appears that all the phenomena with which physics can concern itself are phenomena which depend upon or are connected with the transference or the transformation of energy. Every change in matter of whatever nature which goes on upon the earth involves a simultaneous energy-change. Hence physics is properly defined as that branch of science which considers matter simply as the vehicle of energy and which concerns itself with the phenomena which attend the transference of energy from one body to another.

107. Conservation of Energy.—The close of the last century was made memorable in science by the discovery of the illustrious Lavoisier that matter is indestructible by human agency; and that consequently the amount of matter in the universe is constant (4). So the first half of the nineteenth century has been made equally memorable in science by Joule, whose investigations have established the same great law for energy. It is the present belief of science that energy is indestructible and unalterable in amount; that the quantity of energy in the universe has never been increased or diminished by any action of which we have any knowledge; and that while it is capable of transformation from one variety into another, the sum total remains absolutely constant. This is the law of the Conservation of Energy (6).

EXAMPLES.—As an illustration, consider the energy which comes to us in sunlight. Impinging upon our earth, it raises water to the sky to fall as rain; which rain, collected in reservoirs, becomes the stored energy of a water-supply. This sunlight falls upon the leaf and, under the marvellous chemistry there acting, effects chemical separations within its cells and so stores up energy there, to be set free, perchance directly as fuel or the food of man, or to remain locked up as coal in the interior of the earth, to be given out centuries later to drive the engines and to heat the houses of subsequent generations of men. But having loitered here a while, this energy is carefully collected and sent on its way into space; not a jot or tittle eventually remaining in a world to which it has given for the moment such unwonted activity.

CHAPTER IV.

ATTRACTION AND POTENTIAL.

Section I.—Universal Attraction.

108. Law of Gravitation.—It has been observed that under certain conditions bodies exhibit a tendency to approach one another; and that if free to move, a motion takes place between them and they come together. Such a tendency is called an **attraction**; and the force with which the bodies tend toward each other is called the **force of attraction**. A distinct and formal recognition of the fact that an attraction is exerted between all material bodies in virtue of their masses was first given by Newton in the *Principia*. From various parts of this great work Tait has collected the law according to which this attraction takes place; a law which has been called the **law of gravitation**. It is as follows:

"Every particle of matter in the universe attracts every other particle with a force whose direction is that of the line joining the two, and whose magnitude is directly as the product of their masses and inversely as the square of their distance from each other."

Common observation teaches us that all the phenomena about us which flow from such mutual relations of masses, are precisely such phenomena as would result from an attraction resident in the masses themselves. But a study of other forms of attraction has resulted

in concentrating the attention more closely upon the medium intervening between the two attracting bodies than upon the bodies themselves. All force, as we have learned, is of the nature of a stress. If, therefore, the medium between any two bodies is in a state of stress, one aspect of the stress is toward the one body, the other aspect is toward the other. Hence the two bodies tend to approach each other. "Such a stress," says Maxwell, "would no doubt account for the observed effects of gravitation." "But," he adds, "to account in this way for the actual effects of gravity at the surface of the earth would require a pressure of 37,000 tons weight on the square inch in a vertical direction, combined with a tension of the same numerical value in all horizontal directions. The state of stress, therefore, which we must suppose to exist in the invisible medium is 3000 times greater than that which the strongest steel could support." Hence, from all the facts in our possession, Tait concludes that all that we are entitled to say is: "That the part of the energy of a system of two particles of matter, of masses m and m', which depends upon their distance r from one another, is measured by $-mm'/r$; and this is not altered by the presence of other particles."

Wherever the seat of the energy, however, whether in the attracting masses themselves or in the surrounding medium, the general attraction which is exerted between masses of matter has received the name **gravitation**; while that exerted between the earth and bodies upon its surface is called **gravity**.

109. Experiment of Cavendish.—Direct experimental proof of the attraction exerted between masses of matter has been obtained by several observers. In 1798 Cavendish used a pair of small masses placed at the ends of a long and light rod suspended by a fine wire, and attracted by a pair of larger masses; the deflection increasing until the moment of torsion in the suspending wire is equal to that due to the attraction. In Fig. 45 mm' are the small masses, O the point of sus-

pension, and MM' the large masses. After the deflection had been noted in the position shown, the large masses were placed on the other side of the small ones, in the position shown by the dotted circles, the new deflection noted, and the mean taken. In the actual apparatus the rod was nearly 2 meters long, and the masses were balls of lead; the smaller ones about 5 centimeters

FIG. 45.

and the larger ones about 30 centimeters in diameter. In 1878 Cornu reduced this apparatus to one quarter of the above size; using an aluminum tube half a meter long as the beam, and masses of 250 grams only at its ends. He also used for the larger masses four fixed spherical vessels of iron, so connected that the two opposing pairs could be alternately filled with mercury. In 1889 Boys still farther reduced the dimensions of the apparatus; the little masses being cylinders of pure lead 3 mm. in diameter and 11·3 mm. long, placed at different levels, and attached to a glass tube so that their axes are 6·5 mm. from the axis of rotation. The suspending filament is a fine quartz fiber. The large masses are two cylinders of lead 50·8 mm. in diameter and of the same length, fastened to the inside of a large brass tube capable of rotation about the axis of suspension. The time of a complete oscillation of the torsion pendulum is 160 seconds, and the deflections are read by means of a mirror and scale. Under these conditions, the apparatus proved to be capable of indicating an attraction of $\frac{1}{200000}$ of a dyne; so that by its means the attraction between a pair of No. 5 shot, or even a pair of dust-shot, could be measured.

Evidently having obtained the deflection produced by

the attraction of known masses, we have only to multiply this deflection by the modulus of torsion (149) of the suspending fiber, in order to obtain the absolute attraction of these masses at the given distance; and from this the absolute attraction of unit mass at unit distance. This may be called the static unit of force; i.e., the force with which unit masses attract each other at unit distance. This is the ultimate object of the researches of Boys. The Cavendish experiment, however, had for its object a relative determination: the ratio between the attraction exerted by the leaden balls and that exerted by the earth. By the law of gravitation, mass-attraction f is proportional to mm'/r^2; or to m/r^2 when m' is unity. In the case of a sphere $f \propto \frac{4}{3}\pi r\rho$; in which r is the radius and ρ the density. For the leaden sphere and the earth $f : f' :: r\rho : r'\rho'$; whence $\rho' = f'r\rho/fr'$. Calling the density of lead 11·3, Cavendish obtained 5·48 and Cornu 5·50 as the value of the earth's mean density.

110. The Earth's Attraction Measured in Kinetic Units.—Like any other force, the attraction of gravity, being proportional to the amount of motion it produces in a unit of time, may also be measured in kinetic units. If a body be allowed to fall freely within so small a distance of the earth's surface that the attraction may be regarded as constant, we find that the acceleration produced is about 980 C. G. S. units in the latitude of Philadelphia. Since a dyne is that force which will generate one C. G. S. unit of acceleration in a C. G. S. unit mass, gravity, which will generate 980 units of acceleration in this unit mass, must be equivalent to 980 dynes. This acceleration of gravity is usually represented by g.

111. Relation of the Static Unit of Force to the Kinetic Unit.—The attraction exerted by the earth on a gram mass is m/r^2 static units. But the mass of the earth is $6·14 \times 10^{27}$ grams and its radius is $6·37 \times 10^{8}$ centimeters; and hence $f = 1·513 \times 10^{10}$ static C. G. S. units. Now the value of the force of gravity in kinetic units, i.e., in dynes, is 980. Hence 980 dynes =

$1 \cdot 513 \times 10^{11}$ static units; and one dyne is equal to $1 \cdot 544 \times 10^{7}$ static units.

112. Weight.—The attraction exerted by the earth upon a given mass is called the **weight** of that mass. Weight is therefore a force and is measured in units of force; i.e., in the C. G. S. system in dynes. As above stated, the force exerted by gravity upon a gram-mass is 980 dynes. But since the force exerted by gravity upon a gram-mass is the weight of that gram-mass, the weight of a gram-mass is 980 dynes. Conversely a dyne is $\frac{1}{980}$ of the weight of a gram-mass; or about 1·02 milligrams. In order to obtain the weight of a body in absolute units of force, therefore, we must multiply its mass in grams by the acceleration of gravity; i.e., $mg = w$. Conversely, $m = w/g$; or the mass of a body is the ratio of its weight, expressed in units of force, to the acceleration of gravity. Indeed all the kinematic and kinetic equations heretofore discussed may be made use of when gravity is the accelerating force, simply by replacing a or f, where necessary, by g.

113. Variation of Gravity on the Earth's Surface.—It will be shown farther on that a spherical mass of uniform density acts upon a particle outside the mass as if the entire mass of the attracting sphere were collected at its center. If the earth were a perfectly homogeneous sphere and at rest, then every part of its surface, being equally distant from its center, would have the same value for g. But the earth is a spheroid, and the diameter through the poles is about 43 kilometers less than the minimum equatorial diameter. Consequently in the equatorial regions, which are farthest from the center, g has a less value than in the polar regions. Moreover, in consequence of the earth's rotation, a stress is developed along the line joining a particle on the earth's surface with its center. The maximum acceleration away from the earth's center is at the equator, and is equal to about $\frac{1}{289}$ of the weight. And this is the amount by which the weight of a body at the equator is diminished by the earth's rotation. Since

the acceleration toward the center varies as the square of the speed of rotation, we see that if the rotation of the earth were 17 times more rapid, the acceleration would be 17^2 or 289 times as great; in which case bodies on the equator would have no weight at all. The actual speed at the equator being 456,510 cm. per second, the acceleration or $s^2/r = 3{\cdot}3908$ cm. per second. Moreover, the value of g varies not only with the latitude of the place of observation; it varies also with its height above the earth's surface. At the sea-level, the value of g at the equator is 978·1028 dynes, and at the pole 983·1084 dynes. The value of g at any place whose latitude is known is easily obtained approximately by the formula

$$g = 980{\cdot}6056 - 2{\cdot}5028 \cos 2\lambda - {\cdot}000003h, \qquad [24]$$

where 980·6056 is the value at latitude 45°, λ is the latitude, and h the height above the sea-level in centimeters. Owing to irregularities both in the form and the density of the earth, the above formula is an empirical one; i.e., its constants are deduced from experiment and not from theory. By its means the value of g has been calculated for the following places:

Place.	Latitude.	Length Sec. pendulum. cm.	Value of g. Dynes.
Hammerfest	70° 40′ N.	99·562	982·58
Berlin	52° 30′	99·422	981·24
Greenwich	51° 29′	99·413	981·17
Paris	48° 50′	99·390	980·93
New York	40° 43′	99·318	980·19
Washington	38° 54′	99·307	980·06
Ascension	7° 56′	99·112	978·17
St. Thomas	0° 25′	99·124	978·06

114. Methods of Measuring the Acceleration of Gravity.—I. The Direct Method.—The most simple method of measuring the acceleration produced by gravity would obviously be the direct one. Thus we might allow a body to fall from a known height and measure

the speed acquired by it during the time of its fall. Since we may regard the acceleration as constant for this distance, we have $a = s/t$; so that by dividing the speed acquired by the time of fall, the acceleration would be obtained. But it is a difficult experimental problem to measure accurately a speed as high as that produced by gravity. So we may make use of the fact proved in kinematics (33) that the final acceleration acquired by a body moving under the action of a constant force is twice the space passed over divided by the square of the time. Now inasmuch as it is quite easy to measure with considerable accuracy the distance through which a body falls, a fair value of g may be thus obtained. Since, however, the time enters into this expression as the square, any error in measuring it is greatly multiplied. Hence it is customary to note and record it electrically on a chronograph.

EXAMPLE.—Thus if a leaden ball be allowed to fall through a distance of 20 meters, and the time of fall be observed as 2 seconds, the value of g would be $2l/t^2$ or $(2 \times 2000)/4 = 1000$ centimeters; a value about 20 centimeters too great. If the observed time be 2·02 seconds, the value of g would be 978 centimeters per second.

115. The Atwood Machine.—From the kinetical equation $a = f/m$, it is evident that by varying the ratio of the weight to the mass to be moved, any acceleration desired may be obtained. In the machine devised by Atwood, a cord passing over a practically frictionless pulley carries at its ends two equal masses which also act as scale-pans. If these masses be represented by $2M$, and a mass m be added on one side, the whole mass to be moved will be $2M + m$, while the moving force is only the excess of weight or m. Hence the acceleration will be $m/(2M + m)$. Thus if M be ten grams and m one gram, the acceleration will be one twenty-first of that of free fall; i.e., will be only about 47 centimeters per second per second. As in this apparatus the acceleration of gravity is reduced without altering its law of action, it serves a very useful purpose for demonstrating the *principles* of accelerated motions in general.

116.—II. The Indirect Method.—The Pendulum.—A much more accurate and hence a much better method of measuring the value of the acceleration of gravity is the indirect method which is based upon the law of the pendulum. A simple pendulum, strictly speaking, is only a theoretical one. It consists of a heavy particle suspended from a point by a light inextensible cord. When at rest the pendulum hangs vertically. If drawn to one side and allowed to fall, it oscillates about its position of equilibrium. A single motion in one direction, either to or fro, is called a simple oscillation; a motion to and fro, a double or complete oscillation. Suppose now such a pendulum (Fig. 46) be displaced through an angle ϕ. If l be its length, the arc of displacement will be $l\phi$. Its weight mg acting vertically at the point M may be resolved into two rectangular components, one $mg \sin \phi$, which acts along MH, the tangent to the curve, and which is proportional to the acceleration; the other $mg \cos \alpha$, which acts simply to increase the tension on the string. If the pendulum swing through a very small arc, the angle ϕ will be very small and the sine may be regarded as practically coincident with the arc; so that $mg \sin \phi$ becomes $mg\phi$. Moreover, since $mg\phi = ma$, $g\phi$ represents the acceleration. Now it has been already shown (54) that in simple harmonic motion, the ratio of the acceleration to the displacement is equal to the square of the angular velocity. But the angular velocity is equal to $2\pi/T$. Hence we have

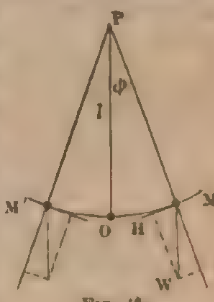


FIG 46.

$$\frac{\text{Acceleration}}{\text{Displacement}} = \frac{4\pi^2}{T^2}; \quad \text{whence} \quad g\phi = \frac{4\pi^2}{T^2} l\phi. \quad [25]$$

From which $g = \dfrac{4\pi^2 l}{T^2}$ and $T = 2\pi\sqrt{\dfrac{l}{g}}$.

Since T in this case is the time of a complete or double oscillation, the time t of a single oscillation will be one

half this; or $t = \pi\sqrt{l/g}$. This equation, which represents the law of the simple pendulum, shows that the time of vibration of such a pendulum is proportional directly to the square root of its length, and inversely to the square root of the acceleration of gravity. It will be noted that no term representing the amplitude of the oscillation appears in the above expression. But this is because of the assumption above made that the sine and the arc may be considered coincident. Isochronism in simple harmonic motion, as already stated (55), requires that the acceleration be proportional to the displacement. In the case of the pendulum the acceleration is $g \sin \phi$, while the displacement is $l\phi$; one being proportional to the sine of an arc, the other to the arc itself. Hence, strictly speaking, the oscillations of a pendulum are not isochronous, and the complete equation shows the time to be a function of the angle:

$$t = \pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin\left(\frac{\phi}{2}\right)^2 + \frac{9}{64}\sin\left(\frac{\phi}{2}\right)^4 + \text{etc.}\right). \quad [26]$$

117. The Compound Pendulum.—Every experimental pendulum is made up of a number of material particles, each at a different distance from the point of suspension, and each tending to oscillate, therefore, in a different time. The time of oscillation of an actual pendulum is the resultant of the action of all the simple pendulums which go to make it up. For this reason all actual pendulums are called compound pendulums. To illustrate this action, suppose the line AB (Fig. 47) to represent the axis of a compound pendulum, in which its entire mass is collected. A material particle at a forms a simple pendulum of length Aa; one at s, a simple pendulum of length As. Since the particles at a and b oscillate more rapidly than those at s and t, it is evident that the former particles tend to accelerate the latter, and the latter tend to retard the former. Obviously this accelerating action diminishes as we go downward, and the retarding

Fig. 47.

action as we go upward. So that at some point, say at O, these opposite actions balance. The length AO is the true length of the compound pendulum.

To obtain the value of this length by calculation we may proceed as follows: Suppose the masses at a and b to be m and m', and those at s and t to be m'' and m'''. Represent the distance of the particle m from the point of suspension (i.e., the distance Aa) by r, that of m' by r', that of m'' by r'', and that of m''' by r'''. If the angular velocity of the pendulum, i.e., the velocity at unit distance from A, be ω, the velocity at distance r will be $r\omega$, and the momentum of the mass m will be $mr\omega$. For an equal mass at O, calling l the distance AO, the momentum will be $ml\omega$. The difference $m(l-r)\omega$ is that part of the momentum which produces an accelerative effect. The moment of this momentum with respect to the center of suspension A will be $m(l-r)r\omega$. For the second accelerating particle at b, the moment of the momentum will be $m'(l-r')r'\omega$. The moments of the momenta of the retarding particles m'' and m''', since r'' and r''' are now greater than l, will be $m''(r''-l)r''\omega$ and $m'''(r'''-l)r'''\omega$. But these accelerating and retarding actions are balanced at O. Equating, therefore, the sum of the moments of the particles above O to those below O, we have

$$m(l-r)r\omega + m'(l-r')r'\omega + \text{etc.} = m''(r''-l)r''\omega + m'''(r'''-l)r'''\omega + \text{etc.}$$

$$\therefore l = \frac{mr^2 + m'r'^2 + m''r''^2 + m'''r'''^2 + \text{etc.}}{mr + m'r' + m''r'' + m'''r''' + \text{etc.}},$$

or, as it is ordinarily written,

$$l = \frac{\Sigma(mr^2)}{\Sigma(mr)}, \qquad [27]$$

in which Σ signifies the sum of these quantities.

But $\Sigma(mr^2)$, as we have already seen (72), is the moment of inertia of the entire pendulum about A. As $\Sigma m = M$, and as we may represent Σr^2 by k^2 (k being the radius of gyration or the distance from A at which the entire mass must be concentrated in order to have the same moment of inertia as the actual pendulum), we may represent $\Sigma(mr^2)$ by Mk^2 or by I. So also $\Sigma(mr)$, the static moment of the pendulum, is equal to MR; in which M is, as before, the entire mass, and R the distance AG from the center of suspension to the center of mass. Hence $l = \frac{Mk^2}{MR} = \frac{k^2}{R}$. Or the length of a compound pendulum is the quotient of the square of the radius of gyration divided by the distance between the center of suspension and the center of mass. Substi-

tuting this value of l in the above equation for the time of oscillation of the simple pendulum, we have

$$t = \pi\sqrt{\frac{k^2}{Rg}} = \pi\sqrt{\frac{Mk^2}{MRg}} = \pi\sqrt{\frac{I}{MRg}} \qquad [28]$$

as equivalent expressions for the time of oscillation of a compound pendulum.

118. Center of Oscillation.—The point indicated by O is therefore a point of importance in discussing the theory of the pendulum. It is called the **center of oscillation**, and is defined as that point on the axis at which if the entire mass of the pendulum were collected it would oscillate in the same time. It has the following properties: **1st, it is interchangeable with** the center of suspension; so that if, in the irregular mass shown in the figure (Fig. 48), A be the center of suspension and C the center of oscillation, the pendulum will oscillate in the same time about A as if inverted and oscillated about C. 2d, its distance from A is the length of the equivalent simple pendulum, and a simple pendulum of length AC will oscillate in the same time. **3d, if struck at a point opposite C, the** mass will oscillate about A without producing any pressure upon the axis through A. Hence the point C is sometimes called the **center of percussion.** 4th, if the body be supported otherwise than from A, as, for example, by floating in water, and the point C be struck, the point A will remain at rest, that part of the body below A moving in the direction of the blow and the part above it in the opposite direction. And 5th, its distance from A is equal to k^2/AB if suspended at A, and to k^2/BC if suspended at C; k being, as before, the radius of gyration and B being the center of mass.

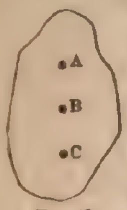


FIG. 48.

119. Energy of the Pendulum.—The formula of the compound pendulum may also be deduced from the energy-relations of its oscillations. We have seen (104) that when at its highest point all the energy of a pendulum is potential, and when at its lowest point all its

energy is kinetic. By obtaining expressions for these two values in terms of acceleration and time, and equating them, we may obtain the formula desired. The kinetic energy of a particle m moving with a speed s is $\frac{1}{2}ms^2$. But when the particle moves about a center, $s = r\omega$; whence the kinetic energy of such a particle is $\frac{1}{2}mr^2\omega^2$, and the energy of the whole body is $\frac{1}{2}\Sigma(mr^2)\omega^2$ or $\frac{1}{2}I\omega^2$. But $\omega = 2\pi\phi/T$,* and hence the kinetic energy in terms of time is $\frac{1}{2}I\frac{4\pi^2\phi^2}{T^2}$. The potential energy is always equal to the mean value of the force multiplied by the distance through which it acts. When the pendulum is at its highest point, the acceleration, as above, is $g\phi$; at its lowest point it is zero. $\frac{1}{2}Mg\phi$ is therefore the mean force. Since it acts through the distance $R\phi$, where R represents the length of the pendulum, $\frac{1}{2}Mg\phi \times R\phi$, or $\frac{1}{2}MRg\phi^2$, is the potential energy of this pendulum. Equating these values,

$$\frac{1}{2}Mk^2\frac{4\pi^2\phi^2}{T^2} = \frac{1}{2}MRg\phi^2; \qquad [29]$$

whence

$$g = \frac{4\pi^2k^2}{RT^2}, \quad T = 2\pi\sqrt{\frac{k^2}{Rg}}, \quad \text{and} \quad t = \pi\sqrt{\frac{k^2}{Rg}},$$

as before.

120. Determination of the Acceleration of Gravity by the Pendulum.—From the equations now given it is evident that by means of a pendulum, if we know its length, the value of g may be obtained, since $g = \pi^2l/t^2$. Two experimental methods of obtaining l have been employed. The one—first employed in 1790 by the French physicist Borda—consisted in making as close an approach as possible to the simple pendulum. For this

* In a circle of radius a, the speed s of a particle moving uniformly is $2\pi a/T$. But in the arc described by the pendulum $a = r\phi$, where r is the length of the pendulum. Hence $s = 2\pi r\phi/T$; and s/r, which equals angular velocity or ω, $= 2\pi\phi/T$, as above.

purpose he used a sphere of platinum supported at the end of a fine wire. The length of this pendulum was approximately the distance from the axis of suspension to the center of the sphere. The other method was employed in 1818 by Captain Kater of the British Navy. In this pendulum, which was a compound one, he availed himself of the principle of reversibility, already mentioned, which had been discovered by Huyghens in 1773. Kater's reversible pendulum consisted of a thin metallic rod of rectangular cross-section, having two steel knife-edges perpendicular to its plane; one of these being at a distance from the center of mass different from the radius of gyration, and the other at the distance approximately assigned to the axis of oscillation. A heavy slide, movable micrometrically along the rod, enabled the center of gravity of the rod to be slightly varied. By oscillating the pendulum first on one knife-edge and then on the other, and adjusting the slide until the time of vibration was the same for both, the distance between the two knife-edges when this point is reached was taken as the length of the equivalent simple pendulum.

In order to determine the time accurately, a method due to Mairan is generally employed, called **the method of coincidences.** The pendulum whose period of oscillation is to be determined is placed in front of another pendulum connected with a clock, whose rate is carefully observed. Attached to the clock-pendulum is a white surface having a vertical black line across its face, exactly behind the position of equilibrium of the experimental pendulum. This pendulum is set swinging, and the time of its coincidence with the black line on the clock-pendulum, when both are moving in the same direction, is carefully noted. At the next oscillation one of the pendulums has gained on the other and the coincidence ceases until one has gained a complete oscillation, when it is again re-established. If again the time be noted, the data are at hand for calculating the time of one oscillation of the experimental pendulum. If n be the number of seconds as given by the clock between

two successive coincidences, then in this time the experimental pendulum will make $n + 2$ or $n - 2$ swings, according as it oscillates faster or slower than the clock-pendulum. Consequently its oscillations are made in $n/(n + 2)$ or $n/(n - 2)$ seconds. Hence the method is not only extremely accurate, but it obviates the tediousness of constantly observing and counting the oscillations. For example, in determining the length of a pendulum beating seconds at Dunkerque in 1809, Biot and Mathieu observed that 7015·5 pendulum oscillations were made in the same time as 7017·5 clock oscillations. Assuming the clock to give seconds, the pendulum made one oscillation in 1·00026251 seconds. If an error of five entire seconds be made, then the time of one oscillation would be 1·00026253 seconds; a difference of only two hundred-millionths of a second.

121. Length of the Seconds-pendulum.—In consequence of the variation in the values of g at different points on the surface of the earth, the length of a pendulum beating seconds also varies. From the equation $t = \pi \sqrt{l/g}$ we have $l = gt^2/\pi^2$, and if t be one second, $l = g/\pi^2$. Hence the length of the seconds-pendulum varies directly as g, and is therefore greater toward the poles and less toward the equator. In the table on page 105 the length of the pendulum beating seconds is given for the places mentioned.

122. Time of Oscillation of a Pendulum independent of its Mass.—By the law of gravitation the attraction between two bodies is proportional to the product of their masses. Hence as the mass of a body increases, so does the force acting upon it, i.e., its weight; and therefore the acceleration, which is the ratio of force to mass, remains constant. It therefore follows, leaving the resistance of the air out of the account, that all bodies, whatever their mass, fall through the same distance under the action of gravity in the same time. Moreover, it will be observed that mass does not enter into the equation of the pendulum, $t = \pi \sqrt{k^2/Rg}$; and this for the same reason. Hence the time of oscillation of

a pendulum is independent of the mass assumed to be concentrated at its center of oscillation. Newton made use of this fact in order to compare the masses of different kinds of matter. He constructed a number of pendulums having light hollow cylinders for their bobs, which in different experiments could be filled with different kinds of matter. One of these cylinders he filled with wood, and as nearly as possible in the center of oscillation of another he placed an equal weight of gold. Suspending both cylinders by wires eleven feet long, thus forming two pendulums equal so far as length, weight, and figure were concerned, he found that the two oscillated in precisely the same time. He then used other substances, such as silver, lead, glass, sand, salt, wood, water, corn, and with precisely the same results. Hence he concludes that this "appears a method both of comparing bodies one among another as to the quantity of matter in each, and of comparing the weights of the same body in different places, to know the variation of its gravity. And by experiments made with the greatest accuracy I have always found the quantity of matter in bodies to be proportional to their weight."

123. Use of the Pendulum as a Measurer of Time.—"Absolute, true, and mathematical time," says Maxwell, "is conceived by Newton as flowing at a constant rate unaffected by the speed or slowness of the motions of material things." To divide up this uniform flow of time into equal parts, and so to indicate these parts that they can be made use of in observing the rate of change of natural phenomena, is the purpose of time-keepers. In early times the flow of water was supposed sufficiently uniform for the practical purposes of life; and clepsydræ or water-clocks were constructed which operated upon this principle. The use of the pendulum to regulate to uniform motion a train of clock-work depends upon an observation due, it is said, to Galileo. At the age of eighteen he chanced to observe a lamp suspended from a long cord in the cathedral at Pisa, as it slowly oscillated to and fro. He noted the variation in the time of

oscillation with the length, and concluded that when the amplitude of oscillation was small these oscillations were performed in sensibly equal times. It is to Huyghens, however, in 1656, that we owe the first actual application of the pendulum to control the escapement of a clock. At each oscillation a single tooth on the escapement-wheel is liberated; and as these oscillations are performed in equal times, the successive teeth are liberated uniformly, and the motion of the hands is also uniform. As has been stated, absolute uniformity in the rate of change of any material phenomenon is yet unknown to science. Thus the adopted standard of time is the mean solar day—the mean interval between successive passages of the sun across a given meridian; or, still better, the sidereal day—the time of the earth's rotation as measured by successive star-transits. "But," says Thomson, "the ultimate standard of accurate chronometry must (if the human race live on the earth for a few million years) be founded on the physical properties of some body of more constant character than the earth." Because, according to Adams, the earth, regarded as a time-keeper, would in a century get 22 seconds behind a perfect clock rated at the beginning of the century. Among the various more constant standards of time which have been proposed are: 1st, the period of vibration of a piece of quartz crystal of specified shape and size, and at a stated temperature. 2d, the period of vibration of a natural standard piece of matter, such as an atom of hydrogen or of sodium which is absolutely independent of its position in the universe. 3d, the time of revolution of an infinitesimal satellite close to the surface of a globe of water at standard density, since it is independent of the size of the globe. And 4th, the time of the gravest simple harmonic infinitesimal vibration of a globe of water at standard density.

124. Attraction in Special Cases.—We have seen (108) that, according to the law of universal attraction, if the masses of two material particles be represented by m and m', and if the distance separating them be r, they

will tend to approach each other with a force expressed by mm'/r^2. And further, that if m and m' be made unity, and if at the same time the distance be made unity, then obviously the attracting force will also be unity. Evidently the unit of force here employed is a static unit, and represents the force with which two unit masses attract each other at unit distance. In the C. G. S. system this unit of force will represent the attraction between two gram-masses placed one centimeter apart. This static unit of force, as already stated (111), corresponds to about one fifteenth of a microdyne.

The case of the attraction mutually exerted between a collection of particles forming a mass of matter is much more complex, since it depends also upon the mutual distribution of these particles. The problem is best attacked by assuming the existence of an independent attraction between each pair of particles, and then adding these attractions together to obtain that of the entire mass. In illustration of this method we may give the two following theorems, which are of great theoretical importance:

1st. *A spherical shell of uniform attracting matter exerts no attraction upon a particle within it.*

Let the point P within the shell (Fig. 49) be the vertex of a double cone of very small angle. The bases of these cones, cut from the spherical shell, will be the attracting masses. If m and m' represent these masses, and d, d' their distances from P, respectively, the attraction in one direction will be m/d^2 and in the other m'/d'^2.

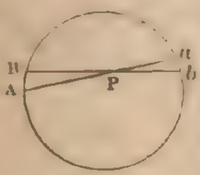


FIG. 49.

But the masses of these segments, since the density is uniform, will be proportional to their volumes; or, since the thickness is the same, to their areas. And the areas are proportional to the squares of their diameters, or as $ab^2 : AB^2$. Hence the attractions are ab^2/Pa^2 and AB^2/PA^2, since the bases are equally inclined to the axis of the double cone. Now geometry teaches us that the products of the two segments of intersecting chords

are equal to each other; i.e., $Pa \times PA = Pb \times PB$; or $Pa : Pb :: PB : PA$. Hence the triangles aPb and APB are similar; and $Pa : ab :: PA : AB$. Squaring, we have $Pa^2 : ab^2 :: PA^2 : AB^2$; or $ab^2/Pa^2 = AB^2/PA^2$. Consequently $m/d^2 = m'/d'^2$, and the attractions are equal and opposite. Hence there is no resultant attraction at the point P, and a particle placed there is in equilibrium. Since the entire shell may be divided into pairs of segments in this way, each of which pairs mutually balance, the proposition is proved. Moreover, the theorem is equally true whatever the thickness of the shell. It is also true when the shell is made up of concentric layers of different densities, provided that the density of each layer is uniform.

2d. *A spherical shell of uniform attracting matter attracts an external particle as if its entire mass were collected at the center of the sphere.*

Let the external particle be at P (Fig. 50), and draw the line PO to the center of the sphere O, cutting the surface at B. On this line lay off a distance OC, such that $OP : OB :: OB : OC$. Suppose the entire surface of the sphere divided into pairs of opposite segments

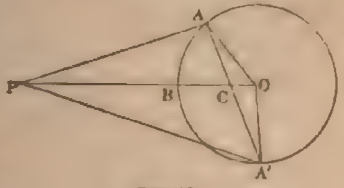


FIG. 50.

with reference to the point C, one of these pairs being at A, A', at the extremities of a chord. Now the volumes of the segments at A, A' are proportional to their areas, since the thickness of the shell is uniform. The area of an oblique section of a small cone is the quotient of the perpendicular section divided by the cosine of the obliquity; and the area of the perpendicular section is the product of the square of the distance from the vertex by the solid angle of the cone. Calling this solid angle ϕ, the area of the oblique segment A is $\dfrac{\phi CA^2}{\cos OAC}$; and of A', $\dfrac{\phi CA'^2}{\cos OA'C}$. If the surface-density of the matter be σ, the masses of A and

A' will be $\frac{\sigma\phi CA^2}{\cos OAC}$ and $\frac{\sigma\phi CA'^2}{\cos OA'C}$, and the attractions will be $\frac{\sigma\phi}{\cos OAC} \cdot \frac{CA^2}{PA^2}$, and $\frac{\sigma\phi}{\cos OA'C} \cdot \frac{CA'^2}{PA'^2}$, respectively. Since the triangles POA, AOC have the angle at C common and the sides about this angle proportional (i.e., $PO : OA :: OA : OC$ by construction), these triangles are similar, the angles OPA and OAC are equal, and $\frac{CA}{AP} = \frac{OA}{OP} = \frac{r}{OP}$. In the same way, by taking the triangles POA', $A'OC$, the angles OPA' and $OA'C$ may be shown to be equal. Hence $\frac{CA'}{A'P} = \frac{OA'}{OP} = \frac{r}{OP}$, and the attractions above given become $\frac{\sigma\phi}{\cos OAC} \cdot \frac{r^2}{OP^2}$ and $\frac{\sigma\phi}{\cos OA'C} \cdot \frac{r^2}{OP^2}$. But the triangle AOA' is isosceles, and hence these two values are equal. Moreover, for the same reason, the angles OPA, OPA', proved above to be equal to OAC, $OA'C$, respectively, are equal to each other. Hence the resultant attraction is the same for the two elements A, A', being $2\sigma\phi \frac{r^2}{OP^2}$, for each; and its direction therefore bisects the angle APA'. The total attraction at P will evidently be the sum of the actions along PO of all the pairs of elements. And since the sum of all the solid angles about a point is 4π, the total attraction will be $\frac{4\pi\sigma r^2}{OP^2}$. But the numerator represents the mass of the shell; being the product of the surface-density by the area of the spherical surface. Hence the attraction at P is the same as that of a mass equal to that of the shell, concentrated at O; which was to be proved.

In like manner it may be proved that a solid sphere will act upon an external particle as if all the mass of the sphere were collected at its center. This mass will be $\frac{4}{3}\pi r^3\rho$, in which ρ is the volume-density. Hence the

attraction on a unit mass at the distance a will be $\frac{4\pi\rho r^3}{3a^2}$. If the particle be just on the surface, $a = r$, and the attraction is $\frac{4}{3}\pi\rho r$ for the solid sphere and $4\pi\sigma$ for the spherical shell. On a solid sphere, therefore, the attraction varies directly as the radius; and hence gravity diminishes uniformly as we descend toward the earth's center. The center of figure, alike of the solid sphere and of the spherical shell, is therefore a true center of mass or center of gravity, in so far as concerns external bodies; the direction of the attraction passing through this point.

The attraction exerted by the arc of a circle of radius r upon a unit particle at its center is equal to $r\sigma/r^2$ multiplied by twice the sine of half the subtended angle; or to $2\sigma \sin \frac{1}{2}\theta/r$. This is the same as the attraction of a mass equal to that of a chord of the arc, and having the same density, concentrated at the mid-point of the arc. If the arc be a semi-circle, the attraction is $2\sigma/r$. A right line exerts upon a unit particle opposite its center the same attraction as a circular arc drawn with the particle as its center and subtending the same angle. The value of this has just been given. If the line be indefinite in extent and the particle be at a distance r, the attraction is $2\sigma/r$ as above. Since the attraction of a hemispherical shell upon a unit particle at its center is $2\pi\sigma$ and is independent of the radius, the attraction of an indefinite plane upon a particle at a finite distance from it must also be $2\pi\sigma$ and be independent of the distance of the particle. This attraction changes sign as the particle passes through the plane, becoming $-2\pi\sigma$ on the other side. Hence the total change from one side to the other is $4\pi\sigma$. If we make the distance zero, the unit particle is now a particle in the plane; and since the surface-density is σ, the total attraction is $2\pi\sigma^2$ per square centimeter. Its direction is perpendicular to the surface.

SECTION II.—POTENTIAL.

125. Attraction and Potential Energy.—Potential energy has already been defined as energy of position. We have seen (99) that whenever two attracting bodies are separated, work is done upon them and energy to a corresponding extent, is stored up in them. Hence energy of position is simply energy stored up in a system of mutually attracting bodies. As these bodies are separated farther and farther, more and more work is done upon them and more and more potential energy is stored up in the system; until, finally, the distance between them approaches infinity, and the potential energy reaches its maximum. On the other hand, as the attracting bodies approach each other, their potential energy diminishes and becomes zero when they are in contact.

It is not difficult to calculate the amount of work which is done in separating two attracting bodies to a given distance. Calling the initial distance of the bodies r, and the final distance R, the distance traversed is $(R-r)$. Now the work done is equal to the product of the mean force through this distance. The initial attraction being mm'/r^2, and the final attraction mm'/R^2, the mean attraction (geometrical mean) is mm'/Rr, and the product of this by $(R-r)$ is $\frac{mm'(R-r)}{Rr}$; which is simply $mm'\left(\frac{1}{r}-\frac{1}{R}\right)$, the work done. Thus, for example, if the final distance be twice the initial, R becomes $2r$, and the work done is $mm'/2r$. If the final distance be infinite, since $1/\infty = 0$, the work done is mm'/r, or double the former value. Hence it follows that if a certain amount of work be done upon a system of two attracting bodies in order to separate them to twice their initial distance from each other, exactly twice this amount of work, and no more, will be necessary in order to separate them to an infinite distance.

126. Attraction and Repulsion.—Hitherto we have supposed that the conditions between two bodies are such as to produce a tendency in them to approach each other; and we have assumed that the stress in the intervening medium consequently is a tension. But obviously, under suitable conditions, bodies may tend to separate from each other, the medium between them in this case being in a state of compression, and exerting an outward pressure upon the bodies. In the former case there is **attraction**, and the bodies are said to attract each other. In the latter there is **repulsion**, and the bodies repel each other. In both cases, however, work may be done upon the system comprising the two bodies; this work being done whenever the force so acts as to overcome the resistance which the system offers to a change in its configuration. In a system of attracting particles, work is done upon the system when force is employed to separate them. In a system of repelling particles, work is done upon the system when the particles are forced closer together. So, conversely, the system itself does work, in the former case when the particles approach each other, in the latter case when they recede from each other. But since, when work is done upon a system, energy is stored up within it, the potential energy in the case of the attracting system reaches its maximum at the maximum limit of the distance; and in the case of the repelling system, its maximum limit is reached when the bodies composing it are in contact.

When the effect of a force is to separate the bodies of a system and thus to increase the distance between them the force is called **positive**. In the contrary case it is said to be **negative**. In accordance with this usage, the force of repulsion, as above defined, is positive and the force of attraction is negative.

In proportion as two bodies which repel each other are allowed to separate, the potential energy contained in the system becomes exhausted. At the distance r, for example, the unexhausted energy is mm'/r. At an infinite distance r becomes ∞ and the potential energy of

the system is zero; i.e., it is entirely exhausted. Obviously, as long as the system possesses potential energy, so long will its particles continue to repel each other and so long will the system be capable of doing work. The potential energy of any system, therefore, always tends to diminish, to become exhausted.

127. Potential at a Point.—It will now be evident that if a material particle be placed at any point in space, its potential energy, if unexhausted, will tend to diminish. And, consequently, that the particle will tend to move from this point to another where its potential energy will be less than before. At a point where it possesses no remaining potential energy, the particle would evidently experience no impelling force whatever in any direction. Hence it follows that the potential energy of a unit mass placed at a given point, measured either by the work which has been done upon it to bring it from an infinite distance to that point, or by the work which it can itself do in returning to its initial position, is an attribute of the given point. It is called the **Potential at the point.** This potential may be either high or low. When the action between the parts of a system is one of repulsion, the particles tend to move from places of higher to places of lower potential; and if they do so move, they will do work. The potential at a point is said to be zero when a unit particle placed there has no potential energy. This is the case when the point is at an infinite distance from all acting masses.

It will be observed that the direction of the force acting on a particle at any point where the potential is not zero is directly opposite to the direction along which the potential increases most rapidly. The value of the force at any such point is simply the rate at which the potential diminishes with the distance. Moreover, the whole work done by the particle in traversing a given distance is measured by the total fall of potential through this distance.

The student should be careful to note the distinction between the potential at a point in space and the poten-

tial energy of a mass placed at that point. The former is only a condition due to the existence of neighboring bodies, defined simply by the fact that if a unit particle were placed there, it would require the expenditure upon it of a definite number of units of work to carry it from this point to an infinite distance; the work being performed either by or against the forces of the system. But this condition, numerically expressible and generally represented by V units of work or ergs, is clearly entirely independent of the fact whether there is or is not an actual unit particle at the point. The potential at a point situated at a distance r from a mass m is given by the equation $V = m/r$.

128. Gravitation-potential.—Since, as above stated, an attracting force is negative, the work done by such a force will be negative and the potential at a point due to an attracting mass will also be negative. Since the work done by the attractive forces is done in approximating the bodies of the system, the work which is done by these forces in separating these bodies is clearly negative. In order, however, to avoid defining the potential due to gravitation as a negative quantity it has been found convenient to change its sign. The gravitation-potential at any point due to any mass is defined, therefore, simply as the amount of work required to remove a unit mass from that point to an infinite distance.

EXAMPLES. Suppose a gram-mass to be placed upon the surface of the earth. Since the earth's radius is $6{\cdot}37 \times 10^8$ centimeters, it would require the expenditure of $6{\cdot}37 \times 10^8$ centimeter-grams of work, done against gravitative attraction, to remove this gram-mass to an infinite distance from the earth. Hence the gravitation-potential at the earth's surface, being the amount of work required to move a unit mass from this surface to an infinite distance, is $-6{\cdot}37 \times 10^8$ gram-centimeters or $-6{\cdot}37 \times 10^8 \times 980$ ergs, a negative quantity. But it has been agreed to call the potential at the earth's surface zero. Hence a mass on this surface possesses no potential energy. Since to remove a gram-mass from the earth's surface to an infinite distance would require the expenditure of $6{\cdot}37 \times 10^8 \times 980$ ergs of work, which work would increase the potential energy of the system by that amount, it is clear that the gravitation-potential at a point at an *infinite distance from the earth* is $6{\cdot}37 \times 10^8 \times 980$ ergs.

129. Difference of Potential.—From the definition of potential above given, it follows that if V be the potential at a point A, and V_1 the potential at a point B, the work done in carrying a unit mass from A to B is $V - V_1$. Hence the work done in carrying a unit mass from one point to another is measured by the difference of potential between these points. Moreover, this result is entirely independent of the path along which the transfer is effected.

It is frequently convenient to assume arbitrarily a zero value for the potential at a given point in the vicinity of acting masses. When this is done, points at a higher potential relative to this zero point are said to have a **positive** potential, and those at a lower potential are said to have a **negative** potential. Obviously, a particle passing from a region of higher to one of lower potential, in this sense, i.e., from a region where the potential is positive to one where it is negative, must pass through a point at zero potential. In other words, its positive potential energy gradually diminishes, becomes exhausted, and then the particle acquires a negative potential energy.

The conception of potential may perhaps be assisted by the analogy of height above a given surface, say the level of the sea. To raise a given mass to a given height above the sea-level requires a definite expenditure of work which is stored up as potential energy in the system. It is in virtue of this energy that gravitation can do an equal amount of work in bringing it down again. The potential energy stored up is proportional to its height. At the sea-level it is zero; and below this level it is negative. Hence the gravitation-potential changes sign as it passes through the zero-point.

It would certainly be possible, though not convenient, to estimate distance above or below the sea-level in terms of gravitation-potential. To say that a given point has a gravitation-potential of 100,000 gram-centimeters might easily be understood to mean that a gram-mass *there placed* would have this amount of potential energy.

in which case the height would be 100,000 centimeters or one kilometer.

130. Equipotential Surfaces.—Equipotential surfaces are surfaces at every point of which the potential has the same value. The potential due to unit mass is the same at the same distance in all directions about this mass. Hence the surface of an enveloping sphere having the unit mass at its center is an equipotential surface. Since such spheres may be constructed indefinitely about the given mass, an indefinite number of concentric equipotential surfaces may be imagined, on each of which the potential has a value numerically represented by the reciprocal of its radius. Evidently these equipotential surfaces may be so placed that one unit of work shall be done in transferring a unit mass from one such surface to the next. Moving a mass along an equipotential surface or parallel to such a surface does not involve any expenditure of work; since it is only when the potential changes that work is done. If, however, a unit mass be moved from one of the equipotential surfaces above supposed to the next, a unit of work will be done. And if m units of mass be so moved, m units of work will be done. If the m units of mass be moved across three of the spaces separating the equipotential surfaces, the work done will be $3m$ units. If it be moved from an equipotential surface of potential V to another of potential V_1, the work done would be simply $m(V_1 - V)$ units.

The amount of work done in all these cases is, as has been already mentioned, entirely independent of the path by which the transfer is effected. It is also independent of the particular points on the given equipotential surfaces from which or to which the unit mass is moved. If this were not so, then it would be possible to carry the mass in one direction by a path requiring less work than the mass would do in returning by the other; in which case there would be an absolute creation of work out of nothing. But this, by the law of the Con-

servation of Energy, is impossible. Hence the work done is independent of the path of transfer.

131. Lines and Tubes of Force.—Since the equipotential spherical surfaces are normal to their radii, along which the force of attraction or repulsion acts, it is evident, 1st, that no component of this force can act along these surfaces, and, 2d, that a free mass placed on such a surface will tend to move only in a direction at right angles to this surface. Such lines, drawn always perpendicular to equipotential surfaces, are called **lines of force**. The tangent to such a line at any point is the direction of the acting force at that point. If the form of the equipotential surface be a simple one, as in the case of the sphere, the lines of force are simply the radii of the sphere. But if this surface be complex, the lines of force, always perpendicular to it, will be correspondingly complex. Just as the space in the vicinity of acting matter may be mapped out in one direction by equipotential surfaces, so it may be divided in a direction perpendicular to this by lines of force, representing the direction in which a free mass would tend to move: i.e., the direction in which the potential diminishes most rapidly.

Thus, for example, the contour lines in a plane-table survey represent differences of level. Where these lines are closest, the slope in a direction perpendicular to them is the greatest. In other words, the fall of the surface is most rapid at these points, and a body free to move would move along these perpendicular lines from points of higher to points of lower level. The analogy of these contour lines and lines of fall to equipotential surfaces and lines of force is quite complete.

If a small portion of an equipotential surface be taken and lines of force be drawn through each point of its outline, these lines, each of them perpendicular, of course, to the surface, will form an enclosing tube. Such a tube is called a **tube of force**. If this tube of force extend through two consecutive equipotential surfaces, the areas thus cut out of these surfaces measure the variation of the acting force from one of them to the other. *In other words*, the force per unit of area at the first

surface is to the force per unit of area at the second as the area at the second surface is to the area at the first. Hence the product of either one of the areas by the force per unit of area is constant. That this must be so appears from the fact that the given amount of force enclosed by the tube is distributed over different areas in the two cases; and hence the force per unit of area must be inversely as the areas themselves. Unit tubes of force contain but a single line of force. And the relative value of the acting forces at any two points may be obtained by noting the relative number of lines of force which traverse unit area of the equipotential surfaces passing through the points.

132. Field of Force.—The region throughout which the action of an attracting or repelling force extends is called a **field of force**. Such a field is traversed by lines of force passing from points of higher to points of lower potential, and by equipotential surfaces cut perpendicularly by these lines of force. The number of lines of force in the field is so drawn that one such line passes through each unit of area of an equipotential surface at a point where the force has unit value. The equipotential surfaces are so placed that a unit of work will be done in moving unit mass from one of these surfaces to the next. By this means (1) the potential at any point in the field of force may be fixed by ascertaining on what equipotential surface the point is found; (2) the mean force along a line of force of unit length cutting any number of equipotential surfaces is equal to the number of surfaces thus cut. If n be the number of surfaces simultaneously cut, then $V = n$. But $V = Fl$, the work done between these surfaces; and since $l = 1$, $n = F$ as stated; and (3) the force at any part of the field may be determined by the closeness of the lines of force; i.e., by the number which cross unit area of a given equipotential surface. If a field of force be uniform, its lines are parallel and its surfaces plane.

133. Gauss's Theorem.—If there be any continuous distribution *of matter*, partly within

and partly without a closed surface, the number of lines or unit tubes of force traversing the surface is numerically equal to $4\pi M$, where M is the amount of matter within the surface.

Suppose, in the first place, a particle of mass m at the center of a spherical surface. The force which is exerted per unit of area is m/r^2; and since the entire area is $4\pi r^2$, the total force exerted is $m/r^2 \times 4\pi r^2$, or $4\pi m$; which is evidently the number of lines of force, i.e., the flow of force, traversing the surface.

In the next place, let the particle m be at any point within a closed surface of any form. Take any small area of the surface, subtending at the particle the angle ω, the normal to which is inclined at an angle ϵ to a line drawn from the particle to the center of the area. The area is equal to $\dfrac{r^2\omega}{\cos\epsilon}$, where r is the mean distance of the particle. The normal force per unit of area is $\dfrac{m\cos\epsilon}{r^2}$. The product $\dfrac{m\cos\epsilon}{r^2} \times \dfrac{r^2\omega}{\cos\epsilon}$ or $m\omega$ represents the number of lines of force, i.e., the flow of force, passing through the small area of the surface. If the particle is completely surrounded by the surface, the solid angle subtended by the whole surface is 4π; and the flow of force due to this particle is $4\pi m$. Or, if the total mass within the surface is M, the flow of force is $4\pi M$.

CHAPTER V.

PROPERTIES OF MATTER.

SECTION I.—GENERAL PROPERTIES.

134. Definition of Matter.—Matter has already been defined (2) as that which can occupy space. It has also been variously defined as that which possesses inertia, that which requires the expenditure of work to put it in motion, that which, in virtue of its motion, possesses energy, and that which is the receptacle or vehicle of energy. As the result of modern investigation, it is believed that matter is absolutely unalterable in quantity by any agency at the command of man. This great principle, which has been called the law of the Conservation of Matter (4), lies at the basis of chemistry and demands absolute equality in mass on the two sides of all chemical equations.

135. Structure of Matter.—Hypotheses almost without number have been made as to the nature of matter. "But," says Tait, "an exact or adequate conception of matter itself, could we obtain it, would almost certainly be something extremely unlike any conception of it which our senses and our reason will ever enable us to form." Consequently in his opinion, "the discovery of the ultimate nature of matter is probably beyond the range of human intelligence."

On the question of the ultimate structure of matter, however, modern physical investigation has thrown some light. Two opposite theories of the constitution of matter had *already been advanced* by the ancients. The

one, that of Lucretius, supposed matter made up of indivisible, hard atoms, infinite in number, of different shapes and in rapid motion. The other, that of Anaxagoras, maintained the homogeneity and continuity of matter, and asserted that matter is infinitely divisible. Neither of these theories, at least in its entirety, is accepted by modern science. Indeed, "the theory that bodies apparently homogeneous and continuous are so in reality," says Maxwell, "is, in its extreme form, a theory incapable of development. To explain the properties of any substance by this theory is impossible." The more common view at present, apparently, regards matter as continuous and compressible, but intensely heterogeneous (Tait). And Thomson has shown that on this assumption it is possible to account by gravitation alone for most of the phenomena now attributed to molecular action.

Another hypothesis of the constitution of matter has recently attracted attention; namely, the hypothesis of vortex atoms. It is based on a remarkable investigation by von Helmholtz upon vortex motion; in which he proved that the rotating portions of a perfect, incompressible fluid maintain their identity forever, being necessarily arranged in continuous endless filaments, forming closed curves. Such vortex filaments are true atoms since they cannot be divided; but they are not hard like the atoms of Lucretius. Thomson was led by these results to offer the theory that matter, as it appears to us, is merely the rotating portions of the medium which fills all space.

EXPERIMENT.—The nearest approach to a vortex filament which can be obtained experimentally is a smoke-ring. If a circular hole be made in one side of a paper box, for example, and the box be filled in any way with smoke, then on tapping sharply the side opposite the opening, a visible vortex ring is projected from it. By using two boxes, the mutual collision of two such rings may be shown. They rebound after impinging on each other, and vibrate to and fro as if they were rings of india rubber.

136. Subdivisions of Matter.—Matter, like every other physical quantity, can be measured; and since

quantity of matter is called **mass**, matter is measured in units of mass. It is sometimes convenient to use the word **mass** in a more restricted sense, as referring to a definite portion of matter; as when we speak of a mass of iron or of granite. Any portion of matter appreciable to the senses may be called, in this sense, a mass of matter. And therefore it is correct to speak of the mass of a mass of lead.

All masses are capable of subdivision, although not indefinitely. A limit is ultimately reached in the process beyond which the identity of the substance is lost. The limiting particle thus reached is called a **molecule.** So that a molecule may be defined as the smallest quantity of a substance which can exhibit the properties by which that substance is identified.

ILLUSTRATION.—Thus, for example, if a mass of salt be ground in a mortar, the smallest particle into which it can be divided by this mechanical process is still visible under the microscope and remains a mass. But if the salt be now dissolved in water, its particles, as such, are no longer visible, though the solution retains the taste and other properties of the salt. Hence the particles into which the salt is now divided are called molecules.

The science of chemistry considers it necessary, in order to explain the phenomena which it investigates, to assume a still further subdivision of matter. The molecule, which is the physical unit, is assumed by the chemist to be also capable of division, this division resulting in a complete change of properties in the substance operated on and two or more different kinds of matter being produced thereby. These still smaller particles which are reached by the subdivision of molecules are called **atoms.** And an atom is defined as the smallest quantity of simple matter which can enter into the composition of the molecule.

ILLUSTRATION.—Thus, for example, the molecule of salt above mentioned when subjected to chemical processes yields two distinct kinds of matter, sodium and chlorine, both entirely different in properties from the salt itself. Hence the salt molecule is said to be made up of sodium atoms and of *chlorine atoms.*

137. Homogeneous and Heterogeneous Matter.—Masses are assumed to be made up of molecules, just as molecules are made up of atoms. If the molecules composing any mass are all of the same kind, the mass is said to be **homogeneous**; if they are of different kinds, the mass is called **heterogeneous**.

138. Simple and Compound Matter.—The chemist makes a similar classification of matter, based on its atomic constitution. Matter whose molecules are made up of like atoms is called **simple** matter; while if it be made up of atoms of different kinds it is called **compound** matter. By combining the two classifications we see that matter may be divided into homogeneous simple, homogeneous compound, heterogeneous simple, and heterogeneous compound.

ILLUSTRATION.—For example, according to this classification, iron, sulphur, oxygen, would be simple homogeneous forms of matter; salt, water, quartz, would be compound homogeneous, brass and air, simple heterogeneous, and glass, gunpowder, and granite, compound heterogeneous substances.

139. Elementary Matter.—Since masses are made up of molecules, the number of heterogeneous substances would seem to be unlimited. The number of homogeneous substances, however, since each has a molecule peculiar to itself, is limited by the number of such molecules. Moreover, if we restrict ourselves to simple molecules, then the number of such molecules is limited by the number of distinct kinds of atoms in existence. Thus far the chemist has succeeded in establishing the existence of only about seventy different kinds of homogeneous simple matter. These substances he calls **elements**. Only elements can have atoms, and each element has its own peculiar atom. Hence there are as many kinds of atoms as there are elements; i.e., about seventy.

In the present philosophy of chemistry it is supposed that in general an atom cannot have a free and separate existence, but that in order so to exist it must combine with another atom either like or unlike itself, and thus

form a molecule. So that whether the matter be simple or compound, it is equally true that the molecule is the smallest quantity of it which can exist in a free or uncombined state in nature. The identity of elementary matter would seem therefore to reside in the atom, while the identity of compound matter would appear to reside in the molecule.

140. Simplicity of Constitution of Matter in the Gaseous State.—Matter is known to us in three states, the solid, the liquid, and the gaseous states, as will appear more fully hereafter. The last of these, the gaseous state, has been proved on investigation to have a constitution of extreme simplicity. In the first place the particles themselves—or molecules, as we may call them—of all the various species of matter, when reduced to the state of vapor, are apparently perfectly similar in shape and equal in size; and in the second, it would appear from the phenomena of the change of volume in gases by change of temperature or pressure that the constitution of the mass is also precisely the same for all substances when they are in the vaporous state. Hence the law of Avogadro: All substances when in the state of vapor contain in the same volume an equal number of molecules. From this it follows immediately, of course, that the molecular mass of any substance is directly proportional to its density in the gaseous state. Now Hofmann has shown that when one volume of chlorine gas unites with one volume of hydrogen gas to form two volumes of hydrogen chloride gas, if we suppose n molecules in the one volume of each gas, there must be $2n$ molecules in the two volumes of the product. And since each molecule of hydrogen chloride contains at least two atoms, one of hydrogen, the other of chlorine, the $2n$ molecules will contain $4n$ atoms; $2n$ of which will be hydrogen and $2n$ chlorine. But the $2n$ atoms came in each case from the n molecules originally taken. Each molecule of hydrogen therefore must have contained at least two atoms of hydrogen, and each molecule

of chlorine two atoms of chlorine. In the gaseous state the molecule is in general a diatomic one.

Upon this reasoning chemistry bases the determination both of molecular and of atomic mass. Density is defined as the amount of matter in the unit of volume. The ratio of the masses of two cubic centimeters of different gases is in the C. G. S. system the ratio of their densities. But since in equal volumes the number of molecules is the same for all substances in the gaseous state, this ratio of the masses of equal volumes is also the ratio of the molecular masses. Knowing the ratio of the molecular masses, the atomic masses must also have the same ratio, if the molecules are similarly constituted.

ILLUSTRATION.—For example, the mass of a cubic centimeter of hydrogen, i.e., its density, is ·0000896 gram; that of a cubic centimeter of oxygen is ·000143 gram. The ratio of these densities, i.e., the relative masses of these two gases contained in unit of volume, is 16. Hence, since there is the same number of molecules in a cubic centimeter of each gas, the molecular mass of oxygen must be 16 times that of hydrogen. And further, since each of these gases contains the same number of atoms in its molecule, the atomic masses of the two are also in the ratio of 16 to 1. If we assume unity for the atomic mass of hydrogen, its molecular mass will obviously be 2, and that of oxygen will be 32.

141. Absolute Size of Molecules.—Perhaps in no direction have our ideas undergone more radical modification in recent times than in that of the molecular constitution of matter. In 1835 Cauchy showed mathematically that in order to account for the dispersion of light, it is necessary to assume a granular structure for matter. Assuming that the diameter of the granulations cannot vary much from one ten-thousandth of the length of the shortest light-wave, and that this light-wave is a two-thousandth of a centimeter in length, we have one two-hundred-millionth of a centimeter on this hypothesis as the distance from the center of one molecule to the center of a contiguous molecule in glass, water, or other transparent substance. Sir William Thomson, from data furnished (1) by the attraction of zinc and copper

after contact, (2) by the extension of a soap-film, and (3) by the kinetic theory of gases, has concluded that the diameter of the molecules of a gas cannot be less than one five-hundred-millionth, and of the molecules of a solid or liquid not less than from the one-hundred-and-forty-millionth to the four-hundred-and-sixty-millionth of a centimeter. Hence the number of such molecules in one cubic centimeter of any gas cannot be greater than six thousand million million million (6×10^{21}); and in one cubic centimeter of the average solid or liquid not greater than from three million million million million as a minimum to a hundred million million million million as a maximum (3×10^{24} to 10^{26}). "Jointly these results establish," says Thomson, "with what we cannot but regard as a very high degree of probability, the conclusion that in any ordinary liquid, transparent solid or seemingly opaque solid, the mean distance between the centers of contiguous molecules is less than the hundred-millionth and greater than the two-thousand-millionth of a centimeter."

ILLUSTRATIONS.—The minimum particle visible to the eye is a cube one four-thousandth of a millimeter on a side. But such a cube contains from sixty to one hundred million molecules. The smallest organized particle visible under the microscope contains about two million organic molecules (Maxwell). And Crookes has calculated that to count the molecules in a pin's head, supposing that ten million are counted every second, would require two hundred and fifty thousand years! Moreover we may contrast the condition of ordinary gas, which contains 3×10^{20} distinct molecules in every cubic inch, with that of space, where it is calculated that there is only one molecule in 10^{214} cubic miles!

142. Divisibility of Matter.—These accepted facts of science may help to set at rest speculations upon the divisibility of matter, by showing that while the maximum division claimed for any form of matter falls far short of the actual reality, yet, on the other hand, there exists a perfectly definite and finite limit beyond which subdivision cannot be pushed without destroying the identity of the substance operated upon. Thus Leslie tells of a grain of musk which *perfumed* a large room

for twenty years and must therefore during that time have been subdivided into 320 quadrillions of particles, as an extraordinary instance of the divisibility of matter. But even if we suppose this number to mean 320×10^{24}, its greatest possible value, we no more than reach the probable size of material molecules as calculated by Thomson.

143. Impenetrability of Matter.—The supposition that two portions of matter cannot occupy the same portion of space at the same time can be made only provisionally. When a stone is immersed in water, the water is displaced; and careful measurements show that the volume of the water displaced is exactly equal to the volume of the stone. Indeed, as Archimedes was the first to show, this is one of the most accurate methods of measuring the volume of an irregular solid. If, however, the body immersed be porous, then the water does penetrate what would ordinarily be called the space occupied by the body. Even in the case of liquids, which are not porous to the eye, there is interpenetration; as is shown when equal volumes of alcohol and water are mixed and the resulting volume is observed to be several per cent less than the sum of the constituent volumes. Hence a distinction has been drawn between sensible and physical pores. Hydrogen gas will pass freely through a plate of the most compact graphite, and carbon monoxide through hot plates of solid iron. Water has been forced through globes of lead, of silver, and of gold. Indeed if by physical pores is meant the space separating contiguous molecules, then there seems room for indefinite interpenetration, since Tait assumes it as probable that the molecule itself does not occupy so much as five per cent of its share of the whole space. Further, whether two molecules can occupy the same space or not depends in the same way upon the character of their atomic constitution; and so on indefinitely. "We have no experimental evidence," says Maxwell, "that two atoms may not sometimes coincide. For instance, if oxygen and hydrogen combine to form

water, we have no experimental evidence that the molecule of oxygen is not in the very same place with the two molecules of hydrogen."

Section II.—Properties of Solids.

144. Definition of a Solid. — In the chapter on Dynamics when discussing the action of force upon matter it was always assumed that the bodies concerned were perfectly rigid; i.e., that they suffered no distortion under the action of the force. Bodies possessing rigidity to any considerable extent are called **solids.**

145. Properties of Solids.—In the solid state of matter the attraction of cohesion between the molecules reaches a maximum. Hence those properties of matter which depend upon cohesion are highly developed in solids. These are:

1. *Hardness*, measured by the power of a body to abrade another. Mohs's scale of hardness consists of the following minerals: 1. Talc, 2. Selenite, 3. Calcite, 4. Fluorite, 5. Apatite, 6. Adularia, 7. Quartz, 8. Topaz, 9. Sapphire, 10. Diamond. Each of these substances will scratch all those placed before it in the scale. An unknown mineral if it will abrade fluorite but not apatite has a hardness represented by 4·5 of Mohs's scale.

2. *Malleability*, the property of being extended in surface under the hammer. Thus gold is beaten out into leaves so thin as to be transparent, the thickness of which does not exceed the 120-thousandth part of a centimeter.

3. *Ductility*, the property of being extended in length by drawing through a plate. The operation of wire-drawing consists in drawing the metal through successively smaller holes in metal plates. Corundum and even diamond plates are used for the smaller sizes. In this way Wollaston succeeded in producing a wire of platinum seventy-five millionths of a centimeter in diameter.

4. *Plasticity*, the property of changing form under continuous stress without developing reaction. Wet clay, cobbler's wax, and ice are examples. The earth itself, though apparently rigid, has been shown to be more plastic under the tidal attraction of the moon than a globe of glass of the same size. (Tait.)

5. *Tenacity*, measured by the stress required for the rupture of a mass of given cross-section. The tenacity of several substances is given in the following table:

Substance.	Tenacity.	Substance.	Tenacity.
Gold	0·27	Steel	0·8
Silver	0·3	Oak	0·1
Copper (hard)	0·4	Teak	0·1
" (annealed)	0·3	Fir	0·07
Iron	0·6	Glass	0·06

By multiplying these values by 980×10^7, the tenacity will be obtained in dynes per square centimeter.

146. Elasticity.—Elasticity is usually defined as that property of a body in virtue of which, after its size or shape has been altered by the action of force, it reacts against the force and returns to its original size or shape more or less completely upon the removal of the force. In other words, it is the stress which is called out in a body when subjected to strain.

Solids may obviously have two sorts of elasticity corresponding to two sorts of strain. In the one case there is a change of size strain, called compression or dilatation; and the active stress produced by it is called **volume-elasticity**. In the other case there is a change of shape strain, called distortion; and the stress which it calls out is referred to as elasticity of form or **rigidity**. The term **rigidity** therefore, in its strict sense, means the resistance which a solid opposes to a change of form, the term **incompressibility** being used to denote the resistance offered to a change of bulk. In ordinary language, however, bodies which offer great resistance to either *kind of strain* are called rigid bodies. In this discus-

sion we shall assume the matter concerned to be homogeneous and isotropic; this latter term signifying that the matter has the same properties in all directions.

147. Volume Elasticity.—When an elastic solid is submitted to a pressure which is equal in all directions and which is always normal to its surface, its volume is altered but not its form. The diminution in volume thus produced is called the **compression**, and the ratio of the compression per unit volume to the pressure applied (measured in units of force per unit of surface) is called the **compressibility**. The reciprocal of this measures the incompressibility; i.e., the **volume-elasticity**.

Thus, if V be the initial volume and $V - v$ the final volume, the compression is v, and the compression per unit volume is v/V. If P be the pressure per unit of area, the compressibility is v/PV; and the reciprocal of this, or PV/v, is the coefficient of elasticity of volume; usually represented by k.

148. Rigidity.—By simple rigidity is meant a resistance to distortion solely; the volume remaining unchanged. A stress which produces change of form only without alteration of volume is called a **shearing stress**, and the resulting distortion a **shearing strain** (49). In the case of a simple shear one plane in the solid is fixed, while all parallel planes are displaced parallel to themselves and in the same direction, through spaces which are proportional to their distances from the fixed plane. The ratio of the displacement to the distance from the fixed plane is taken as the measure of the shear; and the ratio of the shearing stress per unit of area to the shear produced by it is called the **coefficient of simple rigidity** of the solid. This coefficient is usually denoted by n.

Thus if a pile of cards of thickness AB (Fig. 51) be forced from the perpendicular position AD into the oblique one BE by a tangential force P applied along AF, then if AB remains unaltered the volume will not be changed, while there will be distortion, a simple shear. The ratio AC/AB will measure the shear;

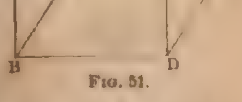


Fig. 51.

and the ratio $\dfrac{P}{AC/AB}$ or $\dfrac{P \times AB}{AC}$ will be n, the coefficient of simple rigidity.

Strictly speaking, however, the volume is changed in the above example. In simple shear the shortened and lengthened diameters always remain coincident with the original diameters; as in the figure (Fig. 52), in which the distorted solid is represented in dotted lines. Obviously the shearing strain may be measured by the change in the right angle of the solid. And the ratio P/θ, in which θ represents this change of angle, is the ratio $\frac{\text{shearing stress}}{\text{shearing strain}}$, or n, the coefficient of simple rigidity.

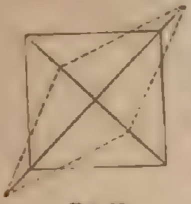


FIG. 52.

149. Torsion.—If the distorting force be applied to one end of a cylindrical solid, and this force be of the nature of a couple, the effect will be to rotate the successive layers about the axis and thus produce a simple shear. Since the volume is not altered, there is change of form only, and rigidity alone is concerned. In this case the displacement is a circular one along an arc of the cylinder, and the shear will be measured by the ratio of the subtended angle to the length of the cylinder. Such a displacement as this is called a **twist** or **torsion**: and the measure of the torsion is the angular displacement divided by the length of the cylinder. Since in the case of an elastic solid strain is directly proportional to stress, the angle of torsion is always directly proportional to the force of torsion. But when the system is in equilibrium the torsional stress called out by the displacement will be equal to the moment of the twisting couple. The **modulus of torsion** is defined as the couple required to twist one end of a given cylinder of unit length through unit angle, the other end being fixed. If τ represent this modulus, then the couple required to twist the given cylinder of length l through an angle θ will be $\tau\theta/l$.

Suppose the radius of this cylinder to be $2r$. Let it be divided into concentric rings of breadth dx, the radius of one of which is x, and its area consequently $2\pi x dx$. The displacement per unit of length is θ/l, as above; or $x\phi/l$, where ϕ is the unit angle; and since displacement multiplied by the rigidity is equal to the stress per unit of area, we have this stress $= nx\phi/l$; and for the entire area of the ele-

mentary ring $= (2\pi x^2 n\phi dx)/l$. The moment of this is $(2\pi n\phi x^3 dx)/l$, which is the moment of the torsional couple for this elementary ring. By integrating this expression between the limits $x = 0$ and $x = r$ we obtain $\pi n r^4 \phi/2l$ for the moment of the torsional couple for the entire cylinder. Calling this moment T, we have for the value of the torsion angle $\phi = \frac{2Tl}{\pi n r^4}$. Whence it appears that the torsion produced is directly as the torsional couple and as the length of the cylinder, and inversely as the rigidity and the fourth power of the radius of the cylinder. Since the torsional stress is proportional to the torsional strain, the stress for an angle θ would be $\frac{\pi n r^4 \theta}{2l}$.

Hence, equating these two expressions for the torsional couple T required to develop this torsional stress, we have

$$\frac{\tau\theta}{l} = \frac{\pi n r^4 \theta}{2l}; \quad \text{whence} \quad n = \frac{2\tau}{\pi r^4}. \qquad [30]$$

Since τ can be determined from the time of the torsional oscillation of the cylinder when a mass of known moment of inertia is attached to it, it is evident that n, the coefficient of simple rigidity of the given substance, may be determined in this way.

150. Young's Modulus.—When the stress developed in a solid is called out by longitudinal extension or compression, the ratio of this stress to the strain producing it is called **longitudinal rigidity**, and is represented by **Young's modulus**. Young himself defined it as the ratio of the simple stress required to produce a small shortening or elongation of a rod of unit section to the proportionate change of length produced.

Thus if a wire or rod of length L be stretched by an amount l, the extension per unit of length is l/L. If P be the force producing the elongation and a the cross-section of the rod, P/a represents the stress per unit of cross-section. Hence

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{P/a}{l/L} = \frac{PL}{al} = M.$$

It is possible to calculate the value of Young's modulus in terms of the volume-elasticity k and the rigidity n. Since the rigidity is P/θ and $\theta = 2(p + q)$

when the angle is small (p being the elongation along one face and q the corresponding contraction along the other), we have $p + q = P/2n$. Moreover, a pressure P applied uniformly to a solid in all directions would reduce the dimensions along three perpendicular axes in the ratio $1 : 1 + p - 2q$; consequently we have $p - 2q = P/3k$. Combining these two equations, we have for the value of the elongation $p = P\left(\frac{1}{3n} + \frac{1}{9k}\right)$; and for the ratio of the elongation per unit of length to the tension P/p, or M, we have the value $\frac{9kn}{3k + n}$; thus giving the value of M in terms of the volume-elasticity and the rigidity.

151. Poisson's Ratio.—When a wire is stretched longitudinally it contracts laterally. The ratio of the proportional diminution of its volume by contraction to the proportional increase by elongation is called **Poisson's ratio.** Its author supposed this ratio to be ¼ for all isotropic solids, but Stokes has shown it to be much greater than ¼ for india rubber, and Thomson and Tait have shown it to be much less than ¼ for cork. On theoretical grounds the value of Poisson's ratio may be shown to be $\frac{3k - 2n}{2(3k + n)}$, in terms of volume-elasticity and rigidity.

COEFFICIENTS OF ELASTICITY.

Substance.	Density.	Volume-elasticity = k.	Rigidity = n.	Young's Modulus = M.
Dist'l'd water	1·000	$0·222 \times 10^{11}$	——	——
Flint glass ..	2·942	3·47- 4·15 "	$2·35–2·40 \times 10^{11}$	$5·74- 6·03 \times 10^{11}$
Brass........	8·471	10·02-10·85 "	3·44-4·03 "	9·48-11·2 "
Steel	7·849	18·41 "	8·19 "	20·2 -24·5 "
Iron (wro't) .	7·677	14·56 "	7·69 "	19·63 "
Iron (cast)...	7·235	9·64 "	5·32 "	13·49 "
Copper	8·843	16·84 "	4·40–4·47 "	11·72-12·34 "

The values in this table are all expressed in C. G. S. units; i.e., in dynes per square centimeter.

152. Elastic Limit.—Just as no actually existing solid is perfectly rigid, so no known solid is perfectly elastic. All solids are strained by the application of an external stress, though the internal stress called out by this strain never exactly equals the external stress in value. Perfectly elastic solids develop a resilience which does not diminish with time, and which restores completely the original size or form when the disturbing force is removed, without the smallest permanent strain or set. Glass, quartz, and steel perhaps approach this condition of things most nearly. Inelastic or plastic solids, on the other hand, develop no resilience on the application of a disturbing force, and hence do not recover in the least their form or volume when this force is removed. Such solids are wet clay, putty, and dough. Most solid bodies lie between these extremes, developing some resilience and yet suffering some permanent strain; as is the case with iron, copper, lead, wood. In most solids, however, the elasticity depends upon the magnitude of the distorting force; so that when this force is very small, the elasticity may be considered practically perfect for these solids. Between the limits of strain thus produced they may be treated as perfectly elastic, therefore; but when these limits are exceeded, the solid is permanently deformed; i.e., takes a "set." And if the stress be increased beyond this, the limit of tenacity of the solid may be reached and it may be broken.

The resistance to deformation experienced in a plastic solid is due, not to its resilience, of course, but to the fact that force is required to slide its particles over one another; i.e., to the internal friction to be overcome. Such bodies are said to be **viscous**. In other cases, such as glass, the solid suffers no deformation, but when the elastic limit is exceeded, the limit of tenacity is very soon reached, and the solid is broken. Such solids are called **brittle**.

153. Fatigue of Elasticity.—A curious phenomenon called the "fatigue of elasticity" has been observed in solids which shows very *clearly the effect* of molecular

friction. When a wire is vibrated torsionally, there is always a displacement of the zero-point to one side or the other, according to the direction of the original torsion, this disturbance requiring hours or even days for its disappearance. If the wire be kept vibrating, however, the molecular friction is greatly increased. Thomson found that, using two equal wires, the arc of vibration fell to one half in 100 vibrations in the case of the wire which had been oscillated only a few times, while in the case of the other, which had become "fatigued" by long vibration, the arc fell to half its value in 44 or 45 vibrations. Tait states that if a wire be kept twisted 90° to the right for six hours and then 90° to the left for half an hour, being allowed gradually to come to zero without oscillation, it will first turn slowly to the right, undoing the effect of the later twist, and then turn more slowly to the left, undoing the effect of the earlier twist. If an electrically vibrated tuning-fork be kept in motion for a long time, its elasticity appears to become fatigued; so that on stopping the current it comes to rest almost instantaneously, like a plastic body.

154. Oscillations produced by Elasticity.—In an earlier chapter we have shown that a body oscillating under the action of a force which is always proportional to the displacement describes simple harmonic motion (54); and that in consequence these oscillations are isochronous, i.e., are performed in equal times whatever their amplitudes. Now by the law of Hooke we see that in all elastic bodies the resilience or force of restitution, which is directly proportional to the distorting force, is directly proportional to the distortion; i.e., the stress called out is directly proportional to the strain produced. Consequently it follows that a body vibrating under the influence of elasticity, of course within the limits of set, will vibrate in periods which are simple harmonic and therefore isochronous.

In illustration of simple harmonic vibration performed under the influence of elasticity, we may take the case of a torsion pendulum consisting of a mass of metal sup-

ported by a wire. If the wire be twisted, though still remaining vertical, it will untwist when the force is removed and the pendulum will perform a series of oscillations around the wire as an axis. The work expended upon it in twisting it is stored up in it as potential energy, which is converted into kinetic energy as it rotates, becoming all kinetic as it passes its zero-point. This energy is expended in twisting it in the opposite direction; and so the system oscillates like a gravity pendulum, but under the action of rigidity.

The formula for the time of vibration of a compound gravity pendulum is $t = \pi \sqrt{I/MRg}$; in which I is the moment of inertia and MRg the static moment expressed in absolute units. As we have seen above, the moment of torsion, active in the torsional pendulum, is T. Hence the time of vibration of a torsional pendulum t is represented by $\pi \sqrt{I/T}$.

155. Propagation of Disturbances in Elastic Media.—If a particle of a solid, held in equilibrium by the attraction of surrounding particles, be displaced from its position, there will be developed a stress tending to bring it back; which stress, being due to the elasticity of the solid, will be proportional to the displacement. Hence the particle will vibrate harmonically about its zero position. In consequence, however, of the original displacement, there will be a compression produced in the direction of motion, and a rarefaction in the opposite direction; which disturbances will be propagated in all directions from the particle as a center, with a speed which is dependent conjointly upon the elasticity and the density of the surrounding medium. Obviously the speed of propagation of a compression in a given uniform solid will depend upon its volume-elasticity, or k; while that of a disturbance of the nature of a shear will depend upon its rigidity, or n.

In simple harmonic motion the speed of the vibrating particle varies directly as the square root of the acceleration (54); i.e., of the force of restitution. Hence the speed of propagation *of a disturbance* in any medium

must be proportional to the square root of k, the volume-elasticity of that medium; or if along a wire, to the square root of M, Young's modulus. Moreover, since the acceleration is the ratio of force to mass, we have $a = f/m = f/V\delta$, where V is the volume and δ the density of the medium. For the case where the ratio of f to V is constant for two media, we have evidently $a : a' :: \delta' : \delta$; or the accelerations are inversely as the densities. But as above $v : v' :: \sqrt{a} : \sqrt{a'}$, and hence the speeds are inversely proportional to the square roots of the densities. Combining these two values representing the speed of propagation of a disturbance in any medium of elasticity k and of density δ, we have

$$\text{Speed} = \sqrt{\frac{\text{Elasticity}}{\text{Density}}} = \sqrt{\frac{k}{\delta}}. \qquad [31]$$

156. Impact.—When a body in motion strikes another at rest, or when two moving bodies mutually impinge, the resultant effect of the impact is determined by the elasticity of the bodies concerned. At the instant of contact a compression is developed, which increases until the speeds become equal, when it reaches its maximum. If the impinging bodies be perfectly inelastic, they will now move on together with this common speed. But as all matter is more or less elastic, there will in fact appear a force of restitution at the place of contact tending to separate the colliding bodies. Newton proved experimentally that the relative speed of separation after impact bears to the previous relative speed of approach a proportion which is constant for the same two bodies. This proportion, which is called the **Coefficient of Restitution**, and is usually represented by e, is always less than unity, though approaching it more nearly the harder the bodies are. The values found by Newton are: For worsted and steel balls, $\frac{5}{9}$; for balls of cork, a little less; for balls of ivory, $\frac{8}{9}$; and for balls of glass, $\frac{15}{16}$.

The entire process, from the instant of first contact

to the instant of final separation, during which the impinging bodies are exerting mutual action upon each other occupies ordinarily an exceedingly brief interval of time. Thomson estimates it as about an hour if the bodies are of dimensions comparable to the earth, and possess a rigidity equal to that of copper, steel, or glass; but if the bodies are spheres of not more than a meter or so in diameter, made of these substances, then the whole operation is finished probably within the thousandth of a second. In the chapter on Kinetics (75) we saw that the impulse of a force is measured by the momentum which it generates; i.e., that $ft = ms$. For a constant momentum generated, then, the force must vary inversely as the time during which it acts. Hence the magnitude of the force exerted in the collision of bodies must be proportionately great in order to generate the given momentum in a thousandth of a second.

Suppose two spherical masses m and m' to be moving in the same direction along the same line with the speeds v and v'. From the instant of contact the force acting between them along this line must, according to the third law of motion, be equal for the two. In consequence, since these forces are in opposite directions, the bodies will, by the second law, tend to move in opposite directions. Or, in other words, one will accelerate the other, and will also be retarded by it, the amount of motion gained by the one being equal to that lost by the other; which action will continue up to the instant at which their speeds are equal. At this instant of time the two spheres will be moving as one mass, whose momentum is the sum of the momenta before impact. If V represent this common speed, then $(m + m')V = mv + m'v'$; whence $V = \dfrac{mv + m'v'}{m + m'}$. If the spheres are inelastic, this is the whole of the phenomenon. If not, a second stage of the process supervenes. The force of restitution now comes into play, and tends to separate the two spheres with a speed proportional to the coefficient of restitution. Each will now have its own speed,

which we may represent by u and u'. The difference in the speeds before impact will be $v - v'$, and after impact $u' - u$; whence, by Newton's law, $u' - u = e(v - v')$. And since the momentum gained by one sphere is lost by the other, we have $mu + m'u' = mv + m'v'$. Combining these two equations, we have for u, the final speed of the mass m, the value $u = \frac{mv + m'v' - em'(v - v')}{m + m'}$; and for u', the speed of the mass m', the value $u' = \frac{mv + m'v' + em(v - v')}{m + m'}$. If the spheres be inelastic, $e = 0$, and $u = u' = \frac{mv + m'v'}{m + m'}$, as above. If they be perfectly elastic, $e = 1$, $u = \frac{(m - m')v + 2m'v'}{m + m'}$, and $u' = \frac{2mv' - (m - m')v'}{m + m'}$. If we suppose $v' = 0$, in which case the body m' is at rest, then we have for the final speed of m, $u = \frac{m - em'}{m + m'}v$; and for the final speed of m', $u' = \frac{m(1 + e)}{m + m'}v$. Hence the body struck, m', moves onward in the direction of the initial motion of m; while m itself, the moving body, goes onward, stops, or moves backward, according as its mass is greater than, equal to, or less than em'. If $m' = em$, then $u = (1 - e)v$, the final speed of m; and $u' = v$; i.e., the final speed of m' is equal to the initial speed of m. Again, the equations $mu + m'u' = mv + m'v'$ and $u' - u = e(v - v')$ enable us to determine the conditions under which the masses should interchange their speeds. In this case $u = v'$ and $u' = v$. Substituting these values, we have $mv' + m'v = mv + m'v'$, and $v - v' = e(v - v')$. Writing the first equation $m(v' - v) = m'(v' - v)$, or $(m - m')(v' - v) = 0$, we see that $m = m'$. From the second equation, e must be equal to 1. Hence the conditions required are that the masses should be equal and perfectly elastic. By giving to the moving masses the proper positive or negative signs to indicate

the direction of motion, all cases of impact may be brought under the above general formulas.

157. Change of Energy by Impact.—If the spherical masses of the last section be assumed perfectly elastic, theory indicates that their relative speeds after the impact will be equal to their relative speeds before it. Since, therefore, the speeds are only exchanged, the total energy of the system remains unaltered. But, practically, no form of matter known is perfectly elastic, in the first place; and, in the second, even if it were, some energy would be expended in producing vibrations in the colliding masses. So that even in the case of the most highly elastic bodies energy is lost on collision. Evidently this loss increases as the colliding masses are more inelastic.

The loss of energy as a function of the elasticity may be determined as follows: From the equation $u' - u = e(v - v')$, above given, we get, by squaring and multiplying both members by mm', the equation $mm'(u' - u)^2 = mm'e^2(v - v')^2$, the second member of which may be written (by adding and subtracting $mm'(v - v')^2$) in the form $mm'(v - v')^2 - mm'(1 - e^2)(v - v')^2$. Squaring both sides of the equation of momenta above given, $mu + m'u' = mv + m'v'$, we have $(mu + m'u')^2 = (mv + m'v')^2$. Expanding this equation and adding it to the one previously obtained, multiplied out, we obtain

$$(m + m')(m'u'^2 + mu^2) = (m + m')(mv^2 + m'v'^2) - mm'(1 - e^2)(v - v')^2. \quad [32]$$

Whence $mu^2 + m'u'^2 = mv^2 + m'v'^2 - \dfrac{mm'}{m + m'}(1 - e^2)(v - v')^2$. Since e is never greater than 1, the expression $1 - e^2$ can never be negative. The initial kinetic energy is therefore always greater than the final kinetic energy of the system, except when e is equal to 1. Then the two are equal. If e be made zero, the loss of energy due to impact is the maximum possible.

SECTION III.—PROPERTIES OF FLUIDS.

158. Definition of a Fluid.—Besides the solid state of matter now described, substances may exist in another and equally important condition, called the fluid condition. As the name implies, the characteristic property of a fluid is to flow. Hence a fluid possesses no rigidity whatever, but is deformed by the action of

any force, however small. In consequence fluids have no shape of their own, but always take the shape of the vessel in which they are contained. They are plastic, and not rigid; are limpid, and not viscous. A perfect fluid is infinitely limpid and infinitely plastic; i.e., its rigidity is zero, and so is its viscosity. Actual fluids, however, are never perfectly limpid. Ether, water, oil, honey, Canada balsam, pitch, sealing-wax, are examples of fluids having progressively greater and greater viscosity.

ILLUSTRATIONS.—It might not at first sight seem proper to call sealing-wax a fluid and jelly a solid. But if we remember that on the one hand the distortion which is produced by a given stress in a perfect solid is perfectly definite, the ratio of stress to strain being constant and independent of time, while in a fluid, on the other hand, the deformation continues as long as the distorting stress continues, the ratio of stress to strain not being constant but increasing with time, the case will present no difficulty. A stick of sealing wax supported at its ends soon yields to its own weight, and is deformed continuously; while a mass of jelly undergoes all its deformation at once on the application of the deforming force. The strain produced is constant, and does not increase with time. Since it does not flow, the jelly is a true solid, although a soft one.

159. Definitions of Liquids and Gases.—Since a fluid possesses no rigidity, the only elasticity it can have is elasticity of volume. The volume-elasticity of fluids is perfect. They recover their size completely when the compressing force is removed. Fluids vary widely, however, in their compressibility; and on this ground are divided into two classes, **liquids** and **gases**. Liquids as a class are highly incompressible; i.e., they have a very high volume-elasticity. A perfect liquid would be an incompressible perfect fluid. Gases, on the other hand, are readily compressible. Indeed they appear to have no elasticity at all of their own, that which they seem to possess being due simply to the pressure which is exerted upon them at the time, and by which their elasticity is measured. Consequently the volume which a gas occupies depends not upon itself at all, but upon the pressure upon it. Liquids take only the shape of

he vessel in which they are contained; gases take not nly the shape but also the size of the containing vessel.

160. General and Special Properties of Fluids.—ince both liquids and gases are fluids, they possess ot only the general properties of fluids, but also other roperties special to themselves. It will be convenient, herefore, to consider first the general properties and hen the special properties of liquids and gases, and o divide the subject for purposes of study into four arts: **A. The Statics of Fluids**; **B. The Kinetics of Fluids**; **Compressibility of Fluids**; **D. Capillarity.** The liquid r gaseous state will be used indifferently in the discussion, according as the one or the other is best suited to the purpose.

A.—STATICS OF FLUIDS.

161. Mobility of Fluids.—The term mobility is the opposite of rigidity. Since a fluid offers no resistance to a deforming stress, its particles are readily moved by the smallest force. Hence fluids are eminently mobile. The peculiar characteristic of fluids, that of transmitting ressure equally in all directions, expressed commonly the form of Pascal's law, is a direct consequence of his mobility. When acted on by force which is not qually applied in all directions, fluids readily suffer istortion unless they are supported upon all sides.

ILLUSTRATIONS.—Thus if a kilogram-weight be placed upon the op of a wooden cylinder, the pressure directly produced by it upon he top of this cylinder is transmitted vertically downward to its upport. But not the slightest pressure is exerted laterally under hese conditions. If, however, the cylinder be of water or of air enlosed in a tube, and the pressure of a kilogram-weight be applied o the upper end of it by means of a piston, it will be found that while this pressure is transmitted unchanged to the lower end as efore, it is also transmitted laterally to the walls of the tube. So hat upon every area of the side equal to that of the end, a presure is exerted equal to that which is produced by the kilogramweight.

Another important principle of fluid equilibrium may e thus stated: At any *point within a fluid* the pressure

is the same in all directions; and hence, if no external forces act, this pressure (which is exerted by the containing vessel) is the same at every point of the fluid. This conclusion evidently follows if we assume a minute portion of the fluid in the interior of the mass to become solid, without other change. Since the entire mass remains at rest as before, the forces acting upon this element must be in equilibrium, and their resultant zero. This can only be true if the pressure at the point is equal in all directions. Moreover, since fluids transmit pressure in all directions equally, the pressure at any one point of a fluid mass must be equal to that at any other point, as above stated.

Again, it follows from the principle of fluid mobility that the pressure upon any surface of a liquid at rest is at every point perpendicular to the surface. For if not perpendicular, the direction of pressure must make an angle with the surface. If so, this direction is capable of resolution into two components at right angles to each other—one perpendicular to the surface, the other along the surface. In virtue of the mobility of the liquid, this latter component would produce motion along the surface. But by hypothesis the liquid is at rest. Hence there is no such tangential component, and the pressure is at every point perpendicular to the surface, as stated.

APPLICATIONS. The principles now given are applied practically in the arts to the construction of hydrostatic presses and of accumulators. In the hydrostatic press two pistons of different areas are provided, having a liquid between them. Now we have seen that the pressure exerted in the interior of any liquid is proportional to the area of the exposed surface. Hence if the smaller piston has a diameter of one centimeter and the larger a diameter of 30 centimeters, the area of the larger will be 900 times that of the smaller, and a pressure of one kilogram applied to the smaller will develop a pressure of 900 kilograms upon the larger. If the smaller piston be that of a pump worked by power, then in moving inward 100 kilograms of pressure might be exerted by it; in which case the larger piston would be forced outward with the pressure of 90,000 kilograms. The development of force in this way is limited only by the strength of the materials employed. But it would be an error to assume that

energy is thus created. The work done by or upon either piston is the product of the mean force acting by the distance through which it acts. Hence, under the conditions above supposed, the quantity of liquid remaining the same, the smaller piston must move through 900 times the distance passed over by the larger; so that fl, or the work done, is the same for both; $1 \times 900 = 900 \times 1$. The function of the hydrostatic press, therefore, like that of every other machine, is simply to transform or transfer energy. Indeed, since in all machines some energy is lost by friction, the total work done by any machine is always less than that which is done upon it—a fact of great practical importance.

162. Fluids under the Action of Gravity.—On the earth's surface fluids exert a pressure in consequence of their weight. Since the pressure in fluids which have a free surface i.e., in liquids, is normal to this surface, it follows that when this pressure is the weight of the liquid itself, acting vertically, the free surface of the liquid must be horizontal. In other words, the surface of a liquid is level; and since the direction of gravity is sensibly parallel to itself over moderate areas, a liquid surface is a flat or plane surface.

163. Fluid Pressure Proportional to Depth.—Since the lower layers of a fluid support the weight of the upper layers, it is evident that the pressure due to gravity must increase from above downward; including in this the lateral as well as the vertical pressure. The downward pressure upon the base of a cylindrical vessel filled with water, for example, is evidently simply the weight of the water. The sides being vertical, sustain and react equally against the lateral pressure; which being horizontal, and acting outward over the whole surface, has a resultant equal to zero.

Since the pressure on one side is equal to that on the other, there is horizontal equilibrium. But if an opening be made in the walls of the vessel, the pressure will be relieved at that point and the water will escape. Moreover, the antagonistic pressure on the opposite wall remains uncompensated, and tends to move the entire system in the opposite direction to that in which the water flows. This principle is illustrated in Barker's mill, in reaction turbines, in rockets, and rotating fireworks. Unbalanced pressure produces the motion.

The pressure exerted by any vertical column of a fluid upon its base is always the continued product of the area of the base, the height of the column, and the density of the fluid. Or, if a represent the area, h the height, and δ the density, the pressure, i.e., the weight of the column, is $ah\delta$; or $ah\delta g$, in absolute units of force. The pressure-intensity, i.e., the pressure in units of force per unit area, therefore, in a fluid of density δ, at a depth h below the surface, is $gh\delta$.

EXAMPLES.—Thus a column of water ten meters high exerts a pressure upon its base, supposed one square decimeter in area, of one hundred thousand grams, or 98 megadynes; since $a = 100$, $h = 1000$, and $\delta = 1$. So a column of mercury 76 centimeters high and one square centimeter in area exerts a pressure of $1 \times 76 \times 13{\cdot}596 = 1033{\cdot}3$ grams or $1{\cdot}0126 \times 10^6$ dynes; which, as we shall prove later, is exactly the pressure exerted by an air-column of the same area, extending to the height of the atmosphere.

If, however, the sides of the containing vessel are not vertical, then the above conclusion does not hold, and the pressure upon the base is not proportional to the amount of water in the vessel. Suppose the vessel be conical, with its smaller base downward (Fig. 53). It is now obvious that, since the lateral pressure is perpendicular to the walls, it may have a vertical component downward which is balanced by the reaction of the walls of the vessel upward. Hence the weight of the water is supported partly by the base, partly by the sides, the part supported by the base being only the vertical column immediately above it.


FIG. 53.

If the conical vessel has its larger base downward (Fig. 54), then the vertical component of the pressure is upward, and the downward pressure on the base is balanced in part by an upward pressure on the sides. In this case the reaction of the walls is downward, and the difference between the upward reaction of the base and this downward reaction corresponds to the weight of the water.


FIG. 54

Hence the pressure upon the base of a containing vessel may be less than, equal to, or more than the weight of the water contained in the vessel, according to its shape; the pressure being in all cases the weight of a column equal in area to that of the base, and having the depth of the liquid for its height; or $ah\delta$, as above.

164. Center of Pressure.—The reasoning above given is quite independent of the particular form which the vessel may have. Indeed, the surface itself upon which the pressure is to be estimated may be immersed in the liquid. If it be parallel to the liquid surface, the pressure upon it is $ah\delta$ as before. If it be not so parallel, then h equals the mean depth below the surface, and the pressure is again $ah\delta$. By mean depth is meant the depth of the center of figure if it be symmetrical, or of its center of mass in any case.

ILLUSTRATION.—Suppose, for example, a thin board, a meter square, be immersed vertically in water, its upper edge being a meter below the surface. The total pressure upon the board will be of course $ah\delta$. Now the area is 10,000 square centimeters. The mean depth or depth of the center of figure is $1\frac{1}{2}$ meters or 150 centimeters; and δ is unity. Hence $ah\delta = 1,500,000$ grams or 1500 kilograms.

If any surface be taken within any liquid, and it be divided up into elements, acted upon by parallel forces, representing the pressures on these small areas, these parallel forces will have a resultant. And the point of the surface through which this resultant passes is called the center of parallel forces, or the **center of pressure.** It is below the center of mass of the immersed surface unless this surface be horizontal.

165. Liquid Equilibrium.—A mass of liquid contained in any vessel is in equilibrium when its surface is horizontal. So, also, if the liquid be contained in a number of communicating vessels the surfaces in them all must be in the same horizontal plane in order for the liquid to be in equilibrium.

APPLICATIONS.—This principle, that "water seeks its level," is made use of in the water-level. This is a metal tube having two glass tubes at its ends at right angles to it, and both in the same plane. Water poured into the tube takes a position of equilibrium

such that the two surfaces are in the same horizontal plane. By sighting across these surfaces a horizontal line may be run between two stations. The spirit-level, which has now almost entirely taken the place of the water-level, consists of a glass tube whose axis is the arc of a circle of very long radius, almost filled with alcohol, and sealed. The air-bubble tends to the highest point of the curve; and when the surface of the liquid is horizontal, this is the middle point of the level. The rise of water in Artesian wells, too, is due to this tendency of liquids to equilibrium. If a well be bored in the middle of a geological basin, through impermeable to permeable strata, the water which may have fallen many miles away at the point where these permeable strata rise to the surface, and may have accumulated therein under pressure, flows to and even above the level of the ground.

If, however, several liquids of different densities are placed in the same vessel, they will be in equilibrium only when they have arranged themselves with their surfaces horizontal, in the order of their densities; the densest being, of course, the lowest. If two liquids of different densities be placed in two communicating vessels, such, for example, as two glass tubes connected at bottom, they will be in equilibrium when the pressures they exert upon equal areas are equal. These pressures are, as already stated, $ah\delta$ and $a'h'\delta'$. At the surface of contact the two columns have the same area; and therefore, when the pressures are equal, we shall have $h\delta = h'\delta'$; whence $h : h' :: \delta' : \delta$. In other words, when in equilibrium the heights of the columns will be inversely as the densities of the liquids.

EXPERIMENT.—If mercury be poured into a U-tube, sufficient to fill somewhat more than the bend, and then water be poured upon the surface of the mercury, it will be seen that the column of water is about 13·6 times as long as the column of mercury, both measured from the surface of contact. But mercury is 13·6 times as dense as water. Hence the product of the density by the height is the same for both liquids, as above stated. This method is often useful for finding the relative density of two liquids.

166. Upward Pressure in Fluids.—When a solid is wholly or partly immersed in a fluid it obviously displaces or takes the place of, a volume of the fluid equal to that of its immersed part. Suppose the solid to be a

to

cube, and let it be immersed wholly in water (Fig. 55). The exactly equal cube of water which it displaces was previously in equilibrium, since it was at rest; and hence the sum of the vertical and of the lateral pressures upon it was zero. The vertical downward pressure on its lower surface is evidently that on its upper surface plus the weight of the cube itself. Since the cube is in equilibrium, this pressure downward must be balanced by an equal upward pressure. But the conditions producing the upward pressure—the law of Pascal—are in no wise altered when the solid cube takes the place of the liquid one. The upward pressure upon the lower face is, as before, the weight of a liquid column whose area is that of this face and whose height is the depth of the face below the liquid surface. But the downward pressure upon this same cubic face is the weight of the cube itself plus that of the column of water above it. The difference is the excess of weight of the cube over the weight of the same volume of water. Hence, when the cube is immersed in water it loses weight exactly equal to the weight of the displaced water; i.e., to the weight of its own volume of water.

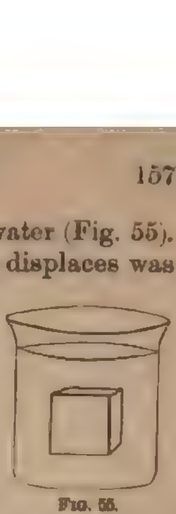


FIG. 55.

167. Principle of Archimedes.—This law of action, which is known as the principle of Archimedes, may be thus stated: When a solid is immersed in a fluid, either wholly or partially, it suffers a loss of weight which is equal to the weight of the fluid which it has displaced. This upward force may be assumed to act at the centre of mass of the fluid displaced, whatever the form of the immersed solid.

If w = the weight of the fluid upon the upper surface of the cube, c_w = the weight of the fluid cube, and c_s = that of the solid cube, the upward pressure upon the lower face, which in the case of the fluid cube is equal to the downward pressure, is evidently $w + c_w$. The downward pressure upon the lower face of the solid cube is $w + c_s$. The resultant pressure is their difference, $(w + c_s) - (w + c_w)$ or $c_s - c_w$. But c_s is the weight of the unimmersed solid

cube; and therefore when immersed it loses the weight c_w; i.e., a weight equal to that of the volume of the fluid which it displaces.

168. Floating Bodies.—Three cases may now be considered, according as the weight of the solid is less than, is equal to, or is greater than, that of the displaced fluid. The resultant pressure, as we have just proved, is $c_s - c_w$. The value of this expression will be negative, zero, or positive according as c_s, the weight of the solid, is less than, is equal to, or is greater than c_w. In the first case, since the negative sign means motion upward, the solid will rise to the surface and float. In the second case there is no resultant, and the solid will remain in equilibrium in any part of the fluid. In the third case the resultant is positive, and the solid will sink in the fluid.

EXAMPLES.—Suppose that the solid, whatever its shape, has a volume of one cubic centimeter, and that it be immersed in water. The weight of a cubic centimeter of water is one gram. Hence the solid when immersed will lose one gram of its weight. If the solid be cork, of which one cubic centimeter weighs 0·24 gram, then the upward pressure exceeds the downward by 1·00 — 0·24 = 0·76 gram; and the cork will move upward. If the solid be of iron, a cubic centimeter of which weighs 7·8 grams, then the downward pressure is in excess by 6·8 grams; and the solid sinks.

Again, suppose the solid to be a rubber balloon filled with hydrogen, and having a volume of one cubic decimeter. Let it be immersed in air. The weight of the hydrogen—one cubic decimeter—is 0·08837 gram. The weight of the same volume of air is 1·2759 grams. Hence the upward pressure, due to the weight of the air displaced, is greater than the downward pressure, due to the weight of the displacing gas, by 1·1875 grams. So that allowing 0·5 gram for the weight of the balloon itself and its attached car, it would still have an ascensional force of 0·6875 gram.

169. Law of Liquid Flotation.—When liquids or solids float on liquids, as, for example, oil or wood on water, the principle of Archimedes is still true and the liquid or solid mass loses weight equal to the weight of the displaced liquid. Now, however, the mass is not wholly but only partly immersed; and hence the volume of the liquid displaced is only a fraction of its own volume. But in order to float at rest, the forces acting must be in

equilibrium; i.e., the upward and downward pressures must be equal. If the former be the greater, the mass will rise; if the latter, it will sink. But the downward pressure is the weight of the mass, and the upward pressure is the weight of the displaced liquid. Consequently the law of flotation is that a floating body will sink in a liquid until it displaces a weight of the liquid equal to its own weight.

Thus a piece of yellow pine, one cubic centimeter of which weighs 0·657 gram, will sink deeper in water than a piece of poplar, weighing only 0·389 gram to the same volume. One cubic centimeter of the former wood would have to sink 657 thousandths of its bulk in order to displace its own weight of water; while the latter would displace its own weight by sinking 389 thousandths. Again, ice weighs 0·918 gram to the cubic centimeter; and therefore the part of an iceberg out of the water is only 82 thousandths of the entire volume of the berg, or about one eleventh of the portion immersed.

A hollow ball of copper may be made to float or to sink in any liquid, except mercury, according to the volume given to it, for the same mass. Let this mass be a kilogram. If solid, the volume of the copper would be 112·4 cubic centimeters; that of the corresponding mass of water being one cubic decimeter. If now the volume of the copper be increased to one cubic decimeter by making it hollow, its weight will be equal to that of the same volume of water, and it will be in equilibrium anywhere in the liquid mass. If its volume be two cubic decimeters, it will float with only one half of its volume immersed.

In the same way an iron ball will float on mercury. While iron weighs 7·8 grams per cubic centimeter, mercury weighs 13·596 grams. Hence to displace its own weight of mercury an iron ball must sink therein to only a little more than half its volume.

170. Equilibrium of Floating Bodies.—In the cases now considered, the floating body was assumed to be in equilibrium when the resultant of all the forces acting upon it was zero. This has been shown already to be the condition for zero motion of translation. If, however, the two equal and unlike forces represented by the upward and downward pressures do not act along the same line, then they constitute a couple and so produce rotation. The condition for complete equilibrium, therefore, is, not only that the resultant of the acting forces

shall be zero, but also that these forces shall act along the same straight line so as to have no moment. Since the downward force is the weight of the body and acts at its center of mass, it is necessary that the direction of the upward force shall also pass through the center of mass. But the upward force acts through the center of mass of the displaced liquid. Hence the center of mass of the displaced liquid must be vertically beneath the center of mass of the solid. The center of mass of the displaced liquid is commonly called the **center of buoyancy.**

171. Stable and Unstable Equilibrium.—The criterion of stable equilibrium is its permanency. If, therefore, the floating body be slightly displaced from its position of rest, and then left to itself, it is easy to determine whether the equilibrium be stable or unstable. In the former case the body will return to its initial position, in the latter it will depart more and more from it; the couple thus formed tending in the one case to restore the body, and in the other to overset it.

The maximum stability of a floating body is of course attained when the center of mass is below the center of buoyancy, the body then oscillating like a pendulum. But this condition of things is not possible when the body is homogeneous. It is accomplished only by weighting it.

EXAMPLES.—Thus a glass tube closed at one end may be weighted with mercury, or a wooden rod with a stone, so as to float in water without danger of oversetting: the center of mass being now below the center of buoyancy.

Homogeneous floating bodies, however, may assume a condition of equilibrium which satisfies the above criterion; though of course, under these circumstances, the center of mass is above the center of buoyancy.

Thus let the block of wood float as shown in the figure (Fig. 56), *M* being its center of mass and *B* the center of buoyancy. Since the forces acting at these points are equal and opposite, the equilibrium is complete. Now displace the block slightly (Fig. 57). The new center of buoyancy will be at *B'*; and the upward force at *B'*

forms with the downward one acting at M a couple tending to restore the body to its initial position. On the other hand, if the

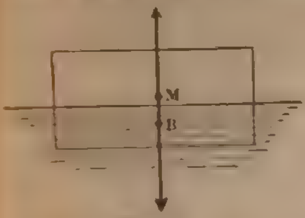


Fig. 56.

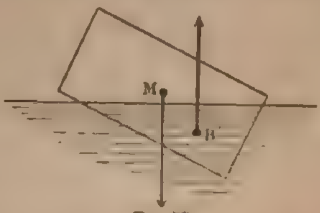


Fig. 57.

block be floating with its longer side vertical (Fig. 58), then on displacement the couple will tend to overset it (Fig. 59).

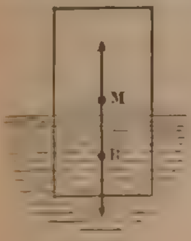


Fig. 58.

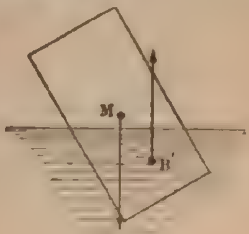


Fig. 59.

172. Measure of the Stability.—Metacenter.—Whenever the equilibrium of such a floating body is disturbed, the position of the center of buoyancy is changed. If the body be made to rotate about its center of mass, the center of buoyancy will move in the same direction as the immersed portion of the body if the equilibrium be unstable, and in the opposite direction if it be stable. Let the vertical line through the new center of buoyancy be prolonged upward until it intersects the original vertical drawn through this center and the center of mass when the body was in equilibrium. The point of intersection is called the **metacenter**, and its height above the center of mass measures the stability of the equilibrium. If the intersection be above the center of mass, the equilibrium is called **stable**. If it coincides with this center, it is **indifferent**. If it is below it, the equilibrium is **unstable**. It will be observed that the displacement of the center of buoyancy, which determines

the position of the metacenter, depends upon the shape of the body immersed, in the vicinity of the water-line. Hence the model of a sailing vessel is so contrived as to keep the metacenter as high as possible. An examination of the figure (Fig. 60) will show that the metacenter C is lowered, and the stability lessened, as the body is turned more and more. To make the metacenter as high as possible, the displacement should be indefinitely small, therefore.

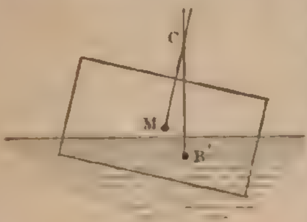


FIG. 60

Since the stability of a floating body is proportional to the height of the metacenter above the center of mass, the stability is lessened as the center of mass is raised. Hence in boats and other vessels the center of mass is kept as low as possible either by ballast or by making the keel of lead and very massive.

DENSITY AND SPECIFIC GRAVITY.

173. Definitions.—Density has already been defined as the amount of matter in the unit of volume; or in the C. G. S. system, the number of grams in a cubic centimeter of any substance. Since in this system the mass of a cubic centimeter of water is taken as the unit of mass, the number representing the density of any substance represents also the ratio of its weight to that of an equal volume of water. But this ratio is commonly called the **Specific Gravity** of the substance. It should be carefully kept in mind that density represents always a definite number of units of mass; i.e., the number contained in one cubic centimeter; while specific gravity is simply a numerical ratio, and represents the number of times a given substance is heavier than water, the volumes being the same.

174. Methods for determining Density or Specific Gravity.—Since $\delta = M/V$, the density is obtained by dividing the mass by the volume; and since the mass

can be obtained by the balance, we have only to find the volume to be able to calculate the density. So, too, specific gravity is equal to W'/W; or to the weight of the body W', divided by that of an equal volume of water W. As before, W' is obtained directly by weighing. It remains then only to find W.

If the body be a symmetrical solid, its volume can be readily calculated.

Suppose it to be a cube of brass three centimeters on a side. Its volume will be 27 cubic centimeters. Suppose further that its mass on the balance is 226·8 grams. This is M and also W'. Hence $M/V = 226·8/27 = 8·4$, the number of grams contained in one cubic centimeter; i.e., the density. In the same way, sp. gr. $= W'/W = 226·8/27$ (since 27 c.c. of water weigh 27 grams) $= 8·4$; or the body is, volume for volume, 8·4 times as heavy as water.

If, however, the solid be not a symmetrical one but be irregular in form, its volume may readily be found by the principle of Archimedes, that the loss in weight of any solid immersed in a fluid is the weight of its own volume of that fluid. Hence if the fluid be water, the excess of weight in air over that in water is W in grams or V in cubic centimeters. Whence the ordinary rule: Divide the weight of the body in air by the loss of weight in water. The quotient is the specific gravity of the body.

The specific gravity of a liquid may be determined in a similar way. Weigh any solid, such as a glass bulb, first in air, then in water, and finally in the given liquid. The difference between the first two weighings is W, the weight of its volume of water. That between the first and the third is W', the weight of its volume of the liquid. Hence $W'/W =$ the specific gravity of the liquid.

175. Equilibrium in Gases.—The Atmosphere.—The earth is surrounded by a vast aërial envelope called the atmosphere, composed essentially of two gases, oxygen and nitrogen, mixed together in the proportion of about one fifth oxygen and four fifths nitrogen. One liter of air weighs 1·2759 grams; and hence the pressure which the atmosphere exerts upon the earth's surface is

very considerable. Did we know the exact height of the atmosphere, the pressure on the unit of area would be $gh\delta$, as in the case of any other fluid. By making use of the laws of fluid equilibrium, however, this pressure may be indirectly measured. If the air be removed from a glass tube the lower end of which is placed in mercury, the mercury will rise in the tube until it reaches a height of about 76 centimeters. Since no pressure is exerted upon the top of the mercury within the tube, we infer that it is the atmospheric pressure upon the mercury in the reservoir which sustains the column and which it balances; in proof of which we find that if we remove the air from the space above this outer mercury surface the column falls. Now this column of mercury 76 centimeters high weighs $ah\delta$ grams; and this, if the area be unity, is equal to $76 \times 13{\cdot}596$ or $1033{\cdot}3$ grams. In consequence, the pressure of the atmosphere upon every square centimeter of the earth's surface is about $1033{\cdot}3$ grams, or $1033{\cdot}3 \times 980 = 1{\cdot}0126 \times 10^6$ dynes; a little more than a megadyne.

176. The Barometer.—The above experiment was first made by Viviani, a student with Torricelli, in 1643, in the following manner: He took a glass tube about a meter long, closed at one end, and filled it with mercury. Closing the open end then with the finger, he inverted the tube and placed the lower end beneath the surface of mercury contained in a jar. On removing his finger the column of mercury fell in the tube, and, after oscillating, came to rest at a height of about 76 centimeters. The experiment was repeated and published by Torricelli, and is generally known by his name. The vacant space above the mercury column is known as the Torricellian vacuum, and the entire apparatus, since it measures the pressure of the atmosphere, is called a **barometer.**

In further proof that the barometric column is supported by atmospheric pressure two experiments may be mentioned, both due to Pascal. In the first the mercury was replaced by other liquids of differing densities, and

it was found that the height of the column was always inversely proportional to the density of the liquid employed; in entire accordance with the law of fluid equilibrium. In the second the mercury barometer was taken by Perier, at the request of Pascal, to the top of Puy-de-Dôme, a hill in Auvergne, about 1000 meters high; and he observed a fall in the column of nearly eight centimeters.

As the total atmospheric pressure is now known, and also the density of the air at the earth's surface, we may calculate the height of the atmosphere, supposing its density uniform, from the formula $h = P/g\delta$; in which P is the pressure of the air in dynes upon unit of surface, and δ the density. Performing the division, we have $\dfrac{1{\cdot}0126 \times 10^6}{980 \times {\cdot}0012932} = 7{\cdot}99 \times 10^5$ centimeters, about, or $7{\cdot}99 \times 10^3$ meters. This, therefore, would be the height of the atmosphere, provided that its density throughout were the same as at the surface. This value is sometimes called the height of the homogeneous atmosphere.

177. Barometric Measurement of Height.—Hypsometry.—The density of the air, however, is not uniform as we ascend, but diminishes very rapidly, in accordance with the law of Boyle, presently to be considered. The law of this diminution may be illustrated as follows: Suppose three air-layers of the same indefinitely small thickness (Fig. 61), between which the density may be considered uniform, though decreasing with each successive layer. These densities, $\delta, \delta_1, \delta_{11}$, will be proportional to the weights of the layers, which weights are themselves proportional to the difference of

FIG. 61

the pressures; and hence $\dfrac{p - p_1}{p_1 - p_2} = \dfrac{\delta}{\delta_1}$. But Boyle's law states that $\dfrac{\delta}{\delta_1} = \dfrac{p}{p_1}$; hence $\dfrac{p - p_1}{p_1 - p_2} = \dfrac{p}{p_1}$, and consequently

$\frac{p}{p_1} = \frac{p_1}{p_2}$, and so on. Therefore the ratio of the densities is constant for each succeeding pair of air-layers. Let n be this ratio, so that $\delta/\delta_1 = n$, $\delta_1/\delta_2 = n$, $\delta_2/\delta_3 = n$, etc. Multiplying these equations member by member, we have $\delta/\delta_3 = n^3$. Or in other words, as the height increases by addition, or in arithmetical progression, the density decreases by multiplication, or in geometrical progression. Since the pressure decreases by an amount $gh\delta$ on unit area for an increase in height h, it will decrease by the value $g\delta dx$ for the variation dx. Hence we have $dp = -g\delta dx$; or, since $p = gH\delta$, $dp = -\frac{p}{H}dx$. Transposing, $\frac{dp}{p} = -\frac{dx}{H}$; whence $H \log_e \frac{p_1}{p_2} = x_2 - x_1$. In this formula H represents the height of the homogeneous atmosphere above given. If the pressures be represented by the barometric heights B and b, x_1 be made zero (i.e., the height be measured from the sea-level), the logarithms be made common, and H be replaced by its value, we have

$$x_2 = 18190 \log \frac{B}{b} \qquad [33]$$

as the equation by which the height of any station above the sea-level can be calculated by means of the barometric readings at the two stations.

Illustrations.—To exemplify the use of this formula, let it be required to ascertain the height at which the barometer would stand at one millimeter. Making $B = 760$ and $b = 1$, we have $18190 \times \log 760 = 18190 \times 2{\cdot}88 = 52400$ meters. Hence at a height of 52·4 kilometers the air is so rare that it will support a column of mercury only about one millimeter in height. Making b one millionth of 760, or ·00076, we find that $x_2 = 109140$ meters, or a little more than twice the former height. Whence it follows that at a height of about 109 kilometers the air would equal in rareness the best obtainable vacuum.

It is evident from the equation just considered that there is no proper sense in which we can use the term

"height of the earth's atmosphere"; since the value of x_2 becomes infinite if b be made zero. In other words, at every finite distance from the earth the height of the barometer will have a finite value. This agrees entirely with what we know of gases, i.e., that they can have no free surface.

In applying this formula practically, various corrections for temperature, moisture, and the latitude of the station are required, so that the complete expression for the height in meters of a given station is

$$\text{Height} = 18404 \log \frac{B}{b} \left\{ \left(1+\frac{2(T+t)}{1093}\right)\left(1+\frac{3e}{8B}\right)\left(1+0{\cdot}00259 \cos 2\lambda\right) \right\} \quad [34]$$

The first correcting term is for temperature, the second for moisture, and the third for latitude. This formula is due to Laplace.

178. Forms of Barometer.—The normal barometer, used always as a standard instrument and therefore fixed in position, consists of a large tube of glass usually about two centimeters in diameter, carefully filled with mercury and firmly supported in a vertical position. Its lower end, narrowed to a small opening, is placed beneath the surface of mercury contained in a suitable tank. The readings are made with a kathetometer.

The most common form of portable barometer is that of Fortin. Its tube is a centimeter or less in diameter, and is enclosed within a tube of brass, to the lower end of which the reservoir is attached. The lower portion of this reservoir is made of chamois leather, against which a screw presses from below; so that when turned in sufficiently the mercury column is raised to fill the tube, and in this condition it may be transported without danger. The scale is engraved on the brass tube, the zero mark being an ivory point within the reservoir, which at top is of glass. Before reading, the level of the mercury in the reservoir is brought exactly up to the ivory point. The reading is made by means of a vernier, the limit being generally 0·02 millimeter.

In Gay Lussac's barometer, frequently called the siphon barometer, there is no distinct reservoir, the tube being recurved at bottom so that the two surfaces of mercury are in the same vertical line. The height of the column is the distance from the lower meniscus to the upper one. And since the diameter of the tube is the same at these two points, Gay Lussac supposed that the capillary effects would neutralize each other and that no correction for capillarity would be needed. This, however, is not the fact; and moreover, this arrangement renders the capillary correction uncertain. The Fortin barometer is preferred, therefore, for accurate work.

The **aneroid** barometer, as its name implies, is a barometer without liquid. Its essential part is a thin circular box of metal having a corrugated top, suitably connected with the index-hand by a system of multiplying levers. This box is partially exhausted of air and then sealed. As the atmospheric pressure varies, the top of the box rises and falls, and this motion suitably magnified is indicated upon the dial. When made with sufficient care and frequently compared with the mercurial barometer, these instruments may be made of excellent service in hypsometry.

179. Use of the Barometer in Meteorology.—The barometric height is not constant, however, even for an instrument fixed in position. Variations are observed in it which at first sight appear entirely irregular, but in which periodicity is readily found by inspection of the barometric curves drawn by a registering instrument. It is then noticed that there are daily, monthly, and annual maxima and minima for every locality, but that the amplitude of the oscillation at different localities is itself variable within considerably wide limits. Maxima of the daily variation occur (in the tropics with great regularity) at about 9 A.M. and 9 P.M.; and minima at about 4 A.M. and 4 P.M. The amplitude of this variation—which reaches its minimum in the winter—varies from 0·2 to 2·36 millimeters according to the latitude.

The mean monthly variation of the daily means attains its maximum in winter, its minimum in summer; the amplitude varying from 3 to 36 millimeters as the latitude increases. Lines drawn through places having the same mean monthly amplitude of barometric variation are called **isobarometric** lines. The annual variation also attains its maximum in January and its minimum in July, the maximum amplitude being about 17 millimeters.

In the temperate zones, however, the irregular fluctuations of the barometer are so great as almost entirely to mask the periodic variations. Since a rise of temperature diminishes the air-pressure, the barometer falls, in general, as the thermometer rises. Consequently with the irregular temperature variations observed in middle latitudes, irregular barometric fluctuations would be expected. Again, moist air being lighter than dry, the height of the barometric column is a function of the amount of moisture in the air. In the ordinary weather-charts, lines are drawn passing through places of equal pressure and called **isobaric** lines or isobars; these lines being separated by a difference, say, of five millimeters of barometric height. Sometimes the lowest isobar encloses an area more or less circular. This is called a **center of depression**. Because of its low pressure, air will flow into it from all directions; and hence the direction of the wind is always toward such areas. Moreover, where these isobars are closest, there the barometric gradient is steepest and there the wind will be strongest. The direction of the wind is, however, modified by the earth's rotation, so that the air moves toward these centers spirally, the direction of rotation in the northern hemisphere being that opposite to which the hands of a watch move. Under certain conditions, a rotating storm of great violence may thus be engendered, called a **cyclone**; in the interior of which, owing to the centrifugal action, there is a still further depression of the barometer. Moreover, the cyclone itself is generally in motion with a high velocity.

B.—KINETICS OF FLUIDS.

(a) Mass-kinetics.

180. Kinetics of Liquids.—The flow of liquids in general is produced by the action of gravity. Suppose, for example, an opening to be made in the bottom of a cylindrical vessel near its center, the vessel being filled with liquid. It is required to determine the speed of the outflow, the depth of the liquid being maintained constant. If V represent the volume of liquid discharged per second, $V\delta$ will represent the mass of liquid thus flowing out, or m. When at a (Fig. 62), the top of a liquid column of height h, the potential energy stored up in this liquid mass is mgh units. Suppose that the liquid issues at b with a speed s; its kinetic energy will be $\frac{1}{2}ms^2$ units. Since to acquire this speed the potential energy of the mass has been transformed into an equal kinetic energy, we may equate these values; $mgh = \frac{1}{2}ms^2$. Whence $s^2 = 2gh$ and $s = \sqrt{2gh}$. From this it will be seen: 1st, that the speed with which a liquid issues from an orifice is the same as that which would be produced in the mass by falling freely from the same height; 2d, that this speed is directly proportional to the square root of the "head," or the depth of the orifice below the surface; and 3d, that, since the speed of efflux is independent of the particular liquid used, it follows that all liquids issue under the same head with the same speed. This law of flow is known as the **law of Torricelli**, its discoverer.

FIG. 63.

EXPERIMENTS.—The law of Torricelli may be readily verified experimentally. If the vessel have a lateral tubulure, in the top of which an opening is made, the liquid will issue vertically, and will rise to a height nearly equal to the level of the liquid in the reservoir. But for the friction at the orifice, the impact of the falling liquid, and the resistance of the air, the height would be the same. Again, if the jet issue horizontally with the speed s, it will immediately begin to fall under the action of gravity, and its path will be a

rabola. If $y = st$ be the horizontal distance passed over in the [tim]e t, and $x = \frac{1}{2}gt^2$, the vertical distance, we have, by eliminating [t b]etween these two expressions, $y^2 = (2s^2/g)x$; which, since the [ab]scissa varies as the square of the ordinate, is the equation of a [pa]rabola referred to its axis and a tangent at the vertex. Since the [foc]al distance is one fourth of the latus rectum $2s^2/g$, it must be [eq]ual to $s^2/2g$. But this is the distance h of the orifice below the [su]rface. Hence the focus of the parabola is as much below the [or]ifice as the surface of the liquid is above it; and the curves are [fla]tter the greater the head under which they issue.

181. Amount of Flow.—Contracted Vein.—It would [s]eem at first that the speed of efflux might be readily as[c]ertained from the volume of liquid issuing per second. [I]f the area of the orifice is a square centimeters, and a [v]olume of b cubic centimeters issues in the time t, then [t]he speed $s = b/at$ centimeters per second. But it is [f]ound in practice that for a circular orifice in a thin [p]late the amount of flow is only about 62 per cent of the [c]alculated value. This arises from the fact that those [p]ortions of the liquid in the vessel which are not in a di[r]ect line with the orifice exert a lateral pressure upon the [is]suing jet; so that instead of being cylindrical in form [t]his jet is conical, diminishing in size as it issues, and [r]eaching a minimum cross-section—about 62 per cent of [t]he area of the orifice—at a distance equal to the diame[t]er of the opening. This conical jet is known as the con[t]racted vein, or **vena contracta**. In order to increase the [f]low, short tubes—called **ajutages**—are fitted to the aper[t]ure. And it is found that a cylindrical ajutage whose [l]ength is two or three times its diameter increases the [f]low to 82 per cent of the theoretical amount. If the [a]jutage be conical, with its smaller end outward, the flow [i]s raised to 92 per cent. And if the larger end of the [a]jutage be outward, it is still further increased, now fall[i]ng only very little short of the theoretical value. If, [h]owever, the ajutage be very long, say 48 times its diam[e]ter, the flow is again diminished, in this case to 63 per [c]ent, owing to the friction due to the viscosity of the [l]iquid. The increased flow produced by the ajutage is [a]ttributed simply to the *adhesion* of the liquid to the

walls of the tube, thus making the issuing jet more nearly cylindrical.

182. Flow of Liquids in Tubes.—If the walls of the tubes be wetted by the liquid, the layer of liquid in contact with these walls does not change, except by diffusion. Hence the mass of the liquid flows by this layer, and the only loss of energy is due to the viscosity of the liquid itself. On the other hand, if the liquid does not wet the walls of the tube, there is friction between the liquid and the walls and a loss of energy is the result. If a continuous flow be maintained at the end of a uniform tube, the speed of issue supposes at that point a certain head h, deduced from the formula $s^2 = 2gh$. This is called the **velocity-head.** In order to maintain this speed at the end of the tube a certain total head must be maintained at the reservoir. The velocity-head being constant throughout the tube, the difference between the total head and the velocity-head at any part of the tube measures the hydrostatic pressure. This is called the **pressure-head.** By placing vertical tubes at different distances along the tube the pressure-head at any point is indicated by the rise of the liquid at that point. The

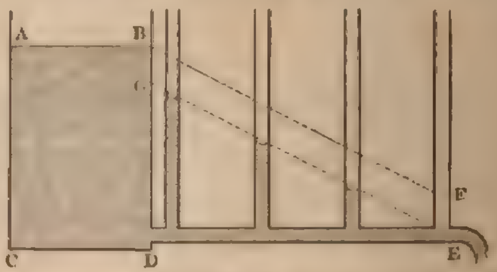


FIG. 63.

fall of pressure in the tube is uniform throughout its length, and is the more rapid the shorter the tube. The pressure-head at any point measures the resistance in the tube beyond this point. This resistance is a function not only of the length and of the diameter of the tube, but also of the speed of the liquid within it. Thus, for

example, in Figure 63, the total head in the reservoir $ABCD$ is the height of the liquid BD, the pressure-head is the height GD, and the velocity-head is the difference BG. The fall of the pressure-head along the horizontal tube DE is indicated in the vertical tubes, and takes place along the dotted line GE; the fall in the total head being represented by the dotted line BF. Since these lines are parallel, their distance apart, which represents the velocity-head, is evidently constant.

If the tubes be of small diameter, the law of flow is materially altered. Poiseuille found that in such tubes the volume of liquid flowing in a unit of time is represented by the equation $V = kr^4H/l$, in which r is the radius and l the length of the tube, H the head, and k an experimental constant. In his experiments the time required for a known volume of liquid to flow through a tube of given length and diameter was noted. He observed that the constant k is independent of the material of the tube if its walls be wetted by the liquid, and is a function of the liquid and of the temperature only. From the above expression it appears that the speed of flow in tubes of small diameter is directly proportional to the pressure and to the fourth power of the radius. The resistance in such tubes is directly as the speed. The size of tube necessary to produce this altered law of flow is determined by the liquid itself. For water the tube must be not over one half a millimeter in diameter. But molasses obeys the law of Poiseuille as well in a tube 25 millimeters in diameter as water does in the half-millimeter tube. Indeed it has been shown that it is not until the ratio of the speed to the viscosity, multiplied by the diameter of the liquid column, reaches a certain critical value that this law ceases to represent the flow. If the speed or the cross-section be too great, the current develops eddies, and the law of flow is that for wide tubes. But if the viscosity of the liquid increases proportionately, the critical value may not be reached and Poiseuille's law of flow may be maintained. Moreover, by suitably varying the speed, the same tube and

the same liquid may be made to show the phenomena characteristic of wide or of narrow tubes.

183. Viscosity of Liquids.—Viscosity, as already stated, is a resistance to flow due to the internal friction of the particles of a liquid upon one another. If this resistance becomes infinitely great, the body is perfectly rigid; if zero, it is perfectly mobile. There is no difference, consequently, between a fluid of infinite viscosity and a rigid solid. All bodies, whether solid, liquid, or even gaseous, are more or less viscous. Tresca has shown that under a pressure of 100,000 kilograms per sq. cm. solids such as lead, silver, clay, ice, and even iron and steel, may be made to flow like fluids. When a viscous body is subjected to shear, there is a change of form without change of volume. If a vessel containing liquid be tipped, the liquid assumes a new position, having undergone a shear. The rapidity with which this new position is assumed varies widely for different liquids; being most rapid in mobile liquids such as ether, least so in viscous liquids such as sirup. When a river flows in its bed, the surface liquid moves most rapidly; so that there is a constant flowing of the upper layers over the lower. And the ratio of the linear displacement to the depth, which is equal to tan α, the angular displacement, and which measures the shear, is constant for all depths. The viscosity of the liquid retards this sliding of the layers and hence may be measured by the ratio of the shearing stress to the amount of shear produced in a unit of time. This ratio is called the **coefficient of viscosity** and it is usually represented by μ. Hence we have $\dfrac{\text{Shearing stress}}{\text{Shear per unit of time}} = \dfrac{F}{\tan \alpha} = \mu$, the coefficient of viscosity. If the displacement in unit of time be unity, and the depth be also unity, tan α becomes unity and the coefficient of viscosity is equal to the shearing stress Hence Maxwell defines this coefficient as "the tangential force on the unit of area of either of two horizontal planes at the unit of distance apart, one of which is (relatively) fixed while the other moves with the unit of speed,

the space between being filled with the viscous material."

Now it can be shown that k, the constant determined for the flow of liquids through small tubes by Poiseuille, as above described, is equal to $\pi\rho g/8\mu$; whence $\mu = \pi\rho g/8k$. By knowing, therefore, ρ, the density of the liquid, and k, which is equal to Vl/Hr^4, the coefficient of viscosity may be readily calculated. In water at 0°, its value is 0·018; at 10°, 0·013; and at 20°, 0·010. In other words, there exists between each of two parallel plane surfaces, one centimeter apart, when one is moving a centimeter per second relatively to the other, the space between them being filled with water at 0°, a tangential stress of 0·018 dyne on each square centimeter.

By oscillating a disk torsionally in its own plane in a given liquid, Meyer has made independent determinations of viscosity. He obtained for water at 0·6°, 0·0173; at 45°, 0·0583; and at 90°, 0·00339. For air he obtained the value 0·00017 $(1 + 0·00733t)$; so that although water is 770 times as dense as air, it is only 100 times as viscous. For brass the value of μ was found to be about 300,000,000,000; i.e., 3×10^{11}.

EXAMPLES.—It is easy now to understand the rapidity with which fluids come to rest after being disturbed. It is the viscosity of water which stills the waves on the ocean, and the viscosity of air which causes the tempest to gradually die out. The speed with which air-bubbles ascend in water and in glycerin is an instructive example of relative viscosity. So, too, the settling of fine particles in water—to which the color of the ocean and of the Swiss lakes is due—is a matter determined by viscosity. Dust, whether composed of solid or liquid particles, is retarded in falling by the viscosity of the air. Stokes has calculated that a water-drop 0·025 of a millimeter in diameter falls in still air only about four centimeters per second, and that if its diameter be reduced ten times it will fall one hundred times slower, or only about 2·5 centimeters per minute.

184. Stream-flow.—Law of Continuity.—When a liquid flows in an open channel, like the water of a river, the conditions of flow are in many respects the same as in a tube. It is common to represent the flow by a series of imaginary lines along which the elements

of liquid are supposed to flow. These lines are called **stream-lines** or lines of flow. So long as the conditions remain the same, the channels not changing in cross-section or slope, these lines remain parallel. They widen as the space enlarges and contract as the space diminishes. If successive cross-sections be taken on any stream, it is evident that when the current has become steady, the amount of liquid which crosses each section in a unit of time must be the same; since otherwise liquid would accumulate somewhere between the cross-sections. This conception when made general constitutes what is known as the **law of continuity**. If we imagine a space fixed in the interior of a fluid, and consider the fluid which flows into this space and the fluid which flows out of it across different parts of bounding surface in any time; then it is obvious that if we suppose the fluid to be of the same density and incompressible, the whole quantity of matter within the given space must remain constant and hence the quantity flowing out in the given time must be equal to the quantity flowing in. Consequently the rate of increase of density of the fluid in unit of time within the fixed space is to the actual density at any instant as the rate of flow of fluid into that space is to the entire quantity of matter within it.

(b) *Molecular Kinetics.*

185. Kinetics of Gases.—In the present view of science, all bodies consist of a finite number of small parts called molecules, each molecule having a definite mass and being, for the same substance, exactly like every other. The kinetic theory of matter supposes that these molecules are in active motion, and that to this motion many of the properties of matter are due. "In gases and liquids," says Maxwell, "this motion is such that there is nothing to prevent any molecule from passing from any part of the mass to any other part; but in solids we must suppose that some at least of the

molecules merely oscillate about a certain mean position." In gases the molecules are not acted upon, for the greater part of their course, by any sensible force, and therefore move in straight lines with uniform speed. But, since a cubic centimeter of any gas contains 6×10^{21} molecules, it is clear that the moving molecule cannot travel very far without encountering another molecule. The mutual action between them is analogous to that between two elastic balls (156), the molecules acting on each other for a finite time during which the centers first approach and then separate. This mutual action is called an **encounter**, and the course of the molecule between one encounter and another is called the **free path** of the molecule. Under ordinary conditions the time occupied by the encounter is very much less than that of the free motion; but as the gas becomes denser the length of the free path diminishes, until finally, as seems to be the case in liquids, no part of the motion can be correctly spoken of as the free path. In a gaseous mass every molecule will change both its speed and its direction at every encounter; so that of the molecules composing the system some are moving very slowly, a very few are moving with enormous speeds and the greater number with intermediate speeds. By adopting a statistical view of the system, however, and distributing the molecules into groups according to the speed with which at a given instant they happen to be moving, Maxwell has deduced some remarkable conclusions as to the molecular kinetics of gases. In order to compare two such gaseous systems, the best method, he says, is to take the mean of the squares of all the velocities. This is called the mean square of the velocity, and its square root is called the velocity of mean square. If two gases having different molecular masses be mixed together, they will exchange energy in their encounters, until every molecule of either gas possesses the same kinetic energy. This average kinetic energy of a single molecule may be represented, therefore, by $\frac{1}{2}ms^2$ if m represents its molecular mass and s^2 the mean square of its speed.

Moreover if we suppose a plane surface within any gas in the condition of equilibrium, the principle of continuity teaches us that if there is no accumulation upon either side of it, the number of molecules which pass through the surface in one direction must be exactly equal to the number passing in the opposite direction. The term "velocity of a gas" may mean, therefore: 1st, mass-velocity, or the velocity of the center of mass of all the molecules composing the system; 2d, molecular velocity, or the velocity of the molecule itself considered as a whole, made up of encounters and free paths; and 3d, atomic velocity, or that of the component parts of the molecule whether of vibration or rotation.

If we consider the wall of a vessel enclosing a gas, it is evident that its internal face must be struck by the molecules moving perpendicular to it, and that the momentum of these molecules must produce a pressure outward upon this face, which is balanced by a counter pressure exerted by the wall itself. If, for example, the vessel be a cube of unit volume, its face will be a unit of surface. If s be the velocity of mean square of the molecules normal to this face, and n the number of molecules in unit of volume moving in this direction, then ns will be the number of impacts in unit of time. If the molecular mass be m, the average momentum of each molecule will be ms and the total momentum expended upon this face of the cube per second will be mns^2. But this is equal to p, the pressure upon this same unit of surface. Since nm represents M, the total mass of the gas, it must equal ρv; or as v, the volume, is unity, $p = \rho s^2$. But s is the velocity-component in one direction only. If we let r and t be the components perpendicular to s and to each other, and V the velocity of mean square in any direction whatever, then, resolving V in these three directions, we have $V^2 = r^2 + s^2 + t^2$; i.e., the mean square of the resultant velocity is equal to the sum of the mean squares of the component velocities. Since the pressure in a gas at rest is the same in all directions, $s^2 = r^2 = t^2$ and $r^2 + s^2 + t^2 = 3s^2$.

Hence $V^2 = 3s^2$ and $s^2 = \frac{1}{3}V^2$. Whence $p = \rho s^2 = \frac{1}{3}\rho V^2$. Multiplying both sides by the volume v, we have $pv = \frac{1}{3}\rho v V^2 = \frac{1}{3}MV^2 = \frac{2}{3}(\frac{1}{2}MV^2)$. Or the product of the pressure by the volume in any gas is two thirds of the molecular energy of translation of that gas, provided that a mass M is contained in the volume v.

We may apply this result practically as follows:

1st. The pressure is the same for all gases at the same temperature. Hence in equal volumes the product pv must be constant for all gases. The total molecular energy of translation, therefore, which equals $\frac{3}{2}pv$, must be the same, at the same temperature, in equal volumes of all gases.

2d. Suppose two gases of molecular masses m_1 and m_2, having V_1 and V_2 for the velocities of mean square, p_1 and p_2 their pressures, and n_1 and n_2 the number of molecules in unit of volume, respectively. Then we have $p_1 = \frac{1}{3}\rho_1 v_1 V_1^2 = \frac{1}{3}m_1 n_1 V_1^2$, and $p_2 = \frac{1}{3}\rho_2 v_2 V_2^2 = \frac{1}{3}m_2 n_2 V_2^2$. If these pressures are equal, $m_1 n_1 V_1^2 = m_2 n_2 V_2^2$. But we have seen above that when the temperatures are equal, all gaseous molecules possess the same kinetic energy; i.e., $m_1 V_1^2 = m_2 V_2^2$. Dividing the former equation by the latter we have $n_1 = n_2$; or, the temperature and pressure being the same, the number of molecules in unit of volume is the same for all gases. This is the law of Avogadro.

3d. Since $m_1 n_1$ is the mass of a gas in unit of volume, it may be represented by ρ_1, the density; and so $m_2 n_2$ may be represented by ρ_2. If then $\rho_1 = m_1 n_1$ and $\rho_2 = m_2 n_2$, we have, since $n_1 = n_2$ as above, $\rho_1 : \rho_2 :: m_1 : m_2$; or, the densities of two gases, at the same temperature and pressure, are proportional to their molecular masses. This is the law of Gay Lussac.

4th. From the equation $p = \frac{1}{3}\rho V^2$ above given, we have $V = \sqrt{3p/\rho}$, from which we can find the velocity of mean square of any gas; a calculation first made by Joule. If we take hydrogen at atmospheric pressure we have $p = 1033·3$ grams or $1·0126 \times 10^6$ dynes. The density of hydrogen ρ is 0·00008957 grams per cubic

centimeter. Hence $\sqrt{3p/\rho} = 184260$ centimeters or 1842·6 meters. This is the velocity of mean square of hydrogen molecules.

TABLE OF MOLECULAR DATA (Maxwell).

	Hydrogen.	Oxygen	Carbon monoxide.	Carbon dioxide
Mass of molecule (hydrogen = 1)...	1	16	14	22
Velocity (of mean square) in meters per second at 0° C..	1859	465	497	396
Mean free path in tenth-meters....	965	560	482	[illegible]
Collisions in a second (millions)....	17750	7646	9489	9720
Diameter, tenth-meters.	5·8	7·6	8·3	9·3
Mass, twenty-fifth grams.....	46	736	644	1012

The values in the second and third lines are placed in the first rank as being known with the highest accuracy, those in the fourth line in the second, and those in the fifth and sixth in the third rank. A tenth-meter is 10^{-10} of a meter; and a twenty-fifth gram is 10^{-25} gram.

(*c*) *Diffusion of Gases and Liquids.*

186. Diffusion of Gases.—The extreme rapidity with which the molecules of gases diffuse through the air is a matter of common observation. A bubble of chlorine gas set free in a large room is perceived by its odor within a few seconds, throughout the entire space. Indeed so prompt and so perfect is this diffusion that Dalton formulated the fact in the statement: Every gas is to every other gas as a vacuum.

The phenomenon of diffusion follows necessarily from the kinetic theory of gases. Since all gaseous molecules move in straight lines with the very high speeds already given, it is evident that, notwithstanding their frequent encounters with other molecules, they must advance with considerable rapidity. Hence the interpenetrating power of two gases when mixed is very great. Indeed the relative speed of diffusion may readily be calculated

from the formula $p = \frac{1}{3}\rho V^2$, which represents the pressure in terms of the density and the mean velocity. For under the same atmospheric pressure we should have for two gases of densities ρ and ρ', $\rho V^2 = \rho' V'^2$; or $V : V' :: \sqrt{\rho'} : \sqrt{\rho}$. The speeds with which two gases diffuse, either into each other or into a third gas, therefore, are inversely proportional to the square root of the densities of these gases. This law has been experimentally established by Loschmidt for the case where no porous partition separates the gases; and by Graham for that where such a porous septum is placed between them. In the experiments of Graham the septum was of compressed graphite, the action being strictly molecular.

EXPERIMENT.—The phenomenon of diffusion through a porous septum may be shown very well (Fig. 64) by cementing an ordinary porous battery-cell to a funnel having a long tube and supporting the whole on a stand, with the end of the tube dipping under some colored water. If now a bell-jar filled with coal-gas by displacement be placed over this porous cylinder the more rapid diffusion inward of the less dense coal-gas will cause an active bubbling of the expelled air from the lower end of the tube. On removing the bell-jar the diffusion outward will now be more rapid than that inward, and the column of water will rise in the tube. If carbon dioxide gas be used, these actions will all be reversed.

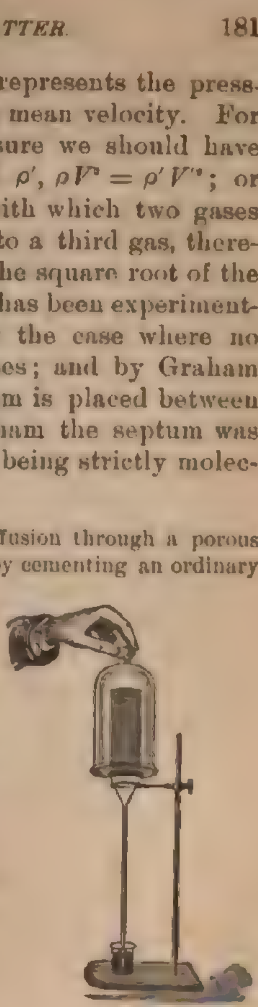


FIG. 64

187. Effusion and Transpiration.—The law regulating the flow of gases through a minute opening in a metallic plate has also been investigated by Graham, who has called the process **effusion**. The phenomenon is of the same character as the efflux of liquids, the speed of effusion into a vacuum being given by Torricelli's theorem $v^2 = 2gh$. In other words, air under the ordinary atmospheric pressure will pass into a vacuum with the speed which it would acquire in falling through the height

of the atmosphere, supposed of uniform density; i.e., through the height of the homogeneous atmosphere, which is $7{\cdot}99 \times 10^5$ centimeters. Hence the speed of effusion, which is $\sqrt{2gh}$, is $\sqrt{2 \times 980 \times 7{\cdot}99 \times 10^5} = 39573$ centimeters per second. For different gases, under the same pressure, $h\delta = h'\delta'$; and hence $h : h' :: \delta' : \delta$. Therefore the speed of effusion, which is proportional directly to the square root of the height, is proportional inversely to the square root of the density of the gas.

Bunsen has made use of this law in an apparatus which he has devised for determining the density of gases from their times of effusion. Since the speed of effusion varies inversely as the time, $t : t' :: \sqrt{\delta} : \sqrt{\delta'}$; or the densities of two gases are directly proportional to the squares of their times of effusion.

A third phenomenon investigated by Graham is that of the passage of gases under pressure, through long and very fine tubes, called by him **transpiration**. He found that the results were much more complex than in the other cases, and, although independent of the material of the tube, were probably due to viscosity in the gas. Thus, for example, the rate of transpiration for hydrogen was only double that of nitrogen, and that for carbon dioxide was greater even than that for oxygen. The times for oxygen, nitrogen, carbon monoxide, and air were found to be directly as their densities; i.e., equal masses of these gases pass in equal times.

188. Adhesion between Gases and Solids.—Occlusion.—Besides the kinetic diffusion just discussed, another form of diffusion exists in which the specific nature of the solid and of the gas plays an important part. The surfaces of all solids appear to possess the power of condensing gases upon them to a greater or less degree. And hence the larger the surface for the same mass, the greater the condensation. A cube one centimeter on a side has a surface of six square centimeters. This cube can be divided into 1000 cubes each a millimeter on a side, and each having a surface of

six square millimeters. By this subdivision, therefore, the total surface has been increased tenfold. According to Mitscherlich, one cubic centimeter of boxwood charcoal exposes a surface within its pores of over 4000 square centimeters. And Saussure found that this charcoal would absorb 90 times its volume of ammonia gas and 85 times its volume of hydrogen chloride. But this even is surpassed by cocoanut charcoal, which, according to Favre, absorbs 172 volumes of ammonia, 97 volumes of carbon dioxide, 99 of hyponitrous oxide, and 165 of hydrogen chloride. In the first case 494 heat-units are evolved. Obviously in these cases the absorption is the greater in proportion as the gas is more easily condensable to a liquid; hydrogen, for example, being absorbed only to the extent of two volumes.

A similar action is observed with metals. The increase of mass observed in a platinum dish when it is allowed to stand after weighing is due to gas condensed upon its surface. Sheet platinum condensed four volumes, silver one volume, and iron 0·44 volume of hydrogen, and silver seven volumes of oxygen, in Graham's experiments. In the finely divided and therefore highly porous forms of platinum sponge and platinum black, this metal is very active and condenses 250 volumes of oxygen into its pores. Palladium, however, of all the metals, possesses this property to the most remarkable extent, Graham having shown that it is capable of condensing 980 volumes of hydrogen into itself. Since the metal is not porous like charcoal, this absorption of gases has received the name **occlusion** (Graham). The absorption of hydrogen may be effected (*a*) by electrolysis, using a strip of palladium as the kathode; (*b*) by heating the metal in vacuo, then admitting hydrogen and allowing it to cool; and (*c*) by passing the gas over heated palladium and cooling it in a current of the gas. By this occlusion the volume of the metal is increased by 0·09827 of its initial bulk; whence the condensation of the hydrogen must have reduced it to 9868 times its normal density, or to 0·88 as compared with water.

EXPERIMENTS.—1. Introduce into a tall glass jar filled with ammonia-gas and standing over mercury (Fig. 65), a piece of charcoal, made from the shell of the cocoanut, which has been heated to redness in sand just previous to use and allowed to cool away from the air. The gas will be absorbed and the mercury be seen to rise in the tube. It is to the oxygen thus condensed in the pores of charcoal that its efficiency as a disinfectant and decolorizer is due.

FIG. 65.

2. Place a fragment of spongy platinum in front of an escaping jet of hydrogen. It will at once glow from the heat evolved by the union of the hydrogen with the occluded oxygen, and finally light the gas at the jet. Platinum black will act in the same way, and even asbestus which has been soaked in a solution of platinic chloride, dried, and afterward ignited.

3. Place a coil of platinum wire or a small platinum spoon in the flame of a Bunsen burner until fully ignited, and then turn off the gas. If now the coil or spoon be suitably placed above the burner and the gas turned on, the metal will rise to full redness and, if placed a little to one side, will finally light the gas.

4. Place in a flat dish containing dilute sulphuric acid two coiled strips of metal, one platinum, the other palladium, the latter being varnished on its outer side and provided with a pointer at its free end. Make the palladium the kathode and send a current through the cell thus arranged. The absorption of the hydrogen will take place on one side of the palladium only, and will cause that electrode to uncoil, as will be shown by the pointer. On reversing the direction of the current, the kathode will return to its former position.

This absorption of gases by metals appears to be of the nature of true solution, in which the metals behave like colloid substances; the phenomenon being analogous to the absorption of carbon dioxide by caoutchouc, which takes up two per cent of it. Graham showed that while a platinum tube 1·1 millimeters thick and having a surface of one square meter will transmit at a red heat 489 cubic centimeters of hydrogen per minute, a caoutchouc film 0·014 millimeter thick and having the same surface will transmit only 129 cubic centimeters per minute at the temperature of 20°. Deville many years ago observed that "the permeability of such homogeneous substances as platinum and iron is quite different from

the passage of gases through such non-compact substances as clay and graphite." In the former case the gas dissolves in the colloid metal, traverses it and evaporates on the other side, as it would do in the case of a liquid film. Thus hydrogen traverses the walls of a rubber balloon, and carbon dioxide those of a soap-bubble, more readily than air; so that in air a hydrogen balloon and a carbon dioxide soap-bubble both contract. But if they both be filled with air and placed, the balloon in hydrogen and the soap-bubble in carbon dioxide, they both will expand. In both cases the gas dissolves in the material composing the septum, passes through it by diffusion and then evaporates on the other side.

Chemical reactions, however, seem to intervene in these cases quite prominently. Graham proved that the occlusion of hydrogen by palladium gives rise to the production of a definite hydride Pd_2H. And the discovery of the fact that sodium unites with 238 volumes of this gas to form sodium hydride Na_2H, in which the hydrogen has a definite pressure corresponding to the temperature, sustains the opinion of Mendeléeff that occlusion "presents a similar phenomenon to solution, based as it is on the capacity of metals of forming unstable easily dissociating compounds with hydrogen similar to those which salts form with water." The transfer of carbon monoxide through red-hot cast-iron becomes explicable through Mond & Langer's discovery of iron-carbonyl $Fe(CO)_4$. And Berthelot considers that the oxygen in platinum black exists in the form of an unstable suboxide, to which is due its ready action upon hydrogen.

189. Diffusion of Liquids.—In liquids, too, molecular motion is active in producing diffusion. Even in homogeneous liquids there is a constant transference of the individual particles from place to place throughout the mass. If a strong solution of any substance be carefully introduced beneath a mass of water, it will be found that the heavier solution diffuses into the lighter one, even against *gravity*. If the denser solution

be a colored one, the progress of the diffusion can be watched by the eye. Graham found that if the time required for a given mass of hydrochloric acid to diffuse into water be taken as unity, that required by common salt is 2·33, sugar 7, magnesium sulphate 7, albumen 49, and caramel 98. So that sugar travels seven times as far as albumen in the same time, and salt three times as far as sugar. In these experiments, Graham used a wide-mouthed bottle nearly full of the solution, placed in a jar of water whose surface was about one or two centimeters above the top of the bottle. After a given time the amount of diffused solid was determined by analysis. It was found to be proportional (1) to the time, (2) to the strength of the solution, (3) to the temperature, and (4) to the coefficient of diffusion of the substance used. This coefficient may be defined as the mass of any substance which passes through unit surface in unit time, in a solution where the fall of concentration for unit of length is unity; i.e., in which unit of mass of substance is contained in unit volume of the solution. Thus measured the coefficient of hydrochloric acid at 5° is 1·74; for common salt at 5°, 0·76, and at 10°, 0·91; for sugar at 9°, 0·31; for albumen at 13°, 0·06; and for caramel at 10°, 0·05. Thus in a solution of salt containing a gram per cubic centimeter less in each successive horizontal layer one centimeter in thickness, from below upward, the rate of advance upward of the salt is, at the temperature of 10°, 0·91 of a gram per day, through each square centimeter of surface.

190. Colloids and Crystalloids. – Graham was led by his experiments to divide substances, according to their diffusivity, into two classes, called colloids and crystalloids. The former, as the name indicates, are glue-like substances having a very low coefficient of diffusion. Such are starch, gum, gelatin, albumen, amorphous silica, and ferric oxide. The latter class includes crystalline substances, which have a high diffusion coefficient. Such are salt, urea, sugar, hydrochloric acid, and the like.

191. Membrane-diffusion.—Osmose. — Crystalloids diffuse readily through colloids; if a layer of pure jelly be placed on a layer of jelly containing a soluble salt, the salt will diffuse into the upper layer as into water. Hence, by separating a saline solution from pure water, by a colloid membrane, the dissolved crystalloid will be readily transferred by diffusion through the membrane, and thus separated from any colloid substance with which it may be mixed. This process of separating colloid from crystalloid matter is called **dialysis** and is of great use in the arts. The best membrane for the purpose is found to be parchmentized paper.

The phenomena which are observed when two miscible liquids are separated from each other by such a membrane are called **osmose**, or osmotic phenomena. Nollet, early in the last century, observed that, on filling a bottle with alcohol, tying a piece of bladder over its mouth, and then immersing it in water, the contents increase so as to distend the bladder almost to bursting. While if the bottle be filled with water and immersed in alcohol, the contents diminish in amount.

EXPERIMENTS. — To illustrate the action here taking place, let a layer of chloroform, a layer of water, and then a layer of ether be placed in a bottle. It will be found after a time that while the ether has traversed the water downward into the chloroform, none of the chloroform has passed upward; evidently a result of the fact that ether is to some extent soluble in the water, and being continually removed from the water layer by the chloroform, is eventually entirely transferred through it. In the same way if a caoutchouc membrane be used to separate alcohol from water, it will be wetted by the alcohol, but not by the water; and hence will allow the alcohol to pass through it into the water. While an animal membrane, like the bladder employed by Nollet, being wetted only by the water, allows only the water to pass through it. If both liquids wet the membrane, but in different degrees, there is transfer in both directions, but the quantities of liquid thus transferred are different, being greatest for the liquid having the greater attraction for the membrane. The subject of osmose is of great importance in physiology, both vegetable and animal.

192. Osmotic Pressure.—Recent investigations have greatly extended our ideas on the subject of liquid diffusion. If a porous battery-cell be filled for a time with copper sulphate solution, then carefully rinsed and filled with potassium ferrocyanide solution, a semi-permeable septum of copper ferrocyanide will be formed in the walls of the cell; so that if a solution of sugar, for example, be placed in it and the cell be immersed in pure water, while the water will pass the septum, the sugar will be retained by it. If the cell be closed and attached to a manometer, it will be observed that a considerable pressure is developed in the cell, this pressure reaching a definite maximum value depending upon the substance dissolved, upon the concentration and upon the temperature. This pressure, thus produced by osmosis, is called **osmotic pressure.** A one-per-cent solution of sugar produces a pressure of 50 cm. of mercury, and a similar solution of potassium nitrate a pressure of more than three atmospheres. Moreover, since osmotic pressure is proportional to density, Boyle's law (194) must be true for liquids as well as for gases. And since osmotic pressure is proportional to the absolute temperature, the law of Gay Lussac (284) is also true for both. Combining these laws, we have $pv = CT$; or the ratio of the product of pressure and volume to the absolute temperature is constant. The value of this constant has been calculated by Pfeffer for a one-per-cent sugar solution and found to agree with the value for gases. He concludes that "the osmotic pressure of a sugar solution has the same value as the pressure that the sugar would exercise if it were contained as a gas in the same volume as is occupied by the solution" (Ostwald). Moreover, as this result is true for other substances, the law of Avogadro appears to be true for solutions as well as for gases.

C.—COMPRESSIBILITY OF FLUIDS.

(a) *Liquids.*

193. Compressibility of Liquids.—A liquid has already been defined (159) as a fluid having a high volume-elasticity. Indeed the fact that liquids are compressible at all was established only in the last century. It was not until 1762 that Canton succeeded for the first time in determining the amount of this compressibility. His apparatus consisted of a large mercury-thermometer, the position of the mercury column in which, at a given temperature, was carefully noted. It was then heated till the mercury filled the stem and sealed. On cooling it now to the same temperature as before, the mercury column was observed to stand higher than at first; owing in part to the expansion of the mercury on removing the pressure of the atmosphere, and in part to the compression of the reservoir. Repeating the experiment with water, he obtained a result considerably larger. He gives the change of volume for one atmosphere of pressure at 10° C. as 46 parts in a million, and notes the fact that the compressibility decreases as the temperature increases. Oersted in 1822 used a similar apparatus—which he called a **piezometer**—but which was not sealed. This was immersed in water contained in a strong glass cylinder in which any desired pressure could be produced by a screw-plug, this pressure being measured by a manometer. His experiments were made at pressures up to 70 atmospheres. Other observers, including Regnault, Grassi, and Cailletet, give 50 millionths for the compressibility of water at 0° for one atmosphere of pressure; which is equal to $4{\cdot}96 \times 10^{-5}$ for one megadyne per square centimeter. Their results also give about 3 millionths for mercury; for ether at 10°, 146 millionths; for carbon disulphide at 8°, 100 millionths; for sulphurous oxide at 14°, 303 millionths; and for

carbon dioxide at 13·1°, 5900 millionths under 50 atmospheres and only 440 millionths at 90 atmospheres. Water has its minimum compressibility at about 63°.

(b) *Gases.*

194. Compressibility of Gases.—Boyle's Law.—Just as a perfect liquid may be defined as an incompressible perfect fluid, on the one hand, so a perfect gas may be defined as a perfect fluid whose elasticity of volume is equal to the pressure upon it, upon the other. Evidently, therefore, gases must be exceedingly compressible. As the volume-elasticity of fluids is perfect, and is of course equal to the stress called out by unit strain, we may readily calculate the relations of volume and pressure in gases. The strain is clearly the ratio of the change of volume to the original volume; i.e., is $(v - v')/v$, if v be the initial and v' the final volume. The stress is the increase of pressure; i.e., is $p' - p$. Whence the elasticity by definition is $\frac{p' - p}{(v - v')/v}$; and this for gases is equal to p', the final pressure upon the gas after compression. From this we get $p'v' = pv$; or what is the same thing, $p : p' :: v' : v$; that is, the volume of a perfect gas varies inversely as the pressure upon it. This is the law of the compressibility of gases which was published in 1662 by Robert Boyle and hence is commonly known as **Boyle's law**. But, since when two quantities vary inversely as each other, their product remains constant, we may express Boyle's law by the equation $pv = C$, where C is a constant depending on the mass and the temperature of the gas. Moreover, inasmuch as the density is inversely as the volume, we may write the above equation $p'\delta = p\delta'$; that is, $p : p' :: \delta : \delta'$; or the density of a gas varies directly as the pressure which is exerted upon it.

EXPERIMENTS.—The law of Boyle may be experimentally illustrated by means of two forms of apparatus originally devised by this philosopher himself. The first of these, employed for pressures greater than one atmosphere, consists of a long glass tube (Fig. 66) recurved at its lower end and sealed. Let mercury be poured into this tube so as to rise to mm' just above the bend and equally on the two sides of it. A definite portion of air under the normal atmospheric pressure will thus be enclosed in the shorter leg. Suppose it occupies a length of 20 centimeters. Add now more mercury until the column in the longer leg stands at B 76 centimeters higher than in the shorter leg. It will be seen that the mercury has risen to nn' and that the air column in the shorter leg has been diminished to 10 centimeters by this increase of pressure. That is, the pressure having been increased from one atmosphere to two, the volume has been reduced from 20 centimeters to 10; or the volume has varied inversely as the pressure, and their product has remained constant. If the mercury-column be increased to 152, to 228, or to 304 centimeters, the pressure will be increased to 3, 4, or 5 atmospheres, and the volume will be reduced to one third, one fourth, or to one fifth of the original volume.

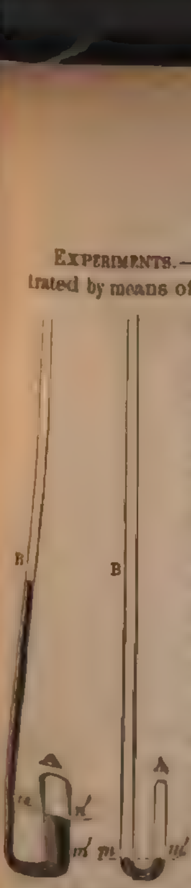


FIG. 66.

For pressures less than one atmosphere Boyle used a glass tube open below and closed above, like a barometer-tube, but containing some air above the mercury. This was immersed vertically in a tubular reservoir CD (Fig. 67). When the tube is depressed so that the mercury within and without stands at the same level, the enclosed air is at atmospheric pressure. If the tube be now raised until the mercury column within it, CB, stands at 38 centimeters above the mercury in the reservoir, this column will evidently support half the pressure of the atmosphere, leaving the elasticity of the gas to support the other half. It will be noticed that, in consequence of this reduction of the pressure to one half, the volume of the air, AB, has doubled. If the tube be still farther raised, so that the mercury column stands at 57 centimeters, evidently the air will now be subjected to only one fourth of its former pressure, and its volume will be quadrupled.

FIG. 67.

In explanation of the expression $pv = C$, above employed, we may take unit mass, one gram, of hydrogen at 0°. In case the pressure upon it is that of the atmosphere, this, as we have seen, is $1{\cdot}0126 \times 10^6$ dynes. The volume of this one gram of hydrogen is 11200 cubic centimeters. The product of these two values is $1{\cdot}135 \times 10^{10}$, the value of C. It is the same for 16 grams of oxygen, for 35·5 of chlorine, and for 14 of carbon monoxide; these numbers being in the ratio of the molecular masses of these gases. Of course for 10 grams of hydrogen or 140 grams of carbon monoxide the constant is $10C$; increasing directly with the mass.

Boyle's law is sometimes expressed thus: $P/\delta = C$; or, the ratio of the pressure to the density is constant. This expression is dependent only upon the temperature and the special gas employed. If the pressure be stated in grams per square centimeter, then C represents for any gas the height of the homogeneous atmosphere of that gas.

195. Variation from Boyle's Law.—Actually existing gases, however, are not perfect and therefore do not conform exactly to the law of Boyle. If the expression $pv = C$ be plotted graphically, taking for volumes distances along the axis of abscissas, and for pressures distances along the axis of ordinates, we shall find that the locus of this equation is a rectangular hyperbola, whose asymptotes are the axes. For a perfect gas, therefore, the volume can be zero only under an infinitely great pressure, and the pressure can be zero only when the volume is infinitely large. But since no gas possesses this property of infinite compressibility or infinite expansibility, no gas follows exactly Boyle's law. The gases which follow the law most nearly are hydrogen, oxygen, and nitrogen; while such gases as sulphurous oxide, chlorine, and carbon dioxide depart from it most widely. But these last-named gases are those which are most readily liquefied by pressure. Hence a gas is the more perfect in proportion as it is at a greater distance, as regards both temperature and pressure, from its liquefying point. The

most extended researches on Boyle's law are those of Amagat, made in a way essentially similar to that above described, but on a very large scale. The shorter leg of the tube was of strong glass. The longer was a steel tube 330 meters long placed in the shaft of a coal-pit. By a powerful pump, mercury was forced in to the bottom of the steel tube until it ran out of a tap placed at a certain height. After measuring the volume of the compressed gas, the tap was closed and the mercury forced up to the next tap. In this way the pressures were extended to 400 atmospheres; and, taking the value of pv as unity for the ordinary pressure, it was found that in the case of air this value decreased up to about 77 atmospheres, when it was 0·9803. At 176 atmospheres it was 1·0113, and at 400 atmospheres 1·1897. Up to a pressure of 152·3 atmospheres, therefore, air practically obeys Boyle's law, being reduced by this pressure to 1/152·3 of the volume which it occupies at one atmosphere.

196. Applications of Boyle's Law.—Manometers.—Instruments for measuring the pressure exerted by liquids and gases are called pressure-gauges or **manometers**. In their simplest form they consist of a closed reservoir containing mercury—though for small pressures water may be used—having a glass tube open at both ends passing through its top and terminating below the surface of the mercury. On opening communication between the vessel containing the pressure to be measured and this reservoir, the mercury is forced up the tube to a height which balances this pressure. The result may be expressed either in centimeters of mercury, in atmospheres of 76 centimeters each, in grams per square centimeter, or in dynes per square centimeter. More lately the custom has been to express the pressure in "atmospheres" of one megadyne per square centimeter; a value very nearly equal to the pressure of 75 centimeters of mercury.

For measuring higher pressures, the manometer-tube is closed at top (Fig. 68) and then the air in it suffers

compression according to the law of Boyle. Under one atmosphere of pressure, the column of mercury is at the same height bc in both legs. But as the pressure increases, the air in the closed leg is compressed and its volume diminishes; the volume being always inversely as the pressure. If l be the length ab of the air-column normally and x the height d to which the mercury rises when the pressure is n atmospheres, $l - x$ or ad will equal the new length of the air-column and we shall have $P : P' :: l - x : l$; or $P' = Pl/(l - x)$. But by hypothesis $P' = nP$, where P represents one atmosphere. Hence $Pl/(l - x) + 2x = nP$; or in other words, the elasticity of the compressed air plus the height of the mercury-column $2x$ is equal to n atmospheres. Solving for x we have

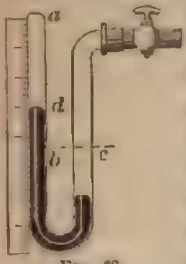


FIG. 68.

$$x = \frac{nP + 2l \pm \sqrt{(nP + 2l)^2 - 8(n - 1)Pl}}{4}. \qquad [35]$$

The lower sign only is admissible, since when $n = 1$, x should equal zero. By replacing n in this equation by successive numerical values, the points on the scale which represent these pressures can be calculated.

For measuring pressures less than that of the atmosphere vacuum-gauges are employed. If the reservoir of a barometer be closed and connected with a vessel from which the air is being removed, the mercury-column will fall in proportion as the pressure decreases. So if the upper end of an open tube dipping in mercury be connected with such a vessel, the mercury will rise as the pressure within the tube decreases; so that the difference between its height and that of the barometer measures the pressure. Another form of vacuum-gauge consists of a U-tube ten to twenty centimeters long, one end of which is closed, the closed leg being entirely filled with mercury. The mercury is sustained in the tube by the atmospheric pressure; and hence when the pressure of the air in the space containing the tube is diminished

the mercury-column falls. The pressure in the space is proportional to the difference of level in the two legs.

Besides these forms of pressure and vacuum gauges there are others used in practice, made of metal. The Bourdon gauge is provided with a thin flattened curved tube, having one end fixed. The motion of the free end as the pressure varies within the tube, multiplied by suitable devices, indicates the pressure to be determined.

197. Methods of removing Air.—Air-pumps.—Air may be removed from any vessel by utilizing the fact that gases are indefinitely expansible. If such a vessel be put in communication with a second vessel from which the air has been entirely removed, the air in the first will expand to fill both vessels, and consequently its amount in the first vessel will be reduced in proportion to the relative capacities of the two vessels. If the two are of equal volume, only one half of the air will be left in the first. If the second is nine times the volume of the first, only one tenth of the air will remain in the first after they are connected. In general, if the volumes are $1 : n$, the amount of air in the first vessel will be reduced to $1/(1 + n)$ of its former amount.

Air-pumps are devices for removing the air from a given vessel by the method above mentioned. In the older forms of the instrument, a piston is made to move air-tight in a cylinder, at the bottom of which is a valve opening inward (Fig. 69). On the top of the piston is a double valve opening outward. On pushing the piston to the bottom of the cylinder, the air escapes through the piston-valves. So that if the valve in the cylinder were kept closed, there would be no air in the cylinder when the piston is again raised. But the vessel to be exhausted is attached to the lower end of the cylinder; and hence as soon as the pressure within this vessel becomes

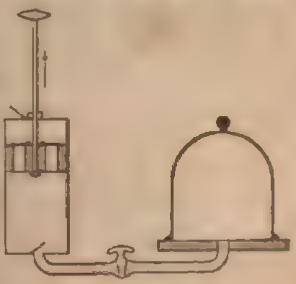


FIG. 69

greater than that in the cylinder, i.e., as soon as the piston has begun to rise, this pressure will lift the valve and the air will expand to fill both vessels as above. By repeating this process, as many expansions may be produced as are necessary to obtain the desired vacuum. It is clear that no further exhaustion is possible after the pressure in the vessel is less than that required to lift the valve. Hence it is sometimes raised automatically by the piston as it moves. Moreover, in theory a perfect vacuum cannot be obtained in this way, since, as we have just seen, the air in the receiver is reduced by each stroke to $1/(1+n)$ of its former amount. After the first stroke there is $1/(1+n)$ of 1, the volume of the vessel; i.e., the mass of air left after the first stroke is $1/(1+n)$. After the second stroke there will be $1/(1+n)$ of this left; which is $1/(1+n) \times 1/(1+n)$ or $1/(1+n)^2$; and so on. At the end of the mth stroke the mass of air in the vessel will be reduced to $1/(1+n)^m$ of the original amount. And this cannot become zero until the number of strokes becomes infinite, or $m = \infty$. If the original pressure be p_1 and the final pressure be p_2, then we have for the final pressure $p_2 = (1/(1+n))^m p_1$.

FIG. 70.

Suppose, for example, that the cylinder of the air-pump has twice the capacity of the vessel to be exhausted. After the first stroke, $\frac{1}{3}$ of the air will remain in the vessel; after the second, $\frac{1}{3}$ of this or $\frac{1}{9}$ of the original amount. So that after 10 strokes $(\frac{1}{3})^{10}$ or about $\frac{1}{60000}$ of the original air only will remain and the pressure will be reduced 60,000 times.

198. Mercury-pumps.—The more modern forms of air-pump are worked by means of liquids, generally mercury; whence they are sometimes called mercury-pumps. The earliest apparatus of this sort was devised by Geissler of Bonn in 1857 and is known as the Geissler pump (Fig. 70). It consists of a stout glass tube two or three centimeters in diameter and nearly a meter

long, having at top a globe, *A*, of about a liter capacity, terminated by a carefully ground glass tap, *C*, known as a three-way tap. When turned as shown in the figure, the globe is in communication with the left-hand tube *d*; and when rotated through 90°, it is connected with the right-hand tube *e*. To the lower end of the tube is attached a piece of stout rubber tubing, which connects this tube to a spherical reservoir, *B*, containing mercury. As the reservoir is raised the mercury rises in the tube, the tap being open through *e*, and fills the globe completely. The tap is then closed and the reservoir lowered. The mercury in the tube falls until the difference of level in the tube and in the reservoir is the barometric height. Evidently now the vacuum in the globe and tube above the mercury is a Torricellian vacuum. On turning the tap *C* so as to put the globe into communication, by the lateral tube *d*, with the vessel to be exhausted, the air in this vessel will expand to fill both. The tap is then closed, the mercury reservoir again raised and the air in the globe expelled through the tube *e*, the tap being turned for the purpose. By repeating this process the exhaustion may be carried to any desired degree. Moreover, by means of the recurved tube shown above *e*, the pump may be used for transferring any gas drawn in at *d*, to a suitable reservoir.

FIG. 71.

Subsequently (1861) Sprengel devised another form of mercury-pump, which is now called by his name. In its simplest form (Fig. 71) it consists of a straight thick tube of glass, *A*, of rather small bore and about a meter long, enlarged at its top into a cylindrical bulb, *B*, provided with a lateral tube, *a*. Into the upper end of this bulb a narrow tube, *b*, is sealed by a ground joint; this tube extending downward to two thirds the length of the bulb. The second tube, *D*, is connected at its lower end, *d*, by means of a stout rubber tube, with a reservoir of mercury placed above *c*, the highest part of the ap-

paratus. The vessel to be exhausted is attached to the lateral branch *a*. On allowing mercury to run down the rubber tube from the reservoir, it flows into the tube *D*, rises through *C*, where it is freed from mechanical impurities (expelling the air through *c*, which is momentarily opened for the purpose), and passes through the tube *b* into the pump *B*. The flow of this mercury down the tube *A* causes a diminution of pressure in *B*, and the air from the vessel flows into the main tube, breaking the column in *A*, into little cylinders of mercury, each acting like a piston to increase the volume into which the air from the vessel can expand. The mercury flowing from the tube is collected in *E*, from which it overflows through *e* into a suitable jar and is returned at intervals to the reservoir.

Both these mercury-pumps have received important and valuable improvements at the hands of Rood, Alvergniat, Crookes, and others. It appears, however, that while the Geissler pump is more rapid in its action, the Sprengel pump is capable of giving the higher vacuum. But the Geissler pump is intermittent in its operation, and the Sprengel pump is exceedingly slow. In many cases, therefore, a partial exhaustion is first obtained by an ordinary air-pump and the vacuum is then completed by the mercury-pump.

Bunsen introduced a modification of the Sprengel pump into laboratories, for the purpose of hastening filtration. It is known as the Bunsen filter-pump, and uses water instead of mercury; of course requiring a fall of ten meters or more.

199. High Vacua.—Fourth State of Matter.—The vacuum obtainable with the common air-pump, in practice, is very far below that which theory indicates, owing to difficulties of construction. It is a good pump which will give a vacuum of one millimeter of mercury; and a vacuum of 0·05 millimeter has rarely, if ever, been obtained in this way. But by means of the mercury-pump it is not at all difficult to reduce the mercury-pressure in a vessel to one millionth of its normal value, or to

0·00076 of a millimeter. The Sprengel form of pump seems to be preferred for the purpose where very high vacua are required.

We are indebted to William Crookes for the extraordinary developments which have recently taken place in this direction; not only for the greatly improved methods for obtaining high vacua which he has devised, but also for the remarkable phenomena exhibited in these high vacua when produced. Beginning his researches in connection with his radiometer, he extended them to the study of the properties of a gas so rare that it would support a mercury-column only one millionth of the barometric height. And so extraordinary did he find these properties that he was led to consider a vacuum of this sort as in a true sense a fourth state of matter; being in its properties quite as different from the ordinary gaseous state as this is from the liquid state. These results we shall refer to subsequently. It is sufficient to say here that the highest vacuum obtained by him was about one twenty-millionth of an atmosphere. This great rareness was determined by an ingenious apparatus known as the **McLeod gauge**. This gauge consists of a closed tube of small bore surmounting a globe at the top of a barometric tube. The small tube is carefully calibrated in terms of the known capacity of the globe, and the globe is connected with the vessel to be exhausted. To ascertain the degree of exhaustion, a separate reservoir of mercury, connected with the lower end of the gauge barometric tube, is raised. The mercury rises to the level of a lateral tube just below the globe, and then cuts off connection with the pump. Continuing to rise, this mercury fills the globe and drives the residual air into the measuring-tube at its top; the volume of which may be read off at atmospheric pressure when the mercury-level is the same in this tube as in the lateral tube. If this volume be one cubic millimeter, the volume of the globe and measuring-tube being one liter, the vacuum is evidently one millionth of an atmosphere. By an improvement in

the Sprengel pump Rood in 1881 succeeded in obtaining vacua as high as a three-hundred-millionth of an atmosphere.

D.—CAPILLARITY.

200. Cohesion in General.—Molecular attraction, as we have seen (145), is called **cohesion**; and in solids its measure is tenacity. The peculiarity of the attraction of cohesion, as compared with gravitation, is that it is exerted only through immeasurably small distances; so that at sensible distances it is inappreciable. Tait has calculated the relative effectiveness of cohesion and gravitation in keeping the earth together; and he finds that if the cohesion be that of sandstone, gravitation is 25,000 times, and if it be that of steel, 100 times as effectual for this purpose as cohesion. In a sandstone sphere of 40 kilometers radius, or in a steel sphere of 640 kilometers radius, gravitation and cohesion are equally effective in holding their parts together. The range of action being so small, it is not easy to bring molecules within the sphere of cohesion. Whitworth has worked steel surfaces, and Barton copper surfaces, so true that when these surfaces were placed together the lower mass could be lifted by raising the upper one even in vacuo. Graphite is compressed hydraulically into blocks which are perfectly coherent. And plastic bodies such as wax and lead at ordinary, and iron at high temperatures, can be made to cohere very firmly by quite a moderate amount of pressure.

201. Liquid Cohesion.—When a solid is immersed in a liquid, it is wetted or not wetted by the liquid according to the relative attraction between the liquid and the solid, on the one hand, and that of the liquid particles for each other, on the other. Or if, to avoid circumlocution, we call the attraction of the solid for the liquid **adhesion**, the solid is wetted if the adhesion be the greater, it is not wetted if the cohesion be the greater. If a plate

of glass be placed on the surface of water and then removed, the glass is wetted and the cohesion of the liquid has been overcome. If it be placed on mercury and removed, it will not be wetted and the adhesion has given way. Gay Lussac hung to one scale-pan of a balance a circular plate 118·4 millimeters in diameter, and brought it just to touch the surface of the liquid to be tested. Weights were then placed in the other pan until the plate was detached from the liquid. He found in this way that water required 59·4, alcohol 31, and turpentine 34 grams to overcome its cohesion over this area. The same values were obtained whether the plate was of glass, of copper, or of other metals; thus showing that the experiment measures cohesion only. If, however, the plate be not wetted by the liquid, then the separation takes place between the solid and the liquid and the experiment measures the adhesion. In this case the material of the plate evidently influences the result. To separate a glass plate of the above diameter from the surface of mercury requires a weight of 158 grams; a measure of the force of adhesion. While when a zinc plate was used which the mercury wetted, nearly 500 grams was required to effect the separation. The cohesion of mercury is therefore nearly nine times that of water; and its adhesion to glass is about one third of its cohesion.

202. Surface-tension of Liquids.—If we consider the molecules of a liquid in equilibrium, it will appear that the conditions under which the molecular forces are balanced are different in different parts of the liquid mass. At any point in the interior each molecule is surrounded by other similar molecules and the attraction is equal in all directions. But at a point upon the surface, although the lateral attractions are balanced, there is no component of attraction acting upward to balance the downward component due to the molecules beneath the surface. Hence the free surface of a liquid acts like an enveloping film, of a thickness equal to the radius of the sphere of molecular action, at every point of which a

force acts toward the interior and normal to the surface. These relations are represented by Tait in the equation

$$E = (M - S . \Sigma t\rho)e_0 + S . \Sigma t\rho e = Me_0 + S . \Sigma t\rho(e - e_0), \quad [36]$$

in which E is the total potential energy and M the total mass of the liquid, ρ its density, S the area and t the thickness of the superficial film, and e_0 and e the energy per unit mass in the interior and upon the surface, respectively. The total energy of the liquid is therefore increased or diminished according as e_0 or e is the greater; and by an amount proportional to the surface. As for stable equilibrium the potential energy of the system must be a minimum, it is evident that when e is greater than e_0, the value of S must be as small as possible in order to increase Me_0 and therefore E, by the minimum amount. This is the case when a surface of water is exposed to air. If, however, e be less than e_0, as when water and glass are in contact, then S tends to take the largest possible value in order that E may still have its minimum value possible. Evidently, e is a function of both the media separated by the surface. For if the surface separate two liquids having the same density and cohesion, e will equal e_0, and the total energy of either liquid will be unchanged. The maximum value of e is reached when the space above the liquid is a vacuum. This tendency of a liquid surface to take a minimum value, as in the case of water and air, can mean only that this surface acts like an elastic membrane, equally stretched in all directions, and by a constant tension. This is called the **surface-tension of liquids.**

ILLUSTRATIONS.—This theoretical conclusion is supported by numerous facts. Whenever a liquid is freed from the action of any force but that of its own cohesion, it assumes the spherical form: like water in the rain-drop and the dew-drop, and like mercury in globules. Now the sphere has the minimum surface for its volume. The phenomenon of the rainbow abundantly proves the sphericity of the rain-drop. So the production of shot by allowing melted lead to fall through the air, and the rounding of the edges of sealing-wax and glass when melted in a flame, illustrate the same fact. By

making a mixture of alcohol and water of the same density as olive-oil, Plateau was able to obtain a mass of this oil, placed within the mixture, entirely freed from the action of gravity, and of course perfectly spherical in form. The same result may be obtained by placing carbon disulphide colored by iodine, in a solution of zinc sulphate of the same density.

The tendency of liquid surfaces to take a minimum value is also well illustrated when oil is placed upon the surface of water. It flashes out at once into a film so thin as to develop iridescent colors. A fragment of camphor placed on water dissolves slightly at the point of contact and lessens the surface-tension there. The greater tension laterally of the unchanged surface-film draws the camphor to one side and active motions are thus produced. Various essential oils when dropped on water produce cohesion-figures which are characteristic of the oils themselves and which have been suggested by Tomlinson as a means for their identification and detection when mixed together.

The phenomena of surface-tension are best seen, however, in the case of films freed from liquid masses, such as those of soap-bubbles. The contractile force of the film often expels the air through the bubble-pipe with sufficient force to blow out a candle-flame. According to Plateau, 5·6 gram-meters of work are required to blow a soap-bubble whose surface is one square meter; and this represents the energy of a bubble of that size. If the mouth of a glass funnel be dipped in the soapy water, and then removed, the finger being held over the narrow end, a flat soap-film will be formed across it; and on removing the finger, the air will be forced out by the contractility of the film which will run rapidly up to the point of minimum area. A flat film may be taken out on a wire ring, having a piece of thread tied to the ring at two points three centimeters or more apart. On breaking the film within this small loop by touching it with a hot wire, the contraction of the rest of the film stretches the thread tense, forming the arc of a circle. If the thread form *a loop suspended* from the wire, and the

film be broken within the loop, the contraction of the outside film in all directions makes the loop a complete circle.

203. Measurement of surface-tension.—Moreover, by means of these films the surface-tension may be directly measured. A wire frame (Fig. 72) consisting of a straight wire AC furnished with

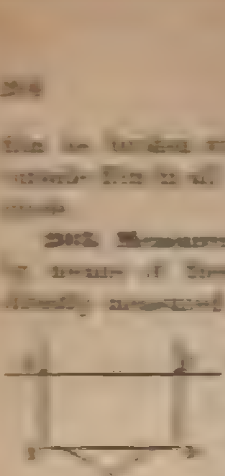


Fig. 72.

guides, has a movable wire $ABDC$ attached to it, which is capable of sliding vertically through the guides. Let a soap-film be formed upon the frame when AC and BD are close together; and let weights be placed in the scale-pan E until the contractile force of the film is balanced, say at the distance AB. Since the length of the film is AC, the quotient of the total weight which it supports divided by this length gives the tension per unit of length, i.e. in grams per linear centimeter in the C.G.S. system; or in dynes if this value be multiplied by g. In the case of a soap-film the surface-tension per linear centimeter is 54·9 dynes. Since the film has two surfaces, one half this, or 27·45 dynes, represents the true surface-tension for soap-solution. In the case of pure water at 20° the tension for a single surface is 81 dynes per centimeter. If, however, the surface of contact be not between these liquids and air, as above supposed, the surface-tension has a different value. Thus between water and olive-oil it is 20·56 dynes; and between olive-oil and air 36·88 dynes. The tension for air-chloroform is 30·61 dynes and for water-chloroform 29·53 dynes per centimeter.

Moreover, the energy-relations of the film may also be obtained with the above simple apparatus. If T be the surface-tension per linear unit, the total tension along AC will be $T \,.\, AC$. Now this force has stretched the film through a distance AB; and hence the work done is $T \,.\, AC \,.\, AB$. Again, if we call S the surface-energy per unit of area, the total surface-energy for the entire area of the film will be $S \times AC \,.\, AB$. But the energy of

the film is the work done upon it. Therefore we have $T \,.\, AC \,.\, AB = S \,.\, AC \,.\, AB$; and of course $T = S$; that is, the numerical value of the surface-energy per unit of area is the same as the numerical value of the surface-tension per linear unit. But we have just seen that for soap-solution and air, the surface-tension is 27·45 dynes per centimeter. Therefore for the same fluids in contact, the surface-energy must be 27·45 ergs per square centimeter. For pure water and air, the surface-energy is 81 ergs per square centimeter.

204. Pressure produced by the Film.—The pressure produced in the interior of a closed film by the contractile force of this film, as in the case of a soap-bubble for example, may be calculated from the surface-tension and the curvature of the film. Suppose a band whose width is unity is stretched over a cylindrical surface of radius r. Let its length ab (Fig. 73) subtend at the center O an angle ϕ. The tension on the band will produce a pressure toward O, which must be balanced by the reaction of the cylindrical surface. Resolving the tension T along OP we have $2T \sin \frac{1}{2}\phi$ for the component toward O. The reaction is $p \,.\, ab$, where p is the pressure outward per unit of surface; and, as $ab = r\phi$, the reaction becomes $p \,.\, r\phi$. Hence $2T \sin \frac{1}{2}\phi = p \,.\, r\phi$, and when the angle ϕ is small, $p = T/r$. Thus the pressure exerted by the film is directly proportional both to the surface-tension and to the curvature. In the case of a spheroid there are obviously two curvatures in planes at right angles to each other; and since they are independent the resultant pressure is the sum of the pressures due to each; or is $T\left(\frac{1}{R} + \frac{1}{R'}\right)$, in which R and R' are the two radii.

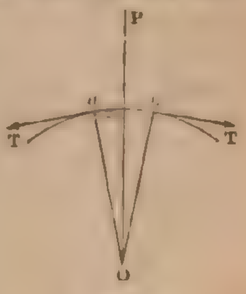


Fig. 73

In the case of a soap-bubble, R and R' are practically equal and the pressure is $2T/R$ for each surface

or $4T/R$ for both. Hence the pressure within such a bubble, of 4 centimeters radius, is equal to T; i.e., is equal to 27·45 dynes per square centimeter of its surface.

205. Angle of Contact between Liquids in Equilibrium.—Knowing the surface-tensions for various pairs of media we may apply to them the principle of the polygon of forces in order to determine their mutual action. Calling T_{ab} the surface-tension between the media a and b (Fig 74), and T_{bc} and T_{ac} the tensions between b and c and between a and c, respectively, and supposing that the surfaces between a and b and between a and c meet that between b and c at the point O, we have evidently the simple case of three forces in equilibrium acting upon a point (84). Hence if a triangle be constructed (Fig. 75) having sides parallel to these tensions and proportional to them in magnitude, the exterior angles of this triangle will be equal to those formed by the three surfaces; and hence $\sin A : \sin B : \sin C :: T_{ac} : T_{ab} : T_{bc}$. **Whenever, therefore, three fluids are in contact and at rest,** the surfaces separating them form angles with one another determined solely by their relative surface-tensions. In case no triangle can be constructed to represent the tensions, no equilibrium is possible. If, for example, one surface-tension be greater than the sum of the other two, no corresponding triangle can be drawn and equilibrium is impossible.

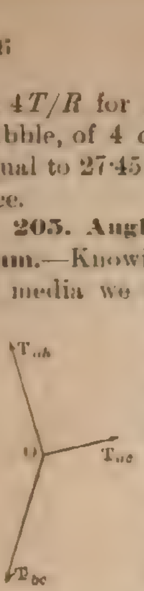


FIG. 74

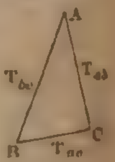


FIG. 75

EXAMPLES.—In illustration of these actions let the three fluids be water, oil, and air. The water-air tension is, as above, 81 dynes, the air-oil tension 36·88 dynes, the water-oil tension is 20·56 dynes. The sum of the last two is 57·44 dynes; very considerably less than the water-air tension alone. Hence a drop of oil on the surface of water cannot be in equilibrium. The superior tension of the water film draws it out indefinitely, until it becomes so thin as to lose tirely the properties of a liquid mass. Conversely, if a drop of

water be placed on chloroform, the water-air surface-tension being greater than the sum of the chloroform-air and the chloroform-water tensions (i.e., $81 > (29.53 + 30.61)$), the water gathers itself at once into a drop. When water, air, and clean glass are placed in contact, the surface-tension between air and glass being greater than the sum of the water-glass and the water-air tensions, equilibrium is not possible and the water spreads itself out indefinitely over the glass, while the air gathers itself into a globule; as is best seen with a bubble of air under water.

206. Capillary Phenomena.—If a glass tube of very small bore be placed in water, after having been previously wetted, the water will rise in the tube against gravity. If a similar tube be placed in mercury, the column within the tube will be depressed below the surface of the mercury outside. Phenomena of this kind are called **capillary phenomena**, since they take place in tubes whose opening is so small that it may be likened to a hair.

We have now to connect the phenomena of surface-tension with those of capillarity. On examining the surface of the liquid in the capillary tube, it will be found to be curved, the concavity being upward when there is elevation and the convexity upward when there is depression. Now we have seen that, owing to the surface-tension of liquids, the tendency of the superficial film to a minimum produces a resultant pressure in the direction of the concavity; upward, therefore, in the case of water and air, downward in the case of air and mercury. The water will rise, therefore, and the mercury will fall until these surface forces are balanced by the weight of the liquid; within the tube in the former case and without it in the latter. Further, if the tension at the surface separating the two fluids be greater than the difference of the tensions between each of them and the solid, the surface of separation of the two fluids will be inclined to the surface of the solid at a definite contact-angle. This angle may be readily calculated, knowing the surface-tensions involved.

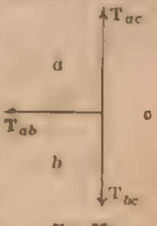


Fig. 76

Supposing the *difference* $T_{ac} - T_{bc}$ (Fig. 76) to be posi-

tive, T_{ac} being the greater, the resultant tension will be such that its vertical component will balance this difference; in other words, T_{ab} will act in the third quadrant, and the angle of contact will be acute (Fig. 77). If T_{bc} be the greater, the difference will be negative, T_{ab} will act in the second quadrant, and the angle of contact will be obtuse (Fig. 78). From these conditions we have $T_{ac} - T_{bc} = T_{ab} \cos \alpha$; whence $\cos \alpha = (T_{ac} - T_{bc})/T_{ab}$. Evidently if the difference $T_{ac} - T_{bc}$ be greater than T_{ab}, $\cos \alpha$ will have a value greater than unity; which is impossible. Hence the tension between a and b must be greater than the difference of tensions between a and c and between b and c, as above stated, in order that there should be a definite angle of contact.

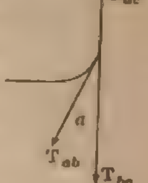


FIG. 77.

FIG. 78.

EXAMPLES. — When water is placed upon clean glass, $T_{ac} > T_{ab} + T_{bc}$. Hence the angle of contact disappears and the water spreads itself over the surface. Mercury on the contrary forms a globule, the angle of contact between its surface and the glass being about 130°. The contact-angle between air, alcohol, and steel is 90°.

207. Jurin's Law.—Suppose, now, a glass capillary tube to be immersed in a liquid whose angle of contact with glass is α (Fig. 79). If r be the radius of the tube, $2\pi r$ will be the circumference; and if the surface-tension be T per unit of length, the entire tension around the tube will be $2\pi r T$. But, since the tension acts at an angle α with the vertical, $2\pi r T \cos \alpha$ is the component acting upward to raise the liquid. Since the area of the tube is πr^2 and the height of the liquid in it h, the volume is $\pi r^2 h$; and if δ be the density, $\pi r^2 h\delta$ is the *weight* of the column of liquid in the tube. Since this

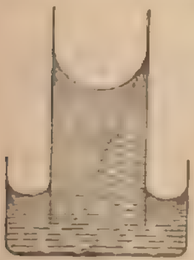


FIG. 79.

...umn is in equilibrium, the upward and the downward ...ces must be equal; or

$$\pi r T \cos \alpha = \pi r^2 h\delta g\text{; whence } h = (2T\cos\alpha)/r\delta g. \quad [36]$$

...water be the liquid used, $\alpha = 0°$ and $\delta = 1$; whence ...$= 2T/rg$. Hence the height of the column is inversely

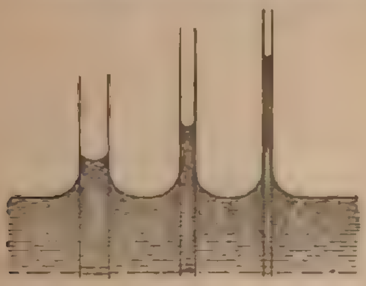


FIG. 80.

as the radius, and consequently the capillary elevation and depression is the greater the smaller the bore of the tube (Fig. 80). This is known as **Jurin's law**. If $\alpha > 90°$, $\cos\alpha$ is negative and there is depression.

If a column of liquid be sustained between two flat surfaces of a solid, as water between two glass plates, we have the column supported on only two sides, the meniscus being cylindrical. Hence if d be the distance between the plates, the vertical component of the tension on both sides per unit of length will be $2T\cos\alpha$ and the balancing weight in dynes will be $hd\delta g$; whence $h = (2T\cos\alpha)/d\delta g$; an expression differing from that for a tube only in having d in place of r. Hence a liquid will rise between plates to the same height as in a tube whose radius is the distance between the plates. The walls of the plates must be separated then by only half the distance that separates the walls of the tube in order to have the elevation the same for both. Putting the above equation in the form $hd = (2T\cos\alpha)/\delta g$ we have the variables in the first member and the *constants in the second*; T, α, δ, and g all being

the same in experiments with the same three media. But when the product of two quantities is constant the quantities vary inversely as each other. Hence the equation $hd =$ constant, represents a rectangular hyperbola, referred to its asymptotes as axes; as may be readily seen by plotting the equation.

EXPERIMENT. - This result is easily shown by experiment. If two wetted glass plates, closer together on one edge than on the other be placed in water with their sides vertical, the water will rise between them to heights which are inversely as the distances of separation, and since these distances vary uniformly from one edge to the other, the heights vary in the inverse ratio of these distances and the liquid surface between the plates takes the form of the rectangular hyperbola (Fig. 81), with the rectangular edges of the plates as axes.

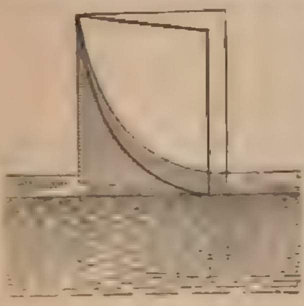


FIG. 81

Obviously, the height to which a liquid will rise in a capillary tube may be calculated if T, α, and r be known. In the case of water $T = 81$ dynes and $\alpha = 0°$. If the tube be one millimeter in diameter, $h = \frac{2 \times 81}{\frac{1}{20} \times 980} = 3{\cdot}32$ centimeters. Experimentally this height may be measured by the kathetometer method of Gay Lussac, and the radius calculated from the length and mass of a mercury column in the tube. This height varies with the temperature, diminishing as the temperature increases.

The law of Jurin is found to hold with great exactness if the tube be wetted by the liquid; and then the height is independent of the material of the tube, and also of its thickness. But if the tube is not so wetted the law is found to hold only approximately, due apparently to the varying conditions between the air condensed on the surface of the solid and the liquid.

208. Value of the Superficial Tension from Capillarity. — Since in many cases it is not possible to meas-

uperficial tension directly, it may be calculated formula $T = (hr\delta g)/2 \cos \alpha$, provided that r, h, known. The following table gives the results e's measurements at the temperature of 20° C., l in dynes per linear centimeter:

SUPERFICIAL TENSION OF LIQUIDS

id.	Density.	Tension of surface separating the liquid from— Air.	Water.	Mercury.
....	0·9982	81	0	418
....	13·5432	540	418	0
isulphide	1·2687	32·1	41·75	372·5
m.	1·4878	30·6	29·5	399
....	0·7906	25·5	——	399
....	0·9136	36·9	20·56	335
n	0·7977	31·7	27·8	284
loric acid	1·1000	70·1	——	377
of sodium phate	1·1248	77·5	——	442·5

Superficial Viscosity.—Besides the surface-ten- quids, there is another property possessed by faces, called **superficial viscosity**, which is inde- f the tension. This is very markedly shown by of saponin, on which if a magnetic needle be l it will remain in any position, instead of turn- and south. Owing to the much greater vis- the superficial film of liquids over that of the his film is hard to break. Soap-solution has ace-viscosity and low surface-tension and hence blown into bubbles. The floating of an oiled water or the walking of water-insects thereon the surface-viscosity of the water, its tension een reduced. To a like increase of superficial and decrease of surface-tension is due the still- of oil upon a rough sea.

CHAPTER VI.

ENERGY OF MASS-VIBRATION.—SOUND.

SECTION I.—NATURE OF SOUND.

210. Definition of Sound.—Sound may be defined either subjectively or objectively; either as a sensation or as the external cause of a sensation. In the physiological sense, sound is an effect perceived normally by the ear. In the physical sense, sound is the antecedent cause of this effect. Since it would seem more philosophical to define a thing by what it is than by any effect it may produce, we shall define sound in the physical sense, as being that special condition of matter in virtue of which incidentally it may affect the organ of hearing.

211. Sound a Mass-vibration.—Whenever we examine a body which is producing sound, we find that it is in a state of vibratory motion. This motion it readily transfers to the surrounding medium, so that vibratory motion in this medium is the condition of sound-propagation. Moreover, sound can be received only by bodies capable of vibration; so that a body receives sound only by taking up vibrations from the source of sound, either directly or through a medium.

EXAMPLES.—If a pencil be held lightly against the edge of a sounding bell, the rattling noise produced shows that the bell is vibrating. If a sounding tuning-fork be made to touch a small ivory ball suspended by a thread, the ball will be projected strongly from the fork. A paper membrane placed in the vicinity of a sounding body will take up the air-vibrations; as may be shown by sprinkling *a little* black sand upon its surface. Indeed, in the case of the lower

note of a large organ, the vibration may be directly felt in the tremor sometimes given to the entire building. In Edison's phonomotor a wheel is made to revolve by means of sound-vibrations.

212. Character of Sound-vibration.—In order that a body may produce or transmit sound it must be elastic. Hence all sound-vibrations are the vibrations of elastic media. For these media Hooke's law holds; and the force of restitution is proportional to the displacement. We have seen (55) that whenever a body vibrates under the influence of a force which is proportional to the displacement, its vibrations are executed in equal times whatever the amplitude; i.e., are isochronous. Such vibrations are called **simple harmonic** vibrations.

For the propagation of sound the elastic medium necessary is generally air. But air is a fluid, and has therefore only volume-elasticity. The only waves which it can propagate consequently are waves of compression and rarefaction. Moreover, since air is an elastic medium, the vibrations of its particles are simple harmonic. Hence, physically, sound may be completely defined as the simple harmonic vibration of elastic masses. A sound-wave is therefore a succession of the particles of such a medium, vibrating harmonically parallel to the line of propagation of the sound, but in progressively different positions with respect to their points of rest; these positions repeating themselves periodically for every complete vibration.

EXAMPLES.—If a tuning-fork be strongly vibrated, the note emitted by it remains the same as long as this note is audible: proving its vibrations to be isochronous. The vibrations of air and water columns are simple harmonic; as is seen when an organ-pipe is sounded in air or under water.

213. Differences in Sounds.—Sounds differ from one another in three essential particulars; namely, in loudness, in vibration-frequency, and in quality. The loudness of a sound depends upon the amplitude of the vibration; i.e., upon the amount of the displacement of the vibrating body from its position of equilibrium. It is proportional to the square of the amplitude. The

vibration-frequency of a sound depends upon the number of vibrations which the vibrating body makes per second. When this number is relatively great, the vibration-frequency is said to be high; when small, it is said to be low. The quality of a sound depends upon its complexity. All harmonic vibrations which are absolutely simple have the same quality. In proportion as the sound becomes more complex, the wave-form changes and the quality changes with it.

214. Characteristics of Sound-waves.—Sound-waves, as we have said, are waves of condensation and rarefaction. If we suppose an elastic fluid to be suddenly compressed, the compression will evidently be relieved by the motion of the compressed particles into the regions beyond; this will produce a compression in these regions which will relieve itself similarly; and so the compression will be propagated through the medium. If the elastic fluid be rarefied, the rarefaction will propagate itself in precisely the same way, the particles moving now in the opposite direction to that in which the rarefaction itself moves. Suppose further, that by vibrating a tuning-fork both these effects are produced; the prong which vibrates to the right producing a compression on the right side of it and a rarefaction upon the left; and the same prong when it vibrates to the left, producing a rarefaction on the right and a condensation on the left. Then it is clear that during one complete vibration of this prong, a complete wave of condensation and rarefaction, i.e., a complete sound-wave, will be produced on each side.

EXPERIMENT.—Take two funnels, one considerably larger than the other, tie over their mouths sheets of india rubber, and connect them together by means of a long rubber tube. Support the smaller funnel in any convenient stand and hang in front of it a small ivory ball so as just to touch the rubber membrane. Stand at a distance from the smaller funnel, and tap upon the membrane of the larger one; a compressed air pulse will be propagated through the tube and the ball will be thrown away from the fixed membrane. If the larger membrane be pulled outward, a rarefied wave will be similarly sent through the tube. And if a to-and-fro movement be given the

ger membrane, a similar to-and-fro motion will be executed by e smaller one.

215. Definition of Wave-length.—In the above ex-eriment the time required for the pulse to pass through he tube depends entirely upon the medium within the ube and not at all upon the agency by which the pulse s generated. So, in general, the speed of propagation f a disturbance in any medium is a function solely of hat medium. Hence the distance to which the disturb-ance is propagated in any medium during the time equired for the vibrating body to make a complete ibration, since it is a function of the particular medium mployed, must be different for different media. This istance is called a **wave-length**. And a wave-length is herefore defined to be the distance over which sound asses in any medium, in the time required for the unding body to make one complete vibration (62). he length of a sound-wave is different therefore not ly for different media, but also for different rates of bration.

216. Relation between Speed of Propagation and ave-length.—Suppose, further, the sounding body ntinues to vibrate. During the second complete vi-ation, which is effected of course in the same time th the first, the distance passed over is the same; so t at the end of the second vibration the disturbance extended twice as far; and so on. It is clear that the end of 1000 vibrations the disturbance will have nded a thousand times as far; and if, therefore, se 1000 vibrations have been made in one second, it vident that in that second the disturbance has ex-ded over 1000 wave-lengths. But the distance over ch sound has passed in one second is the speed of nd; and, as we shall see presently, is constant for same medium at the same temperature. In the case posed the speed of the sound is equal to 1000 wave-gths; and in any case the speed of sound is the prod- of the number of vibrations per second by the wave-gth; i.e., $s = n\lambda$, if *we represent* by s the speed of

sound, by n the number of vibrations, and by λ the length of the wave. Thus we get the speed by multiplying together the vibration-frequency and the wave-length. We get the vibration-frequency by dividing the speed by the wave-length. And we get the wave-length by dividing the speed by the vibration-frequency. Knowing any two of these values we can obtain the third.

It should be kept in mind, however, that there is no necessary relation between the amplitude of vibration of the sounding body and the wave-length of the sound which it emits. The only effect which amplitude produces is upon the loudness of the sound.

217. Simple and Compound Waves.—It is not easy to represent graphically an actual wave of condensation and rarefaction. A row of equidistant dots may stand for the particles in equilibrium and a row of dots successively displaced according to the harmonic law may represent one instantaneous position of these particles when transmitting a sound-wave (Fig. 82).

Fig. 82.

It will of course be understood that the particles vibrate along the line in which the sound is propagated; and that a wave-length is the distance from one particle to the next particle in the same phase; i.e., in the same relative position and moving in the same direction.

Evidently, however, the displacement of a wave-particle from its zero position may also be represented by an ordinate drawn perpendicularly to the line of propagation, which now answers to the axis of abscissas. An entire wave constructed in this way corresponds to the harmonic curve already described. This construction greatly facilitates discussion, and is not liable to mislead if the principle on which it is based be carefully kept in mind.

A single sound-wave, then, may be said to have the

form of a simple harmonic curve (56). A complex sound-wave may have the form of a compound harmonic curve, obtained from its simple harmonic constituents by the method already described for compounding such curves (61). The form of the resultant wave will depend not only upon the relative lengths and amplitudes of the component waves, but also on the particular phase in which they are when united.

EXAMPLES.—It is evident that a sound will be the louder the greater the amount of energy expended in producing it; as when we shout or whisper. But this increased energy simply increases the amplitude of vibration. Again, since a short string vibrates more rapidly than a long one, it gives a higher note in vibrating. Moreover, the product of vibration-frequency and wave-length being constant for the same temperature, the wave length and vibration-frequency must vary inversely as each other. Hence a high note must have a shorter wave than a low note. The average wave-length of the sounds used in ordinary conversation is, for a man's voice, from two and a half to three meters, and for a woman's voice, from 60 to 120 centimeters. Difference in quality is observed whenever a note of the same vibration-frequency is sounded on two different instruments; as, for example, on the flute and on the bassoon. The tuning-fork gives the simplest, a vibrating reed the most complex, vibrations. The precise character of these complex waves will be discussed later.

218. Direct Measurement of Vibration-frequency.—The vibrations of audible sounds are too rapid to be counted; at least in the way we count the oscillations of a pendulum. They may be ascertained, however, by producing a second note of the same vibration-frequency, by means of some device so constructed that its vibrations can be determined. One of the earliest of these devices is Savart's toothed wheel. This is simply a toothed disk which is caused to revolve rapidly by means of a multiplying wheel, a card being held just in contact with its edge. The speed of rotation is increased until the note emitted by the disk is the same as that to be measured; and this speed is maintained for one minute. Then, multiplying the number of rotations made by the large wheel per minute by the ratio of the radii of the wheel and the

disk, and this by the number of teeth on the disk, we have the number of taps per minute received by the card. This number divided by 60 gives the vibration-frequency. If the large wheel turns 100 times in a minute, and its radius is 5 times that of the disk, the disk will turn 500 times in a minute; and if there are 30 teeth upon the disk, it will tap the card 15000 times in one minute or 250 times in one second.

Seebeck modified this apparatus by using in place of the toothed wheel a disk perforated with holes near its edge, and directing a jet of air against this part of the disk. A series of air-puffs was thus produced, which, as the speed of rotation was increased, coalesced to form a continuous note, of any vibration-frequency required. In 1819, Cagniard de la Tour improved the instrument by boring the holes obliquely through the revolving disk and directing the air perpendicularly against the sides of the openings; thus driving the disk by means of the issuing air. At the same time he placed this disk close to a metallic plate, through which an equal number of holes were bored obliquely, but inclined in the opposite direction to those in the disk. Whenever the holes in the disk came opposite those in the plate, air passed simultaneously through them all; thus increasing proportionately the loudness of the sound. On the axis of the revolving disk is a counter to indicate the number of revolutions. This apparatus is called a **siren**. To use it, it is supplied with air from a bellows, the pressure being increased until as before the note which it emits is the same as that which is to be determined. This speed is continued for a minute, and the number of rotations is noted. This value divided by sixty gives the rotations per second; and this multiplied by the number of holes in the disk gives the vibration-frequency.

219. Direct Measurement of Wave-length.—The wave-length of a musical note may also be directly measured by the use of an ingenious method due to Kundt. To a glass tube about two centimeters in diameter and twenty long a piston is fitted so as to

move easily backward and forward. In the tube is placed a light powder such as lycopodium or, better, precipitated silica. The note whose wave-length we desire, suppose, for example, that of a whistle, is sounded near the open end of the tube and the position of the piston gradually varied. When the piston reaches a certain point the light powder will be disturbed and will vibrate in segments, between which it will remain quiescent. After adjusting to get the best effect, measure the distance either from the middle of one segment to the middle of the next, or between the middle points of the interspaces. Twice this distance is the wave-length, in air and at the temperature of the experiment, of the note which is emitted by the whistle.

SECTION II.—SOUND-PROPAGATION.

220. Theoretical Speed of Sound.—The theory of the propagation of disturbances in elastic media was discussed in an earlier chapter (155). In simple harmonic motion, the speed with which a particle vibrates in an elastic medium, and therefore the speed of propagation in such a medium, was found to be directly proportional to the square root of the elasticity of the medium and inversely proportional to the square root of its density. This formula, $s = \sqrt{E/\delta}$, we owe to Newton, who calculated by means of it the speed of sound in air. Since the elasticity of a gas is measured by the pressure upon it (194), we have $s = \sqrt{P/\delta} = \sqrt{10^6/.0012759} = 28000$ centimeters per second, as the speed of sound in air under the pressure of a megadyne. But this is only about five sixths of its true value. Newton accounted for the discrepancy by the remarkable hypothesis that sound required no time to pass through the particles themselves, the whole time observed being that occupied in moving through the spaces between the particles. Assuming, further, that only one sixth of the space through which the sound moves consists of the *air-particles themselves*, he was in this

way able to explain the value which he had calculated. Laplace showed subsequently, however, that it was to the increased elasticity of the air due to the heat developed by its sudden compression, and to the corresponding cold produced by its rarefaction, taken together, that the increased speed of propagation over that obtained by Newton's formula was due. By multiplying the expression $\sqrt{P/\delta}$ by $\sqrt{1{\cdot}41}$, the square root of the ratio of the specific heat at constant pressure to the specific heat at constant volume, we have $\sqrt{1{\cdot}41 \times 10^6/{\cdot}0012759} = 33243$ centimeters per second, the true speed-value.

Since P/δ is equal to gh (176), we may write the equation for speed of sound $s = \sqrt{gh}$. But the speed acquired by a body in falling freely through a height h is $\sqrt{2gh}$. Hence the speed of sound in air is proportional to the speed which would be acquired by a body in falling in vacuo through half the height of the homogeneous atmosphere. This in fact is the mode of stating the speed of sound used by Newton.

221. Experimental Speed of Sound in Air.—Experimentally the speed of sound in air has been directly determined many times. The most noted of the earlier experiments are those of the French Academy in 1738, of the Bureau des Longitudes in 1822, and of the Dutch physicists Moll and Van Beek in 1823. In the last case two hills were selected near Amsterdam, 17·7 kilometers apart, and cannons were fired at stated intervals, simultaneously at the two stations, the period intervening between the time of seeing the flash and that of hearing the sound being noted on the chronometers at these stations. Properly reduced, their results give a speed of 33225 centimeters per second. In more recent times, the experiments of Regnault in 1865, made in the water-pipes of Paris, and those of Stone, made at the Cape of Good Hope in 1871, in both of which electrical recording apparatus was used, are among the best. According to Regnault's results, the speed is 33104 centimeters; according to Stone's, 33240 centimeters; the speed in air *in tubes* appearing to be somewhat less than in free air.

The speed of sound is unaffected by any agency which causes the elasticity and density of the medium to vary together in the same direction. Hence the speed of sound is independent of the barometric height. Stampfer in 1822 found the speed of sound at a height of 1304 meters to be 33244 centimeters; and Bravais in 1844 found it to be 33237 centimeters at a height of 2116 meters. Temperature, however, affects the density of the air only; so that the speed of sound is different in air at different temperatures, even under the same barometric pressure. Thus the speed observed by Parry at Melville Island, in the polar regions, at a temperature of $-38.5°$, was only 30900 centimeters per second. The sound-speeds above given have all been reduced to 0° by means of the formula

$$S_t = S_0 \sqrt{1 + \cdot 003665t}, \qquad [37]$$

in which S_t and S_0 represent the speeds at the temperatures $t°$ and 0° respectively. The increase corresponds to about 60 centimeters for each degree centigrade. Moreover, theory indicates a higher speed for loud sounds than for low ones. This Jacques has confirmed by experiments made at the Watertown Arsenal. And Parry asserts that during artillery practice in the arctic regions the report of the gun was often heard by a distant observer before the command of the officer to fire. The moisture of the air also affects the speed, for the reason that moist air is less dense than dry air. Finally, the speed of sound in free air is affected by the motion of the air, being always increased in the direction of the wind.

APPLICATIONS.—The speed of sound may often be made use of in calculating distances. Suppose that a clap of thunder is heard six seconds after seeing a flash of lightning. Then it is evident that the flash began at a distance of $6 \times 332 = 1992$ meters or nearly two kilometers from the observer. Again, let a stone be dropped into a well and the time noted until it is heard to strike the water. This time is made up of two parts: one that required for the stone to fall, and the other that required for the sound to return. If the depth

of the well be x centimeters, then the former is obtained from the equation $t = \sqrt{2x/g}$, and the latter $t' = x/s$. Whence $T = t + t' = \frac{x}{s} + \sqrt{\frac{2x}{g}}$ and $x = \frac{s^2}{g}\left[1 + \frac{gT}{s} + \left(1 + 2\frac{gT}{s}\right)^{\frac{1}{2}}\right]$.

222. Speed of Sound in Liquids.—The compressibility of liquids being so slight, the heat-changes during their rarefaction and condensation consequent upon the transmission of a sound-wave are inappreciable. The formula of Newton $s = \sqrt{E/\delta}$ therefore gives results which agree closely with those found experimentally. For example, in the case of water, we have seen (193) that at 0° it is compressed ·0000496 of its volume for a pressure of one megadyne per square centimeter. The elasticity, being the ratio of the pressure to the compression, is therefore $10^6/{\cdot}0000496 = 2{\cdot}026 \times 10^{10} = k$; and the sound-speed, since the density of water is unity, is $\sqrt{2{\cdot}026 \times 10^{10}}$, or 141340 centimeters per second, at the temperature of 0°. An experimental determination of this speed was made in 1826 by Colladon and Sturm in the Lake of Geneva. Two boats were moored 13487 meters apart, one carrying a heavy bell immersed in the water, the other supporting a trumpet-shaped tube having a membrane over its larger end. By means of a double lever, a hammer on one arm was made to strike the bell, while at the same instant some gunpowder was fired by a slow match placed on the other arm. The observer in the second boat, who was listening at the tube, noted carefully the interval between the time of seeing the flash and the time of hearing the sound. Dividing the distance, 13487 meters, by the time required to traverse it, 9·25 seconds, the sound-speed in the water was obtained. The speed thus measured was 143500 centimeters at the temperature of 8·1°; corresponding closely to 143870 centimeters, the calculated value for that temperature.

223. Speed of Sound in Solids.—The theoretical speed of sound in solids may also be calculated by the formula of Newton, if we use Young's modulus for the elasticity. Let it be required, for example, to calculate

the speed of sound in iron. The value of this modulus for wrought-iron, as given in the table on page 142, is $19{\cdot}63 \times 10^{11}$ dynes per square centimeter. Its density is 7·677. Hence the speed is $\sqrt{19{\cdot}63 \times 10^{11}/7{\cdot}677} = 505600$, or $5{\cdot}056 \times 10^{5}$ centimeters per second. A rough experimental determination of the speed of sound in solids was made in Paris by Biot, using a series of water-pipes 951·25 meters long. He observed that on striking the iron at one end with a hammer, two distinct sounds were observable at the other, about 2·6 seconds apart. One of these had been propagated by the air, the other and more rapid one by the material of the pipe. At the temperature of the experiment, 11°, the sound would require 2·86 seconds to traverse this distance in air; and therefore to traverse the line of pipes the sound required only about 0·26 second. Whence the speed in the water-pipes, as arranged, is $3{\cdot}49 \times 10^{5}$ centimeters per second. As the material of the pipes was not homogeneous, this result does not, of course, represent the speed of sound in iron. Accurate experiments by Wertheim give the speed of sound in iron at 0° at $5{\cdot}016 \times 10^{5}$ centimeters per second. The propagation of sound in solids is made use of in the string or wire telephone.

224. Indirect Methods of measuring the Speed of Sound.—Several methods have been employed for measuring the relative speed of sound indirectly. One of the simplest is a modification of the method already described for measuring wave-length, devised by Kundt. In order to measure the relative speeds of sound in hydrogen and air by this method a glass tube about 1·5 meters long and not less than a centimeter in diameter is selected, some precipitated silica is shaken into it, its ends are closed with corks, it is grasped at its middle point with the thumb and forefinger and rubbed longitudinally with a wet cloth. The tube will emit a musical note, and the silica will be thrown into the vibrating segments already mentioned (219). The distance from the middle of one of the segments to the middle of the next but one, is the *wave-length* in air corresponding to

the note given by the tube. Replace now the air in the tube by hydrogen and repeat the experiment. The segments will be greatly increased in length, being nearly four times those in air. In both cases the speed is the product of the vibration-frequency and the wave-length. If this speed be represented by $s = 2l \, . \, n$ in the case of air and $s' = 2l' \, . \, n$ in the case of hydrogen, where l is the length of a segment, the note being the same for both, we have, by dividing the first of these equations by the second, $s : s' :: l : l'$; or the speed in air is to the speed in hydrogen as the length of a segment in air is to the length of a segment in hydrogen. The observed ratio gives a value for the absolute speed in hydrogen of 126950 centimeters per second. Moreover, since the absolute speed in any gas is inversely as the square root of the density of that gas, it is evident that the relative speed for two gases must be inversely as the square roots of their densities; i.e., that $s : s' :: \sqrt{\delta'} : \sqrt{\delta}$. Now oxygen is 16 times as dense as hydrogen; and hence the speed of sound in it should be only one quarter of that in hydrogen. One quarter of 126950 centimeters is 31737 centimeters. The observed experimental value for oxygen is 31717 centimeters.

By taking a glass tube of a somewhat larger diameter, placing silica in it, fitting a movable piston to it, and holding the end of a metal, glass, or wooden rod, vibrating longitudinally, just within the open end of the tube (the end of the rod being enlarged if necessary by placing upon it a cork or a disk of card-board), the ratio of the length of a segment to the length of the rod will be obtained; but this is also the ratio of the speed of sound in air to the speed of sound in the rod. Thus, for example, a pine rod will be about ten times as long as one of the silica-segments; and hence the speed of sound in pine is ten times the speed in air.

SPEED OF SOUND.

Substance.	Temperature.	Speed. Meters per second.	Observer.
Air	0°	332	Stone
Oxygen	0°	317	Dulong
Hydrogen	0°	1269	"
Water	15°	1437	Wertheim
Alcohol	20°	1160	"
Lead	20°	1228	"
Gold	20°	1743	"
Silver	20°	2707	"
Copper	20°	3556	"
Iron	20°	5127	"
Cast steel	20°	4986	"
Pine (with the fiber)	—	3322	"
" (across the rings)	—	1405	"
" (with the rings)	—	794	"

225. Change in Direction of Sound-waves.—So long as a sound-wave continues in the same unchanged medium, it is propagated rectilinearly. But when it encounters an obstacle then its direction changes. It may be thrown back from the surface upon which it strikes, or be reflected; or it may pass into the second medium with an altered direction, or be refracted. Reflection of sound is a matter of common experience. The sound of a church-bell is often reflected from the solid walls of a building, and the voice of a speaker from the walls of a room. Tyndall has shown the reflection of sound from a sheet of paper, a pocket-handkerchief, a gas-flame, and even a sheet of heated air. But in these cases a portion of the sound is also transmitted. Because of the want of uniformity in the outer air due to currents, or to the presence of fog or snow, sound is often speedily dissipated by these repeated changes of direction. In rare cases, however, sound is heard better in rainy or snowy weather than in clear; the fact being that the air-currents are now destroyed by the equalizing effect of the rain or snow upon the temperature.

226. Sound-reflection.—Echos.—Echos are due to sound-reflection. If a sharp sound is produced so as to impinge perpendicularly upon the face of a cliff, the sound strikes the cliff and is reflected back over the same path. If the distance to the cliff be 166 meters, the reflected sound will be heard after an interval of about one second. If it be 332 meters, nearly two seconds will elapse before the return. If a person utter five sounds in a second, then, in the first case above supposed, the last sound will be emitted before the first one returns; and the five sounds will be heard distinctly repeated. The echo is then pentasyllabic. By standing before a reflecting wall and clapping the hands periodically, varying the distance until the reflected sound of one clap exactly coincides with the direct sound of the following one, a rough measurement of the speed of sound may be made; since this speed is evidently twice the distance to the wall multiplied by the number of claps made in one second. If, for example, the wall be 83 meters distant, then the sound would pass over this distance and back, or through 166 meters, during the interval between two claps. And if two claps be made per second, it would pass over 166 × 2, or 332, meters in the entire second.

Where the reflecting surfaces are multiple, the echos are also multiple. Addison speaks of an echo which repeated the report of a pistol fifty-six times. An echo at the old Simonetta Palace near Milan, according to Kircher, repeats a sound forty times; and Herschel says that an echo in Woodstock Park, England, repeats seventeen syllables by day and twenty by night. The echo of Verdun, formed between two large towers fifty-two meters apart, repeats a word twelve or thirteen times. The long-continued roll of thunder is due simply to multiple reflection from the surfaces of clouds.

In case the surfaces are curved the sound may be reflected by them to a common point called a focus. A room with elliptically-curved walls or ceiling is therefore a whispering-gallery; whispers uttered at one focus

distinctly audible at the other, but at no inter-te point. Such whispering-galleries are contained Colosseum and in the Cathedral of St. Paul's in n, and in a room in the Conservatoire des Arts et s in Paris.

7. **Sound-refraction.**—Refraction of sound is a of direction in the transmitted wave, and is due, large scale, either to differences of temperature or wind. If, in consequence of receiving heat from ound, the temperature, and hence the density, of diminishes from the surface upward, then sound ing such a varying medium will be deflected d, as Reynolds has shown; and hence will be in- to a person on the surface. If, on the other the upper layers of air be the warmer, then the will be thrown downward. The same effect prac- has been proved by Stokes to be caused by the of the air. If the sound be moving with the its direction will be inclined downward, and if t the wind, it will be inclined upward.

8. **Sound-shadows.—Diffraction.**—Interposing an le between the observer and a sounding body, en- the sound but does not destroy it. The sound- are bent round the obstacle, or inflected, as it nd so reach the ear. There is here evidently a und-shadow. In order, however, that this appa-nding inward should occur, the aperture through the sound passes, or the obstacle around which it should be small relatively to the wave-length. und-waves, in general, have a considerable length; ence to produce sound-shadows, the obstacles lves must be proportionately large. When a mix- long and short sound-waves strikes an obstacle, ite possible that the long waves may sweep round ducing no shadow; while the short waves, being pted, will cast a sound-shadow. The change in ality of the music of a brass band, for example, as und comes round a street-corner, illustrates this ont. This deflection of a sound-wave inward into

the shadow-space is called **diffraction**. It is in fact an interference phenomenon, as explained in treating of waves (66).

229. Interference of Sound.—That two sounds may be made to interfere and thus destroy each other either partially or wholly, must be accepted as conclusive proof that sound is a wave-motion. When treating of wave motions, it was shown (66) that whenever two waves of the same length but of different amplitudes are superposed, the resulting wave, while of the same length, will have an amplitude equal to the algebraic sum of the component amplitudes. If the amplitudes are equal and the phases opposite, the algebraic sum will be zero; and the waves will completely destroy each other.

EXPERIMENT.—To show this experimentally, let a tuning-fork vibrate in front of a resonator *a* (Fig. 83), connected with the tube *b* by two paths, one of which, *c*, can be made longer than the other by half the length of the sound-wave emitted by the fork. The two waves will start together at *a*; but the one which traverses *c*, having half a wave-length farther to go than the other, will reach *b* half a wave-length behind it, and so the two will be in opposite phases. In consequence the ear placed at the end of the tube *b* will hear no sound whatever.

FIG. 83

230. Sound-intensity modified by various Causes.—When sound originates at a point, it radiates in all directions from that point. So that at any instant the sound-wave is spherical in form, having the point as a center. If the radius be unity, the surface of the sphere will be 4π, and over this surface the amount of sound emitted by the sounding body will be spread. At the end of a second instant the sound will have moved out to twice the distance, the radius of the sphere will be doubled and its surface will now be 16π. The same amount of sound will now be spread over four times the surface

and therefore a unit of the second surface will receive only one quarter as much sound as a unit of the first, i.e., the intensity of the sound there will be reduced to one quarter of its former value. In other words, the distance from the sounding body having been doubled, the intensity of the sound has been reduced to one fourth; or the intensity has varied inversely as the square of the distance. For the same reason four precisely similar sounding bodies at a distance of two units will produce the same effect as one such body at unit distance.

Again, in order to produce audible sound the condensation and rarefaction of a considerable mass of air is required. Therefore a stretched string, since it produces about it when vibrating only a very small flow of air alternately in one direction and the other, produces scarcely any sound. But let such a string be supported upon a sounding-board, as is done in the piano and in the violin; and then the vibrations, being communicated to this larger surface, are sufficient to generate air-waves of great volume. In the same way the vibrations of a tuning-fork may be reinforced by resting it upon a table-top. This property of reinforcing sound is called **consonance**, and bodies which thus act, such as the sound-boards just mentioned, are called **consonators**. If, however, the reinforcement of the sound is effected by a body tuned to vibrate in unison with the sounding body, this effect is called **resonance**, and the tuned body is called a **resonator**. The resonant box on which the tuning-fork is generally mounted acts in this manner.

Further, the intensity of sound depends upon the amount of matter in the region where it is produced; i.e., upon the density of the medium there. Thus, the sound of a bell in rarefied air or in hydrogen is extremely weak, since the energy of the small vibrating mass is unequal to the task of setting a sufficient mass of the outside air into vibration. For the same reason, the ticking of a watch, which can be heard in air only 300 centimeters, can be heard in alcohol at a distance of 400

centimeters, in oil of 500 centimeters, and in water of 700 centimeters.

Section III.—Production of Sound.

231. Sources of Sound.—Sound may be generated in various ways: by impact, as when a hammer strikes the anvil, or a drumstick strikes the drum; by explosion, as in the cannon and the singing flame; by electric action, as in thunder and the musical telephone; by friction, as in the sharpening of a saw, the playing of a violin, and the weird music of the eolian harp. In short, whenever by these or by any other means elastic bodies are displaced from their position of equilibrium and then released, sound is produced.

232. Musical Sounds and Noises.—We may distinguish audible sounds as musical or non-musical according as the effect produced upon the ear is or is not agreeable; the latter sounds being generally called noises. This distinction, however, is not absolute; there is no sound which is not to some extent rhythmical, and there is none which is purely so. For example, the rattle of a cab upon the pavement is clearly a noise. But Dr. Haughton has calculated that in the streets of London, where the granite paving-blocks are ten centimeters across, a speed of about thirteen kilometers an hour produces a succession of thirty-five shocks per second, corresponding to a low musical note easily recognizable. The difference between musical sounds and noises, however, is apparently purely one of degree depending solely upon the complexity. Fourier's theorem teaches us that any arbitrary periodic curve whatever can be produced by compounding a sufficient number of simple harmonic curves of different wave-lengths. "It appears, then," says Deskin, "that a noise and a simple tone are extreme cases of sound. The former is so complex that the ordinary powers of the ear fail to resolve it. The latter is incapable of resolution by reason of its absolute simplicity." He therefore defines noise in general to be the combination of a number of [illegible]

ones too near one another in the frequency of their vibrations to be distinguished by the unaided ear.

233. Production of Musical Sounds.—Since the chief object in producing continuous sounds is an esthetic one, we may restrict our consideration of the methods of producing sound to those instruments which serve for the production of music. These may be classified as depending: 1st, upon the vibration of strings, as in the violin and piano; 2d, upon that of rods, as in the tuning-fork and music-box; 3d, upon that of plates, as in the gong and cymbals; 4th, upon that of membranes, as in the tambourine and drum; and 5th, upon that of air-columns, as in the organ, in the cornet, and in wind instruments in general.

234. Vibration of Strings.—A string, in the acoustic sense, is a perfectly uniform and flexible thread of solid matter, stretched between two fixed points. Such strings are capable of three kinds of vibration: transverse, longitudinal, and torsional. We shall consider only the first of these.

A complete transverse vibration is simply the movement of a transverse wave along the string and back. Hence the time of vibration of a string is the time required for the wave to move over twice its length; i.e., the length of the wave is twice the length of the string. Consequently we may obtain the law of transverse vibration in a string from that of wave-propagation along it. Suppose a stretched string to be drawn aside at its middle point. The force of restitution will be proportional to CO, the component of the tension perpendicular to the string at that point (Fig. 84). But the displacement is OC;

FIG. 84.

therefore the force of restitution is proportional to the displacement, *and the string will vibrate* harmonically.

The speed of propagation (155) $s \propto \sqrt{a} = \sqrt{F/M} = \sqrt{F/\pi r^2 \delta}$ for unit of length. But $n = s/\lambda$ and $\lambda = 2l$. Whence

$$n = \frac{1}{2l} \cdot \frac{1}{r} \cdot \sqrt{\frac{F}{\pi \delta}}; \qquad [38$$

in which n is the vibration frequency per second, l the length, r the radius and δ the density of the string, and F the stretching force in dynes. It therefore appears that the number of transverse vibrations made by a stretched string is proportional: 1st, to its length, inversely; 2d, to its radius, also inversely; 3d, to the square root of the stretching force directly; and 4th, to the square root of the density of the string, inversely.

The instrument used in verifying these laws, in its earliest forms as used by Euclid and Pythagoras, is called a **monochord**; in its later forms, a **sonometer**. It consists of a long narrow resonant box of wood provided at its ends with steel pins, to which one or more wires can be fastened. These wires pass over two bridges placed near the pins, and generally a meter apart. This distance between the bridges is the length of the vibrating wire. In some instruments a pulley is placed on one end, to facilitate stretching any wire by a direct weight. The wires proper of the sonometer are stretched by turning the pins with a key, as in the piano.

Experiments with the Sonometer.—If such a wire be made to give a certain vibration-frequency as a whole, then by placing a movable bridge under its middle point it may be made to vibrate in halves. The number of vibrations is now doubled, since with the same speed of propagation the wave has only half as far to go. The number of vibrations therefore is inversely, and the period or time of one vibration is directly, as the length of the string.

In a second experiment, place a third string on the sonometer between the other two, both of which have been tuned to give the same note. Let this third string pass over the pulley and hang weights upon it until it has the same vibration-period as the other strings. Note the weight, and then add to it until the note emitted is the same as that given by one half of either of the other strings, i.e., a note of twice the vibration-frequency. It will be found that if the first weight be called unity the second will be four times this

other words, to double the number of vibrations the tension has en quadrupled; i.e., the number of vibrations is proportional to e square root of the tension.

To determine the law of the diameters, two wires of the same aterial but of different diameters, say as 1 : 2, are stretched sucssively upon the sonometer. If for a given stretching weight the aller vibrates with the same frequency as half the fixed wire, the ger will give the note of the entire wire. All other conditions ing the same, then, a wire of the diameter two will give half as any vibrations per second as a wire of the diameter one.

By employing a wire of platinum, density 21·5, and one of steel, nsity 7·8, the law of densities may be established in the same way; e ratio of the vibration-frequency of the platinum to that of the el, other things being equal, being as $\sqrt{7 \cdot 8}$ is to $\sqrt{21 \cdot 5}$.

Obviously, by using the equation $n = 1/2l \cdot \sqrt{F/M}$, the last two periments may be condensed into one and the vibration-frequency proved to vary inversely as the square root of the mass of the ing per unit of length.

235. Reversal of Phase in Reflection.—If a pulse sent along a stretched string, it will not only be reected at the fixed end, but it will be reversed in phase 5). So that if at the instant at which it starts on its turn a second pulse be started outward, the two will eet in the center of the string and will destroy each ther by interference. The center of the string will be a oint of rest, provided that the two pulses are alike and ual; if not, it will be a point of minimum motion. ch a point is called a nodal point or **node**. Half-way tween this central node and the ends the motion of string attains a maximum. These points of maxium motion are called antinodes or **loops**. The sta-

FIG. 85.

nary wave in the string consists of two loops with a dal point between; i.e., it is a complete wave. In rating, the loops on the two sides of the nodal point *oppositely directed*, as the figures *A* and *B* (Fig. 85

show; the two states being successive. The waves in such a vibrating string are stationary. There may be as many vibrating segments produced as is desired; since for three, four, five, six, etc., such segments the pulses must succeed each other so rapidly that the interferences shall take place at one third, one fourth, one fifth, one sixth, etc., the length of the string.

EXPERIMENTS.—This subdivision of a vibrating string into segments may be readily shown on the sonometer. If the string be touched with a feather at its middle point while the bow is drawn across it, both halves will vibrate, but in opposite directions. If the feather be placed at one third, one fourth, one fifth, or any other exact fraction of its length, there will be three, four, five, or more vibrating segments, the alternate segments moving in opposite directions. To show this subdivision into segments, small strips of folded paper may be placed astride the string. If any of these are at nodal points they will not be disturbed; but if they are at the loops, where the motion is at a maximum, they will be promptly thrown off. It will be noticed that a node is produced wherever the string is damped, and a loop wherever it is struck or bowed.

But not only may these different modes of subdivision exist separately upon a string; they may and do coexist. Whenever, therefore, a string is emitting its lowest musical note, corresponding to its rate of vibration as a whole, it is also emitting notes of frequencies represented by the natural numbers 2, 3, 4, 5, 6, 7, etc. Such accompanying notes are called the **upper partial tones**, or sometimes the **overtones** of the string. It is to them that the quality of the tone is due. When concordant, they are frequently called **harmonics.**

Strings are generally excited in one of three ways: either by plucking them with the finger, sometimes armed with a quill or plectrum, as in the harp; by drawing a bow across them, as in the violin; or by striking them with a hammer, as in the piano.

The quality of the note emitted by a string is markedly affected by the method of exciting it; the vibrations of a piano-string in consequence differing very much from those of a violin-string. In the former case the string is struck suddenly at one point, and it departs

greatly in its form from the sine curve. If the hammer be soft and the blow be gradual, however, the fundamental tone is stronger, the higher upper partials weaker, and the whole tone purer. The string is also distorted in the violin. The fundamental tone is distinct, but is rendered penetrating in quality in consequence of the mass of high associated upper partial tones.

236. Vibration of Rods.—Transversal.—Like strings, rods may vibrate transversely, longitudinally, or torsionally. The force of restitution in these cases, however, is the elasticity of flexure of the rod itself; and this whether it be supported in the middle, at one end, or at both ends. The transverse vibrations of rods are quite complex, but they may be represented by the equation

$$n = \frac{\tau}{l^2} \cdot \sqrt{\frac{M}{\delta}}\, C, \qquad [39]$$

in which n is the vibration-frequency, τ the thickness, measured in the direction of vibration, l the length, M Young's modulus, δ the density, and C a constant depending on the method of supporting the rod. If it be clamped at both ends or free at both ends, this constant is 1·78; if free at one end only, 0·28. It will be seen that the number of vibrations is directly proportional to the thickness and to the square root of the elasticity, and inversely proportional to the square of the length and to the square root of the density of the material. It is independent of the width of the rod. Hence the vibration-period of a square rod is the same in two perpendicular directions parallel to the sides. While if the section be rectangular, the vibration-periods in these perpendicular directions will be in the ratio of the dimensions in these directions. Wheatstone's kaleidophone consists of a series of rods, each having a different ratio of thickness in the two directions, and hence giving when vibrated the figure characteristic of their two vibration-periods, as already explained (59). These figures are seen by the

reflection of light from a silver bead carried on the end of each rod.

A special case of the vibration of rods free at both ends is that of the tuning-fork. A straight rod of this sort when sounding its lowest note has two nodal points, each at a distance from the end equal to about one fourth the distance between them (Fig. 86). As this rod is gradually bent into the form of a tuning-fork, these nodal points approach each other. It will be seen that while the prongs of the fork vibrate laterally—and in opposite directions—the base of the fork vibrates vertically (Fig. 87), and thus through the stem communicates its vibrations to the resonant box on which it is ordinarily mounted.

FIG. 86.

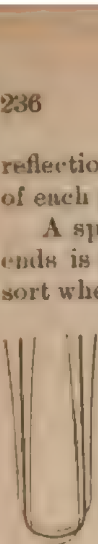


FIG. 87.

The harmonics of rods rise much more rapidly than those of strings. Even for a rod which is fixed at both ends like a string, as well as for one free at both ends, the vibration-frequencies for the fundamental and its three first harmonics are as 3^2, 5^2, 7^2, and 9^2; or as 9, 25, 49, and 81. When fixed at one end only, the ratios of vibration for the fundamental and the first harmonic are as 2^2 to 5^2, or 4 to 25. The first harmonic of a tuning-fork giving 256 vibrations as its fundamental, is a note of 1600 vibrations, as was first proved by Chladni.

EXAMPLES.—As instances of the use of transversely vibrating rods in musical instruments, the reeds of an accordeon or harmonium, the tongue of a jew's-harp, the similar tongues of a music-box, the triangle, and the steel spirals often used in place of bells, may be cited. For rods free at both ends, the glass or metal harmonicon, the claque-bois or xylophone, and the tuning-fork may be mentioned.

237. Vibration of Rods.—Longitudinal.—The longitudinal vibrations of rods are much less complex in character, since their lowest vibration-period is simply the time required by the sound to pass through four times the length of a free segment of the rod. Hence the vibration-frequency for rods free at both ends is

$n = s/2l$, and for rods free at one end $s/4l$, in which s is the speed of sound in the material of the rod. The period of vibration of a rod, therefore, is independent of the area of its cross-section. Thus the vibration-frequency of a steel rod one meter long when clamped in the middle is $(5{\cdot}056 \times 10^5)/200$, or 2528 vibrations per second. Conversely, since $2nl = s$, the product of the vibration-frequency by twice the length of the rod gives the absolute speed of sound in the substance of which the rod is made. Moreover, the relative speeds in two rods of different substances are proportional to their vibration-frequencies if they are of the same length; or to their lengths if they have the same vibration-frequency.

238. Vibration of Plates.—A rod when extended sufficiently in width becomes a plate. The mode of vibration of plates may be readily studied by strewing sand upon them, a method originally used by Chladni. If a square brass plate thus prepared be clamped at its center, and damped at the middle point of one of its sides by touching it with the finger, and then the bow be drawn across the edge near a corner, the plate will emit its lowest note and the sand will collect along two lines through the center parallel to the sides (Fig. 88).

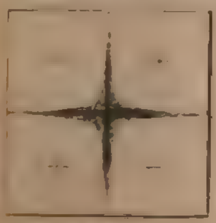


FIG. 88.

FIG. 89.

But if it be damped at a corner and the bow be drawn across the middle point, the sand-lines will be found along the diagonals (Fig. 89); and the note emitted by the plate will be higher than before in the ratio of 3 to 2. The plus and minus signs in the segments indicate the directions of vibration. Consequently the sand collects

simply at the nodal lines between two segments vibrating in opposite directions.

Wheatstone explained these results as due simply to the superposition of two plates whose axes of vibration are perpendicular to each other. Suppose three square plates *A*, *B*, and *C* (Fig. 90) vibrating transversely, after

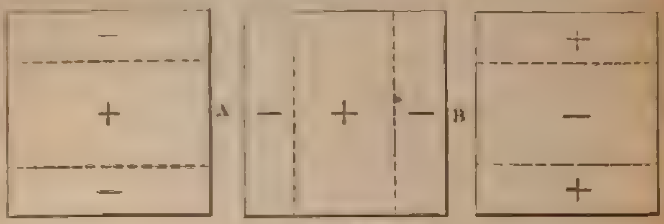


Fig. 90.

the manner of rods free at both ends. Let two of these modes of vibration, say *A* and *B*, be impressed at the same time upon a plate, and we shall have *D* (Fig. 91),

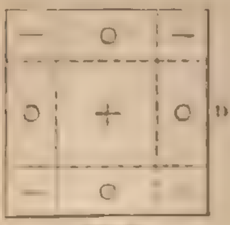


Fig. 91.

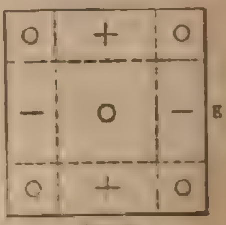


Fig. 92.

or *B* and *C*, and we shall have *E* (Fig. 92); which are the two cases above given. In *D* the nodal lines are parallel to the sides and intersect at the center where the plate is clamped. In *E* the nodal lines are the diagonals. The laws of the vibration of plates, then, are the same as those of wide rods similarly clamped.

By varying the position of the point of damping and the point at which the bow is drawn, sand-figures of great complexity and great beauty may be produced. ...ight powder, such as lycopodium, be mixed with ... this powder will be observed to collect in the

ddle of the segments; i.e., at the places where the tion is most violent. This result was shown by Far-y to be due to the air-currents produced by the rating segments. The phenomenon disappears en-ly in vacuo.

When circular plates are used, the nodal lines are ial and divide the plate into sectors proportional in mber to the vibration-frequency (Fig. 93). The lowest e corresponds to four sectors, the next to six, the

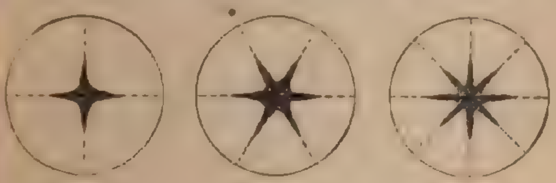


FIG. 93.

t to eight; the vibration-frequencies being as the ares of these numbers. In general the vibration-quency of a disk is directly proportional to its thick-s and inversely proportional to the square of its meter.

Bells, like circular plates, divide when vibrating into even number of sectors, the alternate ones moving ositely. Hence one diameter at the mouth of the l elongates while another at right angles to this tracts, for the lowest note; the bell becoming ellipti-alternately in these two rectangular directions.

239. Vibration of Membranes. — Membranes are ets of thin parchment, of india-rubber, or of paper, tched upon suitable frames. They are set in vibra-either by a blow, as in the drum; or by the vibrations he air in their immediate vicinity. If the tension on embrane is the same at all points of its edge, the nbrane vibrates as a whole. But if the tension be equal, the membrane has two rates of vibration, portional to the tensions; as in the case of a string. ibrates strongly along the line of greatest tension, ly along the weakest line.

240. Vibration of Air-columns.—That the air is the effective vibrating body in organ-pipes and wind instruments generally, is shown by the fact that, provided the walls be thick, the vibration-frequency of the note emitted is independent of the material of which the instrument is made. In discussing the production of sound by the vibration of air-columns, we will consider first the means by which the vibration is set up in the tube; and second the specific character of the air-vibration itself; using the organ-pipe as an illustration.

That part of an organ-pipe by means of which the vibrations are set up is called the **mouthpiece.** Two kinds of organ-pipe are in common use, known as the flue-pipe and the reed-pipe, respectively, according to the method employed in them for generating the vibrations. In the flue organ-pipe the air issues from a narrow slit in the form of a thin sheet and strikes upon a sharp edge placed immediately above the opening; as is seen in the common whistle. That portion of the air which is thus deflected into the pipe produces a pulse of compression, which is propagated up the tube, and is reflected at the outer and open end. This reflected pulse meets a second pulse coming from the lower end, and thus the air-column is thrown into vibration. So that in flue-pipes the air-column itself determines the vibration-frequency.

In a reed-pipe, the vibrating body is initially a thin tongue of metal called a **reed,** placed over the orifice at which the air enters. It may be narrow enough to pass through the opening; in which case it is called a **free reed.** Or it may be so broad as to cover the opening completely when forced down upon it; then it is called a **beating reed.** When a current of air is directed against a free reed it is forced into the opening; and the air-pressure thus being lessened it springs back by virtue of its elasticity, and so is thrown into vibration. It is the rate of vibration of the reed which fixes the vibration-frequency of the note emitted by the pipe;

quality only of the sound being affected by the air-umn above it.

The time of vibration of the air-column in a tube imply the time required for a sound-wave to pass ough twice the length of the tube. Hence the tube lf is one half the length of the sound-wave rresponding to the note emitted by it. Since th ends of the organ-pipe are open, there be no compression at either end, and both ends of the column therefore are loops. e middle point of the pipe being intermedi- between two loops must therefore be a le (Fig. 94). The vibration-frequency n an open organ-pipe is $s/2l$, in which s is speed of the sound in air and l the length the pipe. Since s, in air at the ordinary perature, is about 34000 centimeters, an n pipe 70 centimeters long will have a ration-frequency of 243 approximately.

FIG. 94.

EXPERIMENTS. — Lower into a wide organ-pipe, sounding its damental note, a small circular membrane strewn with sand. it enters the pipe the sand will rattle loudly upon the membrane, vibrations growing less as it goes down, and ceasing entirely at middle point of the pipe. They increase again, however, as the tambourine descends further, becoming as lively as ever at mouth. A node therefore is a point of no motion, although it point of maximum condensation and rarefaction. Consequently, opening be made through the tube at its middle point, which in case is a node, the sound emitted immediately changes, the becoming a loop, and the pipe being practically reduced one in length. This is the action of the openings in the flute and the onet. On the other hand, if a diaphragm be placed across the of an open pipe thus vibrating, the condensations and rarefac- are not interfered with, and the vibration-frequency of the is unchanged.

A stopped organ-pipe, as the pipe in the last experi-t is generally called, gives the same note as an open of twice its length. Hence the wave-length of the nd emitted by a stopped organ-pipe is four times length of the pipe itself.

The upper partial tones of an open organ-pipe are represented by the natural numbers 2, 3, 4, 5, 6, etc., like those of a string vibrating transversely (Fig 95).


Fig. 95.

The middle point is a loop for the even modes, and a node for the odd modes of subdivision. Hence if an open pipe be stopped at its middle point, only those overtones are possible which have this point for a node; i.e., the uneven overtones, 3, 5, 7, etc. In consequence, these are the only overtones possible in the note of a closed pipe (Fig. 96). The quality of the note which such a pipe emits consequently is quite distinct in character from that emitted by an open pipe.

When an open organ-pipe is sounded in any other medium than air, the equation $n = v/2l$ still holds for its fundamental note. Since at 0° the speed of sound in hydrogen is 126950 centimeters and in water 142340 centimeters per second, the vibration-frequency of the note emitted by the 70-centimeter pipe above mentioned will be nearly 907 when sounded with hydrogen, and nearly 1017 when sounded with water.

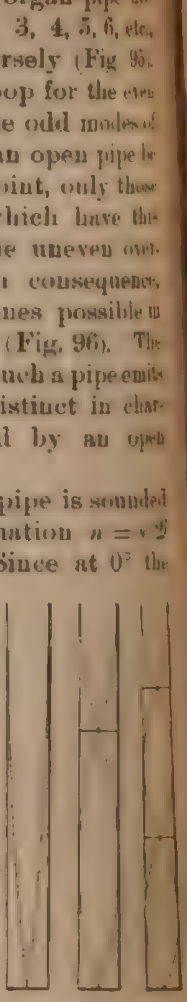


Fig. 96.

The pipes used with reed mouthpieces are generally called **cornets**. They differ widely in shape, their function being to reinforce certain upper partials of the vibrating reed more strongly than others, and thus to develop a certain quality of tone. The clarionet, oboe, and bassoon are reed instruments, the reeds being made of cane; as are also the brass instruments known as the trumpet, French horn, cornet, trombone, etc. The flute, the fife, the piccolo, and the flageolet are flue-pipes. The quality in all these instru-

ments depends on the form of the air-cavity, which may be modified by various devices.

SECTION IV.—RELATION OF SOUND TO MUSIC.

241. Limit of Musical Sounds.—Musical sounds have already been defined as those which are agreeable to the ear. But the sensation of hearing, like the other sensations, varies widely in different persons; and hence there is not only a wide difference of opinion as to the musical character of sounds, but even the range of audibility itself differs in different cases. Tyndall tells us that the chirping of the insects in the Wengern Alp, which was so shrill as to be almost intolerable to him, was entirely inaudible to a friend walking by his side. Preyer gives 16 and von Helmholtz 34 as the vibration-frequency at the lower limit of audibility. The upper limit is 32000 (Despretz), 38000 (von Helmholtz), and 40000 (Appunn) vibrations per second. But of course all these sounds are not available for musical purposes. The extreme range employed in music extends in general from 32 vibrations a second given by the lowest pipe of an organ (4·88 meters long) to 4224 vibrations in the piano-forte, or 4752 vibrations in the piccolo; the average being from 40 to 4000 vibrations. The average compass of the human voice is included between about 87 vibrations for a bass voice, and 775 for a treble one. Exceptionally, the soprano voice has reached 1305 vibrations.

242. Musical Pitch.—Absolute Vibration-frequency.—What is called vibration-frequency in acoustics, in music is called **pitch**. The former is stated numerically as the number of vibrations per second; the latter is generally indicated by one of the first seven letters of the alphabet. Thus the pitch of the musical note C_1 is about 256 vibrations in a second. Since the period of vibration of a note is inversely as its frequency, the pitch is said to be high or low, *according* as its period is rela-

tively short or long. The absolute pitch employed in music is regulated somewhat arbitrarily by musicians; not only does it vary from time to time, but the standard concert-pitch varies considerably at the same time in different countries. Thus Ellis has shown that in England concert-pitch for C_4 has risen in the course of 130 years from 467 to 546 vibrations per second. Lissajous found for the note A_3 in the Turin opera 444·75, the Paris opera 448, the Milan opera 450·3, the Berlin opera 450·75, and the St. Petersburg opera 451·5 vibrations. In Paris in 1826 the A_3 fork of the French opera gave 445, that of the Italian opera 449·5, and that of the Opera Comique 452 vibrations. And Cross has found for C_3 vibration-frequencies varying from 259·1 to 273·9 among the musical-instrument makers in this country; Chickering's standard fork being 268·5, Weber's 270·3, and Steinway's 272·2. The Thomas concert-pitch in 1879 was 271·1, and that of the Boston Music Hall organ 271·2. The French Academy has adopted 435 vibrations for the note A_3. In Germany the A_3 fork of the Stuttgart congress, 440 vibrations, is the standard. In England the Society of Arts adopted this, but the pitch used in acoustic instruments, $C_4 = 512$, is also in quite general use.

243. Relative Pitch.—Intervals.—Von Helmholtz has pointed out the fact that in the music of all nations, so far as known, alterations of pitch in melodies take place by intervals, and not by continuous transitions. Hence arises a number of distinct musical notes between which these intervals occur. But all tones which are musical when sounded alone do not produce musical intervals when sounded together. Consequently melody, which consists of a succession of notes, is more ancient and more primitive than harmony, which is obtained by simultaneously sounding tones which together produce an agreeable effect. We owe to Pythagoras the first experimental investigation of the laws of harmony. By suitably subdividing a stretched string, he proved that

the harmony of the two sounds is the more perfect, the simpler the ratio of the two parts into which the string is divided. Subsequent investigation established the fact that the vibration-frequency of a string is inversely, and its period is directly, proportional to its length; and therefore that those intervals are the most musical which have the simplest vibration-ratios. Calling the vibration-ratio of two notes their relative pitch, we see that the intervals 1 : 1, 1 : 2, 2 : 3, 3 : 4, etc., are the intervals producing the maximum harmony, since their relative pitch is expressible by the simplest numbers. Why this should be so, remained for a long time an enigma. Even the eminent mathematician Euler accepted it as an ultimate fact, contenting himself in the belief that it was because the human mind takes a peculiar pleasure in simple ratios. The problem was ultimately solved by von Helmholtz, in a way which we shall presently explain.

244. Musical Chords and Scales.—But not only may two notes whose relative pitch is expressible by a simple ratio be sounded together with good effect; three or more such notes may also be sounded together with the same result, provided that these notes have vibration-frequencies which bear to each other simple ratios. Thus four notes having the vibration-frequencies 4, 5, 6, 8 have the ratios 1, $\frac{5}{4}$, $\frac{3}{2}$, 2, which are simple or consonant ratios with reference to each other, being $\frac{5}{4}$, $\frac{6}{5}$, $\frac{4}{3}$. While the notes 6, 8, 9, 12, giving the intervals 1, $\frac{4}{3}$, $\frac{3}{2}$, 2 are dissonant, because their ratios are more complex, $\frac{4}{3}$, $\frac{9}{8}$, $\frac{4}{3}$. The sound produced by the simultaneous production of more than two separate notes is called in music a **chord**. A consonant chord like the first just given is called a **concord**; a dissonant chord a **discord**. Now on comparing together the notes which have been used for musical expression, and to which the first seven letters of the alphabet have been given as names, it appears that this harmonic triad, i.e., the three notes whose ratios are 4 : 5 : 6, is repeated three times in the series. Thus we

have $C:E:G::4:5:6$; $G:B:2D::4:5:6$, and $F:A:2C::4:5:6$. Whence we have

$$\begin{array}{ccccccccc} C & D & E & F & G & A & B & C & D \\ 4 & & 5 & & 6 & & & & \\ & & & & 4 & & 5 & & 6 \\ & & & 4 & & 5 & & 6 & \end{array}$$

Combining these together and clearing of fractions, we have for this series of eight notes the relative pitch of each note as follows:

$$\begin{array}{cccccccc} C & D & E & F & G & A & B & C \\ 24 & 27 & 30 & 32 & 36 & 40 & 45 & 48 \end{array}$$

Such a succession of notes is called a **musical scale**, the first note being repeated to form the eighth. There are seven intervals in this scale, these intervals being characterized by their position in the series. Thus the interval from the first or fundamental note to the second note is called the interval of the **second**; to the third, fourth, fifth, sixth notes, the interval of the **third**, **fourth**, **fifth**, **sixth**, etc. And the interval from the first to the eighth an **octave**. Since the interval is determined by the ratio of the vibration-frequency of the upper note to the lower, it is evident that intervals and chords have to do with relative pitch alone. The intervals in the musical scale above given reckoned from the first or key note are therefore as follows: Interval of the second $\frac{27}{24}$ or $\frac{9}{8}$; of the third $\frac{5}{4}$; of the fourth $\frac{4}{3}$; of the fifth $\frac{3}{2}$; of the sixth $\frac{5}{3}$; of the seventh $\frac{15}{8}$; and of the octave 2. Between the successive notes we have

$$\begin{array}{ccccccccccccccc} C & & D & & E & & F & & G & & A & & B & & C \\ 1 & & \frac{9}{8} & & \frac{5}{4} & & \frac{4}{3} & & \frac{3}{2} & & \frac{5}{3} & & \frac{15}{8} & & 2 \\ & \frac{9}{8} & & \frac{10}{9} & & \frac{16}{15} & & \frac{9}{8} & & \frac{10}{9} & & \frac{9}{8} & & \frac{16}{15} & \end{array}$$

It will be observed that the intervals in this scale thus constructed are not equal; three being represented by $\frac{9}{8}$, two by $\frac{10}{9}$, and two by $\frac{16}{15}$. The first of these is called *a* **major tone**, the second a **minor tone**, and the last a **major**

one or **limma.** The interval between the major and minor tone, $\frac{81}{80}$, is called a **comma.** Moreover, the val between C and E, called the **major third,** and C G, the fifth, is $\frac{3}{2} \div \frac{5}{4}$ or $\frac{6}{5}$; and this, which is the val between E and G, is called a **minor third.** The val between the major third $\frac{5}{4}$ and the minor third $\frac{25}{24}$, a **minor semitone** or **diesis.**

45. **Minor Scale.**—If a musical scale be constructed ich the minor third precedes the major instead of ving it as above, its musical character is entirely d, and it becomes a minor scale. If A be taken as ey-note, a minor third below C, the key-note in the scale, the ratios of the intervals are as follows:

$$
\begin{array}{cccccccccccccccc}
A & & B & & C & & D & & E & & F & & G & & A \\
1 & & \frac{9}{8} & & \frac{6}{5} & & \frac{4}{3} & & \frac{3}{2} & & \frac{8}{5} & & \frac{9}{5} & & 2 \\
& \frac{9}{8} & & \frac{16}{15} & & \frac{10}{9} & & \frac{9}{8} & & \frac{16}{15} & & \frac{9}{8} & & \frac{10}{9} &
\end{array}
$$

are therefore in the minor scale three major and inor tones, and two major semitones, just as in the scale, but they are differently distributed. The cteristic interval is the minor third, and this slight e produces a marked effect; so that while music n in the major key is cheerful, open, and bold in its cter, that written in the minor key is sad, melan- and above all undecided. A second form of the scale is in use in which the intervals are as fol- $\frac{9}{8}$, $\frac{16}{15}$, $\frac{10}{9}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{9}{8}$, $\frac{16}{15}$. This form is preferred for cending scale, while the form above given is gener- mployed for the descending scale.

46. **Transposition of Scales.** — The scales now ibed, which are called **diatonic** scales, are quite ient for any music written in the particular key on they are based. But since any note of the scale be used as a point of departure, it is obvious that der that the succession of intervals shall remain ame, certain changes must be made in the scale . **Suppose G to be made the key-note.** Then of wo semitones necessary, one between B and C, the between F and G, only the former exists; while

at the same time a semitone exists between E and where it is not required. Hence simply by raising a semitone the intervals on the scale of G will beco the same as in the scale of C. This process of raising note by a semitone is called **sharpening** or sharping the converse process, lowering it a semitone, being call **flatting** it. Thus F raised a semitone becomes F sha or $F\sharp$; while C lowered a semitone becomes C flat $C\flat$. The number of vibrations of a sharped note obtained by multiplying the vibration-frequency of natural note by $\frac{25}{24}$, the interval of a minor semitone, the vibration-number of a flatted note by dividing by same fraction.

Since every transposition of this sort from one key another requires the sharping or flatting of some of notes of the scale, it is evident that upon an instrum like the piano, for example, new keys must be added the new notes. Since, for example, $C\sharp$ and $D\flat$ are in fact the same note, the number of such new key greater than the number of whole tones in the scale. play in perfect tune in the seven keys of the major sc requires the introduction of twelve notes; and of minor, of seven notes more, making in all twenty notes to the octave. But since the key-note may be it a sharped or flatted note, still more notes will be requi for these scales. So that Ellis fixes 72 notes in octave as the number essential to theoretically compl command over all the keys used in modern music.

247. Temperament.—In practice, such a number notes in the octave is clearly impossible. A compro is then necessary between the pure intervals required theory and the possibilities of execution. Such a proc of accommodation is called "**tempering**" the scale, the various methods which have been proposed accomplishing it are called "**temperaments.**" That generally adopted is a system of equal temperam developed by Bach. It ignores the distinctions betw the major and minor tones and semitones, and consid the sharp of one note as the same as the flat of the

above it; thus reducing the notes in the octave to twelve only; i.e., to the seven original ones represented by the white keys and the five interpolated ones by the black ones. "Music founded on the tempered scale must be considered as imperfect music, and far below our musical sensibility and aspirations. That it is endured and even thought beautiful, only shows that our ears have been systematically falsified from infancy." (Blaserna.)

248. Analysis and Synthesis of Composite Tones. —The term **quality**, as applied to musical sounds, has reference to the form of the wave; or since, by Fourier's theorem, the form of a compound wave depends upon the character of its components, the quality of a composite tone is determined by the number and character of its component waves. As we have seen, the tones emitted by musical instruments are all more or less composite, the fundamental or lowest note in the combination being associated with higher notes, rising in pitch by definite intervals, these associated notes being called overtones or upper partials; sometimes also harmonics. When, for example, a piano-string or the air in an open organ-pipe vibrates, not only do we hear the note corresponding to its maximum period, which is called the pitch of the string or air-column; but accompanying it we hear the overtones which are also present. In the case of the string, the fundamental note is given when the string vibrates as a whole in a single segment. But when it vibrates in two segments, or three or four, simultaneously, there are superposed upon this fundamental note, other notes whose relative pitch is two, three, or four times that of the note proper of the string. The intervals thus produced are therefore $\frac{2}{1}$ for the first added tone, $\frac{3}{2}$ for the second, $\frac{4}{3}$ for the third, $\frac{5}{4}$ for the fourth, $\frac{6}{5}$ for the fifth, and so on. When the series of overtones is complete, the vibration-frequency follows the order of the natural numbers 1, 2, 3, 4, 5, etc., and the first overtone is the octave of the fundamental, the second is the fifth of that octave, the third is the double octave, the fourth is its major third,

the fifth is the fifth of the double octave, and so on, up to the limit of audibility. If the fundamental note be C_1, for example, the harmonic series will be as follows:

C	C_2	G_2	C_4	E_4	G_4	X
128	256	384	512	640	768	896
m	$2m$	$3m$	$4m$	$5m$	$6m$	$7m$

The note marked X is intermediate between A_4 and B_4.

249. Methods of analyzing Sounds.—Two methods have been employed for the analysis of composite sounds, both based on the principle of sympathetic vibration. When two vibrating bodies, tuning-forks, for example, are in exact unison, either of them will set the other in vibration. In the method of Mayer, the sympathetic vibrations are excited in tuning-forks; in that of von Helmholtz, they are excited in the air of resonators. The reed-organ pipe, for example, whose tone is to be analyzed by the former method, has an opening in its wall opposite a node, and this opening is covered with a thin inelastic membrane. To the center of this membrane is attached a bundle of silk cocoon threads a meter or more long, their outer ends being fastened each to a carefully adjusted tuning-fork of the harmonic series. When the organ-pipe sounds, the vibrations of the membrane are all transmitted along the tense threads to the forks; but each fork is influenced only by vibrations of its own rate. So that, after the experiment, by noting which of the forks are sounding, it is easy to tell the vibration-ratios actually existing in the composite tone. Moreover, the proportionate strength of any harmonic in the original tone is faithfully reproduced on the corresponding fork.

In von Helmholtz's method resonators are employed. These are hollow vessels whose air-cavity has been carefully tuned to a definite pitch and the air in which therefore is readily thrown into vibration by a note of the same period. The older resonators were spherical, but the later ones are cylindrical in form. One end is drawn down to a small opening which may be placed in

the ear. If the sound to which the resonator is tuned exists in the air in the vicinity, it will be reinforced and become audible. Hence by sounding a composite tone, and applying successively different resonators to the ear, t is possible to indicate at once the simple notes which xist in the compound tone, and thus to analyze this one into its constituents. Moreover, by educating the ar by practice with these instruments, it becomes possi-le to distinguish many of the different harmonics in a mpound tone even without the aid of resonators.

250. Methods of synthesizing Sounds.—In Mayer's pparatus just described, it is evident that as all the rks whose notes exist in the composite tone of the ed-pipe are simultaneously sounding, they must to-ther reproduce this composite tone, and thus synthe-e the sound. Another special form of apparatus for the nthesis of sounds has been devised by von Helmholtz. consists of a set of eleven tuning-forks, one giving e fundamental note and nine others its harmonics; the eventh fork acting as an interrupter, to control elec-ically the vibrations of the others. Each of the ten rks is provided with a resonator, and is kept in vibration means of an electro-magnet. The aperture of each sonator is closed with a disk, movable from a key-board means of a lever. When all the forks are vibrating e sound is feeble; but on depressing any key the cor-sponding fork speaks out loudly. If all the keys are essed down at once, the full set of harmonics, like ose of an open organ-pipe, for example, are heard. hile if only the odd keys are so depressed, the sound sembles that of a closed organ-pipe.

The results of von Helmholtz's researches with these struments have not only abundantly demonstrated the ct that the peculiarity of sound which we call quality due solely to the degree and to the character of the mplexity of that sound; but they have enabled him to assify musical sounds. Thus he finds, 1st, that simple nes, like those of tuning-forks on resonant boxes, are ft and smooth, but feeble and dull. 2d, that com-

pound tones containing a moderately loud series of harmonics up to the sixth, are harmonious and musical. While rich and splendid, they are also sweet and soft if the higher harmonics are absent. Such are the tones of the pianoforte, of open organ-pipes, and the softer tones of the voice and of the French horn. 3d, that when the uneven harmonics only are present, as in narrow closed organ-pipes, piano-strings struck in the middle, and clarionets, the quality is hollow, and if the upper notes are present, nasal. When the fundamental tone is strong, the tone is rich and full; when weak, the tone is poor and empty. 4th, that when higher harmonics than the sixth or seventh are prominent, the tone is rough and cutting; as in the violin and reed-pipe, the oboe, bassoon, and accordeon.

251. Theory of Dissonance.—Resultant Tones.—We have noticed already (61) that when two or more harmonic curves are compounded the resultant is also a harmonic curve, whose amplitude is the algebraic sum of the component amplitudes. If the two harmonic curves be equal in length and in amplitude but opposite in phase, they will mutually destroy each other. But if one has a wave-length greater than the other, the one will gradually gain on the other so as to be alternately in the opposite and in the same phase with reference to it (66). When in the same phase, the resultant ampli-

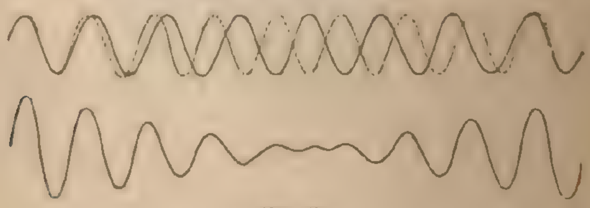


FIG. 97.

tude is doubled and the sound quadrupled; when in opposite phases, the sound is zero. Hence under these conditions a rise and fall of the sound is heard, producing the phenomenon known in music as **beats** (Fig. 97). If, for example, one note has 64 vibrations a second

and the other 65, then, if they start together, the latter, at the end of half a second, will have gained half a vibration over the former. Hence the two will be in opposite phases and will oppose each other. At the end of a whole second, the latter note will have gained an entire vibration over the other and the two will again be in accord. There will therefore be one increase and one decrease of the sound each second; or in other words, there will be one beat per second. In general, the number of beats has been shown by Koenig to stand in a simple relation to the vibration-frequencies of the interfering sounds. Primary beats, i.e., the beats of fundamental tones, fall naturally into two sets, called respectively a **superior** and an **inferior** set. If on dividing the higher vibration-frequency by the lower there is a positive remainder, the primary beats thus produced belong to the inferior set; while if the remainder is negative, they belong to the superior set. As a rule, the inferior beat is heard best when its number is **less** than half the frequency of the lower primary; while when its number is **greater** than this the superior beat is more distinct.

EXAMPLES.—Suppose two forks of vibration-frequencies 100 and 512, respectively, to be sounded together. Since 100 goes into 512 five times, with a positive remainder of 12, there will be produced 12 beats per second, belonging to the inferior set. But since also 100 goes into 512 six times, with a negative remainder of 88, i.e., six times 100 minus 88 is equal to 512, another set of beats will be produced, this time the superior set, having 88 beats per second. As the number of beats in the inferior set, 12, is less than half of 100, the frequency of the lower primary, the inferior set of beats in this case will be heard more distinctly. If the second fork be raised to 588 vibrations, the superior beat will be 12 per second and the inferior beat 88; so that in this case the superior beat will be very strong and the inferior beat almost inaudible.

Evidently the inferior beat, when produced between two notes in the same octave, corresponds simply to the difference of the vibration-frequencies of these two notes. If, for instance, the fork C_1 of 64 vibrations be sounded with D_1 of 72, the positive remainder 8, which determines

the inferior beat, is also the number which represents the difference of frequency.

This fact that the number of beats per second corresponds to the difference in the frequency of vibration between two notes has been utilized in determining pitch. Scheibler's tonometer consists of a series of tuning-forks, each of which is carefully tuned to give four vibrations a second more than the fork next below it, so that when sounded together they produce four beats per second. To ascertain the pitch of any given sound by means of this tonometer, it is only necessary to find, first, between what two forks in the series the sound comes, and second, the number of beats which it makes with each of these forks.

As the frequency of the beats increases, a point is finally reached where they cease to be recognized as distinct sounds and blend into a more or less pure tone. This tone was first observed by Sorge (1745), and it is known as the **grave harmonic** of Tartini. Since its pitch corresponds to the difference of frequency of the two tones, Young called it a **difference-tone.** To this, von Helmholtz added another resultant tone whose pitch is the sum of the two frequencies, and which he called a **summational tone.** But while maintaining that these combinational tones play a very important part in determining the harmonious character of chords, the summational tones of the primaries beating with their upper partial tones and so making the interval more or less harmonious, he yet denies that beats can blend so as to form a true tone. The researches of Koenig appear to have settled these matters conclusively. In the first place, he finds that when pure tones are used, no tone is heard under any circumstances the frequency of which is the sum of the frequencies of the two component tones. And in the second place, he has apparently proved that **beats do coalesce to produce beat-tones.**

EXAMPLES.—Thus Koenig sounds the forks C_6 giving 2048 vibrations, and D_6 giving 2304, and obtains the inferior beat corresponding to 256 vibrations; these beats blending perfectly and giving the

clear note C_3 corresponding to this frequency. If the note B_6 of 3840 vibrations be combined with C_6 of 2048, the superior beat is now 256, and precisely the same note as before is obtained. But in this case the beat-tone is neither a differential nor a summational tone, and yet it corresponds to the calculated vibration-frequency.

Koenig's investigations seem then to have established three facts: 1st, that beat-tones are in fact produced, and that they correspond in pitch to the number of the beats; 2d, that these beat-tones can themselves interfere and produce secondary beats; and 3d, that the same number of beats will always give the same beat-tone whatever be the interval between the two primary tones.

Mayer has made an important contribution to the theory of dissonance by showing that the duration of the sensation of a sound depends upon its pitch, this duration being less the higher the pitch. Thus, for example, he finds that while the duration of the sound C_1 of 64 vibrations is 1/16 of a second, that of C_3 of 256 vibrations is 1/47, that of G_3 of 384 is 1/60, that of E_4 of 640 is 1/90, and that of C_5 of 1024 vibrations is 1/135 of a second. Hence while the note C_1 may be intermitted 16 times in a second without ceasing to appear continuous to the ear, G_3 must be intermitted 60 times per second in order to preserve its continuity. In consequence, if C_1 and G_1, having vibration-frequencies of 64 and 96, respectively, be sounded together, the inferior and superior beats will both be 32 in number; and as this is greater than 16, the blending value, the two sounds are harmonious. On the other hand, if C_3 of a frequency of 256, and D_3 of 288, be sounded together, the inferior beat will be 32, the superior beat 224 per second. The former being below the blending value 47, the resultant tone will be dissonant.

While simple sound-waves can differ only in length and in amplitude, complex sound-waves may also differ in form. It is the view of von Helmholtz not only that "every different quality of tone requires a different form of vibration," but also that "different forms of vibration may correspond to the same quality of tone." In other

words, that "differences in musical quality of tone depend solely on the presence and strength of partial tones, and in no respect on the differences of phase under which these partial tones enter into composition." Koenig, on the other hand, seems to have shown experimentally that differences of phase do produce a distinct effect upon the quality of compound tones; and further, that combinations in which the constituents of the sound vary in their relative intensity and phase from wave to wave, are recognized by the ear as possessing true musical quality.

Section V.—Speaking and Hearing.

252. The Human Voice.—All vocal sounds are produced within a cartilaginous prismatic box placed upon the summit of the trachea, and called the larynx. Its vibrating parts, called vocal membranes, consist of two sharp folds or ridges, which project into the cavity and which are formed of elastic tissue, and are covered with the mucous membrane which lines the air-passages. The fine smooth edges of those vocal membranes nearly meet; so that between them there is a narrow slit called the glottis. In front, the vocal membranes are attached to the thyroid or principal cartilage of the larynx, and behind, to the two arytenoid cartilages which are pyramidal in shape and movable in position. Ordinarily the glottis is a V-shaped opening, through which the air passes during respiration without producing sound. By means of the muscles attached to the arytenoid cartilages, however, the vocal membranes may be made tense, and the glottis narrowed to any desired extent. So that when a sound is to be produced, these membranes, thus stretched, are readily thrown into vibration by the current of air which is sent through the glottis from the lungs. Evidently, the pitch of the note emitted will depend, first upon the length of these membranes, and second upon their tension. Hence in the first place we find them longer in the male than in the female larynx,

and secondly, an examination with the laryngoscope shows them to be tenser when high notes are sung. This condition of things may be roughly imitated by cutting the end of a hard-rubber tube obliquely, stretching two strips of soft sheet-rubber over the edges so as to have a small slit between them, and tying them with a string. The membranous tongues thus formed may be thrown into vibration by the passage of air through them in either direction, and their action resembles closely that of the vocal membranes.

The air-cavities connected with the larynx have a not less important part to play in the production of vocal sounds. These cavities, and particularly that of the mouth, act by their resonance to reinforce the sounds produced by the vocal membranes. Since these sounds are rich in overtones, it follows that the quality of the voice depends mainly upon the shape of these air-cavities and upon the particular overtones which they reinforce. The richness of the voice in harmonics appears from the fact that, with the aid of resonators, it has been possible to recognize harmonics as high as the sixteenth in the notes of a powerful bass voice. Moreover, in singing, notes are always sustained on a vowel-sound; and every vocal sound has in it something of the vowel quality. Now von Helmholtz has shown that the vowels can be arranged in three series according to the shape assumed by the mouth as a resonant cavity in producing them. The vowel *a* as in "father" forms the common origin of the three; and with it are associated *o* as in "more," and *u* as in "sure." The second series consists of *ä* nearly as in "bat," *e* as in "there," and *i* as in "machine." The third includes *ö* like the *eu* in "peu," and *ü* like *u* in "pu." If a musical note be sung and the different vowels be pronounced at the same time, their characteristic quality may be easily determined by means of resonators. Thus, for example, in *U* the fundamental note is strong, and the third harmonic well defined; in *O*, there is, besides the fundamental, a strong second harmonic and weak third and fourth harmonics;

in A the second and fourth harmonics are feeble, the third is strong; in E, the fundamental note, the third and the fourth harmonics are feeble, the second and the fourth harmonics are strong, the latter the most so; and in I the high harmonics, especially the fifth, are strongly marked. These results are complicated by the fact that the vowel-sounds are dependent, not only upon the quality of voice which sounds them, but also upon the pitch of the note taken as the fundamental, and upon the language employed.

Two forms of apparatus for producing the vowel-sounds synthetically have been contrived by von Helmholtz. A set of tuning-forks in one of those, and a set of stopped organ-pipes in the other, each carefully tuned to give one of the required harmonics, are so associated together that any note or assemblage of notes can be obtained by depressing suitable keys. The synthesized vowels are readily recognized, and the result is a striking confirmation of the correctness of the theory. Koenig has made a set of tuning-forks with corresponding resonators, so tuned as to give the vowel-sounds. If the mouth-cavity be adjusted so as to sound any one of the vowels, and the corresponding fork be held near the opening, the sound of that vowel will be heard, the cavity reinforcing by its resonance the sound emitted by the fork.

Speaking differs from singing chiefly in the manner in which the vocal sounds are modified. In the former as well as the latter, sustained sounds are always vowel-sounds. The inflections of the voice in conversation take place in musical intervals. When a question is asked, the voice rises a fourth. When a word is emphasized, it rises a fifth. In ending a statement it falls a fifth. The fundamental modifications of the voice, however, are effected by means of consonants. In some of these, vibration of the vocal membranes plays no part, the effect being produced solely by the cavity of the throat and mouth. Consonants are frequently classified according to the place where the characteristic modification takes place. Thus, labial consonants like B and

P, dental consonants like *D* and *T*, and guttural consonants like *G* (hard) and *K*, are so called because the interruption takes place at the lips, the teeth, and the throat, respectively. Moreover, they may be divided into explosive (like those just given), aspirate, resonant, or vibratory consonants, according to the suddenness or other special character of the motion producing them. In whispering, no vocal-membrane sounds are employed, and therefore the distinctions between consonants requiring such sounds, as *B*, *D*, and *G*, and those not requiring them, such as *P*, *T*, and *K*, are for the most part entirely lost.

253. The Organ of Hearing.—The ear is the organ through which sounds are able to affect our consciousness. Its delicacy, its range, its sensitiveness, are most surprising. Rayleigh heard a whistle at so great a distance that, as he calculated it, the amplitude of the sound-wave was only $8{\cdot}1 \times 10^{-8}$ (or 0·000000081) centimeter. We have seen that the limits of audibility range from 16 to 40000 vibrations per second. And highly trained musical ears are said to be able to distinguish seven hundred sounds in a single octave.

The external ear consists of the auricle, serving probably only to collect the sound, and the auditory canal by which it is transmitted, partly direct, partly by reflection, to a membrane which separates the auditory canal from the drum of the ear. The ear-drum is a cavity closed on the external side by the membrane just mentioned, and on the internal side by two others, called the oval and the round membranes, respectively. Within it is a chain of three small bones, called the malleus, the incus, and the stapes. The malleus is attached to the first-mentioned membrane; and also to the incus, which connects it to the stapes. Since the flat portion of this stirrup-shaped bone is fastened to the membrane closing the oval opening, it is evident that the vibrations received by the outer membrane from the air will be transmitted through these bones to the membrane of this oval *opening*. Beyond the ear-

drum is the inner ear, enclosed entirely in bone. It consists of three portions termed, respectively, the vestibule, the cochlea, and the semicircular canals. The round and the oval membranes separate the ear-drum from the vestibule. The inner ear is filled with liquid which serves to distribute the vibrations which are received through the oval window. The cochlea is a canal, in the shape of a conical helix like a snail-shell, divided lengthwise into three cavities, two of which communicate at their extremities, while the third is closed. The partition separating the two former cavities from each other is bony, while that separating each of these from the third is membranous. This basilar membrane is triangular in form, and is capable of vibrating in parts through a considerable range. Upon it rest two series of fibers known as Corti's rods or arches, wound round and covered with hair-cells and fibers, which are the terminations of the auditory nerve. Each nerve-fiber is sensitive only to the vibrations of the arch with which it is connected; i.e., to those of that part of the basilar membrane on which this arch rests. Since the rods of Corti number ten thousand or more, and the hair-cells are even more numerous, it is clear that all the various rates of vibration within the limits of audibility may by their means be separately detected and transmitted to the brain.

The drum-membrane is slightly concave externally, and is kept stretched by a muscle which acts on the malleus. It has no vibration-rate of its own, and can therefore readily take up from the air vibrations of various periods. These vibrations are transmitted by the ossicles to the liquid contained in the middle ear and through it to the basilar membrane, certain part or parts of which are thus thrown into vibration, according to the pitch of the sound; the organ of Corti, with its rods, hair-cells, and hairs, serving to convert these vibrations into excitations of the nerve-endings. The radial dimensions of the basilar membrane have a range from about five tenths to about four hundredths of a millime-

ter; and hence being tense radially but loose longitudinally, may act as a series of radial strings, each capable of vibrating at a given rate and therefore thrown sympathetically into vibration by a note of the same pitch. This explains the remarkable power of analysis possessed by the ear. Whenever a composite sound is heard, the ear is able, within certain limits, to resolve this sound into its constituent vibrations. The compound wave throws different portions of the basilar membrane into simultaneous vibration, corresponding to the different component simple vibrations. And these nerve-impulses acting together, produce the same impression collectively that is produced successively when the components are sounded one at a time.

Intensity of sound also is appreciated by the cochlea as well as pitch and quality. It is possible, however, that the vestibule has some function to perform in this direction. The semicircular canals have nothing to do with hearing, as it is believed, but are concerned only in the preservation of bodily equilibrium.

Section VI.—Optical Representation of Sounds.

254. Visible Sound-ratios.—Various methods have been proposed for representing to the eye the phenomena of sound-vibration. If a vibrating tuning-fork, for example, having a pointed strip of thin metal attached to one of its prongs, be drawn across a plate of smoked glass, it will trace a nearly simple harmonic curve. If the smoked glass be a disk, and the speed of its rotation be noted, the vibration-frequency of the fork may readily be determined in this way. Conversely, a fork of known rate may be used to measure the speed of rotation of the disk, and may act therefore as a chronograph. If two forks be employed, one of which carries the glass and the other the style, the trace is the result of their mutual action; and by suitably varying the relative vibration-rate of the two forks, their interference-curves may be

beautifully shown in this manner both when their vibrations are parallel to and when they are perpendicular to each other The glass should not be smoked too heavily.

A second optical method is that of Koenig, known as the manometric flame method. A hollow wooden cylinder, divided transversely in the middle, has its two cavities separated by a thin elastic membrane. Through one of its ends a tube passes for the conveyance of the sound; through the other a smaller tube from a gas supply passes, and also a fine jet. On lighting the gas at the jet, a luminous flame an inch or more in height is produced. If sound-waves be made to enter the cylinder, the membrane, and consequently the gas-column, is thrown into vibration, causing a vertical oscillation of the jet. If the image of this be viewed in a revolving mirror, it will be drawn out into a series of serrations characteristic of the sound employed.

A third method, proposed by Lissajous, depends upon the optical combination of two harmonic vibrations, on the principles already explained (59). If a tuning-fork be provided with a mirror, from which a beam of sunlight is reflected to a screen, then on vibrating the fork the spot of light will be drawn out into a line. If a second similar fork be interposed, so as to receive upon its mirror the beam reflected from the first fork, and so placed that its plane of vibration is perpendicular to that of the first fork, then on causing both forks to vibrate the figure on the screen will be a Lissajous curve, characteristic of the relative period of the forks. If their vibration-ratio be unison and their amplitudes equal, the curve will be a circle, an ellipse, or a straight line inclined 45° to the original directions, according to the difference of phase between them. If the ratio be 1 : 2 or an octave, the curve will be the lemniscate or figure of 8; and so on. If the ratio be exact, the figure at first produced is permanent, except that it decreases as the amplitude of vibration diminishes. But if the tuning is not exact, one fork gradually gains on the other, and the figure passes gradually through its complete cycle of changes

in the period which is required by one fork to gain a complete vibration on the other.

255. The Phonautograph and the Phonograph.—For the purpose of securing a record of the air-vibrations produced by sound, Leon Scott contrived an apparatus called a phonautograph, by means of which the sound is made to write its own curve. This apparatus consists of two distinct parts: 1st, of a hollow paraboloid, cut away at its apex so that a membrane stretched across this open end is in its focus. To the center of this membrane is attached a style made of a bristle; the vibration of the point of attachment being regulated by means of a suitably placed damping-screw. 2d, of a brass cylinder rotating spirally on a horizontal axle upon which a screw-thread is cut; and upon which is secured a sheet of smoked paper. The apparatus is adjusted so that the style is just in contact with the smoked surface, and so that the motion of the membrane causes the end of this style to vibrate with its maximum amplitude. If no sound is produced, rotation of the cylinder causes the style to trace a spiral line on the smoked surface. But when a sound is sent into the mouth of the paraboloid the vibrations of the membrane and of its attached style cause a curve to be traced, the form of which corresponds to that which characterizes the particular sound employed. By the use of this apparatus the character of any simple or composite tone may be conveniently studied in relation to its quality as well as to its pitch and amplitude.

It occurred to Edison in 1877 that by using a grooved cylinder covered with a sheet of tinfoil, and by placing perpendicular to its radius a thin metal diaphragm provided with a steel needle-point as a style, it would be possible to indent the tinfoil as the membrane vibrates, and so to produce the wave-curves corresponding to the given sound in a plane normal to the cylindrical surface. And further, that by causing the style to move again over the wave-surface thus traced, it would throw the diaphragm into corresponding vibration, and

thus reproduce sound-waves in the air analogous to those which originally produced the record. Subsequently he greatly improved the **phonograph**, as he called the instrument, using a composition of wax for the receiving-cylinder, rotating it by an electric motor, and providing a much better form of style; so that in the new form it has become a commercial instrument, and is used in place of stenography, the correspondence being dictated to the instrument and then reproduced by means of a typewriter. It is stated that as many as 40000 words can be recorded on a space not greater than 65 square centimeters. A modification of the phonograph has been invented by Tainter and Bell, and called the **graphophone**. Both of these instruments have proved of great use in studying the peculiarities of spoken sounds.

256. Acoustic Attraction and Repulsion.—It has been long known that a body in a state of vibration exerts an attractive or repulsive action on light bodies in its vicinity. Dvorák in 1875 vibrated a long wooden rod, and found that a small square of paper or a pith-ball suspended near it by a filament of silk was attracted at certain points and at certain other points repelled; this effect being due to the currents of air flowing toward or from the vibrating body. Similarly a vibrating tuning-fork will attract a piece of suspended card-board. Dvorák showed that near the closed end of a resonant cylinder, which is a node, the pressure is greater than the atmospheric pressure when the air within it is vibrating. Hence a balloon filled with hydrogen is repelled from the mouth of a resonance-box on which is a vibrating tuning-fork; while a balloon filled with carbon dioxide is attracted. Placing an ordinary brass Helmholtz resonator (the smaller opening in which is closed) upon the end of a light rod provided with a glass cap, resting on a needle-point, and counterpoising it by a piece of lead, Dvorák observed a strong repulsion when the mouth of the resonator was placed opposite the open end of the resonant

box of a tuning-fork tuned in unison with it. Mayer placed four small glass resonators on cross-arms suspended by a thread, and found that when an organ-pipe in tune with the resonators was sounded opposite the mouth of one of them, the repulsion produced caused rotation of the apparatus. A remarkable series of experiments was devised by Bjerknes in 1880 for exhibiting similar phenomena in liquids; these experiments being exhibited at the Paris Electrical Exhibition in 1881, as showing striking analogies with electrical and magnetic attraction and repulsion.

257. Energy of Sound-vibration.—The energy of a vibrating body which emits sound, like the energy of an oscillating pendulum, is alternately all kinetic and all potential (104). For an elastic body vibrating in accordance with Hooke's law, the work done in displacing the system, and therefore the energy stored up in it when displaced, is the product of the mean force by the displacement. To determine this mean force, let O

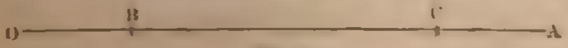


FIG. 98.

(Fig. 98) be the position of equilibrium, and B and C points equally distant from O and A. Let μ be the force required to produce unit displacement, then $\mu . OA$ will be the force developed when the body is at A, and $\mu . OB$, $\mu . OC$ the force developed at B and C, respectively. Let λ be an element of the path at any point, then the work done in moving the body over this distance at B will be $\mu . OB . \lambda$; and at C, $\mu . OC . \lambda$; the sum being $\mu . (OB + OC)\lambda$ or $\mu . OA . \lambda$. Now this is evidently the same result as if the force had had the constant mean value $\frac{1}{2}\mu . OA$; since in that case the work done in each of the two elements would have been $\frac{1}{2}\mu . OA . \lambda$, and in both $\mu . OA . \lambda$, as before. For all the elements of the line, $\Sigma\lambda = OA$; and hence the whole energy in the displaced body at A will be $\mu . OA . OA$ or $\mu . (OA)^2$; *which is proportional to* the square of the

amplitude. But when a body is thus vibrating, its total energy is constant, being the sum of its kinetic and potential energies at any instant; and this of course is equal to the potential energy at the extreme point of its swing, where its kinetic energy is zero; i.e., is equal to $\frac{1}{2}\mu \,.\, a^2$, where a is the amplitude. In the case of a Koenig tuning-fork, Wead has shown experimentally that the value of $\frac{1}{2}\mu \,.\, a^2$, the potential energy when the fork is bent a centimeters, is $(Ebd^3a^2)/2l^3$ ergs; in which l is the length of the prong, b its breadth, and d its thickness, and E is Young's modulus. For Koenig's forks $b = 1{\cdot}4$ and $d = 0{\cdot}65$ cm., and $E = 2{\cdot}14 \times 10^{12}$ C.G.S. units. Substituting these values, the value of the energy becomes $(1{\cdot}03 \times 10^{11} \,.\, a^2)/l^3$ ergs; or for the fork Ut_2, giving 128 double vibrations per second, the energy value is $\frac{1}{2} \times 23{\cdot}94 \times 10^6 \,.\, a^2$ ergs for one prong. If the fork give the h harmonic of Ut_2, the energy for both prongs will be $23{\cdot}94 \times 10^6 \,.\, h^{\frac{3}{2}} \,.\, a^2$ ergs. In the case of a string vibrating transversely, it may be shown that the mean value of the square of the speed of vibration is $\frac{1}{2}(4\pi a/T)^2$, in which a is the half amplitude at a loop and T is the period. Hence the energy or $\frac{1}{2}ms^2$ is $m \,.\, (2\pi a/T)^2$. For each of the two moving waves composing this stationary undulation, and moving in opposite directions, the energy is $m \,.\, 2(\pi a/T)^2$; or one half the energy of the stationary wave. The same reasoning is applicable to longitudinal vibrations. In the case of the air in an organ-pipe, for example, the energy of the sonorous vibration is equal to the mass of the air multiplied by $(2\pi a/T)^2$, where $2a$ is the amplitude at a loop.

PART THIRD.

MOLECULAR PHYSICS.

CHAPTER I.

MOLECULAR KINETIC ENERGY.—HEAT.

Section I.—Nature of Heat.

258. Definition of Heat.—Heat may be defined either as a sensation or as the objective cause of a sensation. In physics the term is generally used in the latter sense to indicate that special condition of matter in virtue of which it can affect our nerves of general sensation.

259. Historical.—During the last half-century a radical change has taken place in scientific opinion concerning the nature of heat. Under the name of caloric, it had been regarded as a species of matter, although of an imponderable kind. In 1798 and 1799, two important experiments were made; the one by Count Rumford, the other by Sir Humphry Davy. The Rumford experiment, which was made in the Munich Arsenal, consisted in boring a cannon by means of a blunt drill, and in proving that the heat generated was practically inexhaustible. The Davy experiment, which was made in the Royal Institution in London, consisted in melting two pieces of ice by rubbing them together at a temperature below the melting-point. From his experiment Rumford drew the legitimate conclusion that "anything which any insulated body or system of bodies can continue to furnish without limitation cannot possibly be a material substance." Since, when a hotter body is placed in contact with a colder one, there is a positive transfer of heat from the one to the other, it is obvious

that heat must be an actually existing quantity capable of exact measurement. In the above example, however, heat was not only produced by friction, but it was produced indefinitely. The mechanical work which was expended disappeared as such, and heat appeared in its place. In consequence this heat can be considered only as the increased energy of a system upon which work has been expended. It follows therefore that heat must itself be energy, either in the kinetic or in the potential form. Moreover, Rumford went still further and succeeded in obtaining a quantitative relation between the work done and the heat produced. He found that the work of a single horse for two hours and twenty minutes was capable of raising to the boiling-point nearly nine kilograms of water; of course in addition to the rise in temperature produced in the metal which was immersed in it. It was not, however, until about the year 1849, when Joule made his elaborate and conclusive experiments on the relation of mechanical work to heat, that the material theory of heat was finally overthrown.

EXAMPLES.—Since energy is indestructible, its disappearance in one form must be accompanied by its reappearance in another. The most common form into which other forms of energy are converted is heat. Indeed, probably no conversion of energy from one form into another takes place without the production of more or less heat, and hence heat is sometimes called the lowest form of energy. When a hammer strikes the anvil, or a rifle-ball the target, the kinetic energy in both is apparently destroyed. But if the experiment be made with care, it will be found that this energy has simply been transformed, the impinging masses being now warmer than before. The same is true in the case of friction, which from this standpoint is only the gradual arrest of motion. In short, whenever mechanical energy is expended, an equivalent heat-energy appears.

260. Heat the Energy of Molecular Motion.—Is heat energy in the kinetic or in the potential form? Davy said in 1812: "The immediate cause of the phenomenon of heat, then, is motion, and the laws of its communication are precisely the same as the laws of the communication of motion." This in modern language

is equivalent to the statement that heat is kinetic energy; not evidently of the mass, since the hot body may be at rest; but of the molecules. We know that one of the ways in which a hot body cools is by transferring its energy to another and a colder body not in contact with it; and we shall study later the mechanism of this radiating process. One thing about it is certain, however, and that is that it consists in a motion of the intervening medium. The hot body communicates motion to the medium, and the cold body receives motion from this medium. We conclude therefore that the surface of a hot body must be in motion; and because radiation may take place as well from the interior of a body as from its exterior, we also conclude that the body must be in motion throughout its entire mass. This view of the case is in entire accord with the kinetic theory of matter already discussed (185), which supposes the molecules of matter to be actively in motion. The motion to which heat-energy is due "must therefore be a motion of parts too small to be observed separately; the motions of different parts at the same instant must be in different directions; and the motion of any one part must, at least in solid bodies, be such that however fast it moves it never reaches a sensible distance from the point from which it started." (Maxwell.)

261. Temperature.—By the temperature of a body is meant simply its thermal condition, considered with reference to its capability of communicating heat to other bodies. If two bodies be placed in contact, one of three results must follow: (*a*) the first body must receive more heat from the second than it gives to it; (*b*) it must give more heat than it receives; or (*c*) the amount given and received must be equal. In the first case, the temperature of the second body is said to be higher than that of the first; in the second case, the first body has the higher temperature; and in the third case, their temperatures are equal. In accordance with the kinetic theory of heat, however, it is evident that temperature has to do simply with the speed of the

molecular motion existing in a body. When heat is transferred to a body, its temperature is raised; i.e., the speed and therefore the energy of its molecular motion is increased. Conversely, as heat is withdrawn from a body its temperature falls, the speed of its molecular motion decreases, and its kinetic molecular energy is diminished. But evidently a point must finally be reached at which this kinetic energy becomes zero and the molecules are at rest. This point is called the **absolute zero of temperature.**

262. Amount of Heat concerned in producing Temperature-changes in a Body.—The amount of heat necessary to raise the temperature of a body is proportional: (1) to the number of molecules which it contains; and (2) to the increase of the kinetic energy of a single molecule. Hence it is measured by the product of the temperature-increase by the mass of the body. In order to raise a doubled mass through the same temperature-range, therefore, a doubled amount of heat is required; and to raise the same mass through a doubled temperature-range a doubled amount of heat is also required, provided of course that the same substance be employed under the same conditions. Conversely, the amount of heat which a body loses in cooling, is also proportional to the product of its mass by its change in temperature. The total amount of heat contained in a body we have no means of knowing. We can measure heat only during its transference or transformation.

Section II.—Measurement of Heat.

A.—THERMOMETRY.

263. Measurement of Temperature.—Temperature cannot be measured directly. It can be measured only by measuring some one of the effects produced by heat, which, other things being equal, is proportional to the temperature. Since substances expand when heated, expansion is such an effect. And as expansion and

temperature go hand in hand, the one may be employed to measure the other. An instrument in which expansion is made use of for determining temperatures is called a **thermometer**; or a **pyrometer**, if the temperatures are high.

Air, mercury, and alcohol are the thermometric substances ordinarily made use of. The air-thermometer has great range and great accuracy and is generally used as a standard. The alcohol-thermometer is used only for very low temperatures. The mercurial thermometer is the most convenient form of instrument, and is therefore commonly employed. It consists of a capillary glass tube called the stem, on the lower end of which a spherical or cylindrical bulb is blown, of such a size that the expansion of the mercury it contains, between the limits within which the thermometer is to be used, exactly fills the stem. When thus filled with mercury the upper end of the stem is sealed, a small enlargement of the bore being made at this point. In order that the indications of thermometers thus made may be comparable with one another, two fixed points of temperature are marked upon their stems. These fixed points are the melting point of ice and the boiling point of water; which, under proper conditions, represent the same temperatures everywhere. For this purpose the thermometer is first surrounded with melting ice and the point at which the mercury stands is marked upon its stem. It is then immersed in the vapor of boiling water and the height of the mercury is again noted. The space between these two marks is divided into one hundred equal parts, and the divisions are continued of the same size above the upper and below the lower point so as to include the whole stem. In thermometers for scientific use, the graduations are etched upon the glass. The lower point is called zero and the upper, of course, one hundred; meaning degrees of temperature. Such a thermometric scale is called the **centigrade** scale. Where great accuracy is required, the stem should be carefully calibrated; i.e., the relative

values of the divisions on the stem should be determined and tabulated. Moreover, the readings of such a thermometer should be compared either with those of a standard instrument, or better, with those of the air-thermometer directly.

Thermometers are said to be **accurate** when they indicate the true temperature. This end is secured by careful construction and comparison. They are said to be **delicate** when they indicate very small differences of temperature; say from the tenth to the one hundredth of a degree. They are called **sensitive** when their indications are prompt. Delicacy in a thermometer depends upon the ratio of the volume of the bulb to the size of the tube. If the bulb be large and the tube small, a minute change of temperature will cause the column to rise through an appreciable distance. Sensitiveness depends upon the surface-area of the bulb. In case the bulb is spherical, the sensitiveness is a minimum, since, for the same volume, the surface of the sphere is a minimum. In general, therefore, thermometer-bulbs are made cylindrical; the cylinder in the case of meteorological instruments being very long and either coiled into a spiral or bent into a zigzag.

The zero point of thermometers is liable to change, owing to the slow contraction of the glass forming the bulb. If, after the boiling point has been fixed, the thermometer be again placed in melting ice, the reading may be too low by a tenth of a degree or more. In the course of a week or two, this effect disappears. There is a second one, however, similar in character, but much slower in disappearing. If the thermometer be graduated soon after it is filled, a continuous displacement of the zero point goes on, sometimes for years; so that the reading in melting ice may be too high eventually by as much as an entire degree. Hence, thermometers intended to be accurate should not be graduated until some months or even years after they have been filled.

264. Registering Thermometers.—By placing the stem of a thermometer behind an illuminated slit, so that the rise and fall of the column varies the length of the slit through which the light passes, and then by allowing this varying line of light to fall on a strip of photographic paper moving perpendicularly to it, the variations of the thermometric height may be continuously registered. If a mercury-thermometer have its stem horizontal, and a bit of steel wire be placed in front of the column, the wire will be pushed forward by the expansion, and will indicate by its position the maximum temperature reached. A minimum thermometer is similar except that it is made with alcohol and has a bit of glass rod within the liquid, which is drawn backward as this liquid contracts and is left in the minimum position.

B.—CALORIMETRY.

265. Measurement of Heat.—The process of measuring heat is called **calorimetry,** and the instruments used for the purpose are called **calorimeters.** As we have just seen, heat can be measured only during its transfer from one body to another; the amount of heat required to raise the temperature of a given body through a given number of degrees being proportional to the conjoint product of its mass and of the temperature-change. On comparing different substances, however, we find that they are very differently heated or cooled by the same increment or decrement of heat even when their masses are the same. That amount of heat which will raise the temperature of one gram of water one degree will raise that of the same mass of ice two degrees, of iron ten degrees, of silver twenty degrees, and of mercury thirty degrees.

266. Unit of Heat.—It is therefore necessary to agree upon a particular substance, such that the amount of heat required to raise unit mass of it through unit temperature shall be called unity. Such a substance is

water. It is readily obtained pure, and it requires a greater amount of heat than any other substance to produce in it a given temperature-change. In defining the unit of heat, however, the limits of temperature must be specified; since the amount of heat required to raise the temperature of unit mass of water one degree increases with the temperature. **A unit of heat** is therefore defined as the amount of heat required to raise the temperature of unit mass of water one degree between 0° and 4°. If the gram be taken as the unit of mass, the quantity of heat required to raise one gram of water one degree in temperature, between the limits 0° and 4°, is the special unit of heat called a **small calory** or a **therm.** It is a water-gram-degree.

267. Capacity for Heat.—Specific Heat.—The term "capacity for heat" or "thermal capacity" expresses the same idea with reference to a body that "specific heat" does with regard to a substance. The thermal capacity of a **body** is simply the amount of heat, expressed in units of heat, which is required to raise the temperature of that body one degree. Thus if it takes half a unit of heat to raise the temperature of the given body one degree, its capacity for heat is said to be 0·5. When we speak of the heat-change of **a substance**, however, then we must limit its mass. The specific heat of a **substance** is defined as that fraction of a unit of heat which is required to raise the temperature of unit mass of that substance one degree. Evidently, therefore, the specific heat of a substance is simply the thermal capacity of unit mass of that substance. We should also distinguish between the mean or average specific heat of a substance and its actual specific heat. The former is the average value between two given temperatures; and is the quotient of the number of units of heat required to raise unit mass of the substance from the lower to the higher temperature, divided by the number of degrees representing the temperature-change. The latter, which is the actual value for a particular temperature, is the limiting value of the mean specific heat as

the range of temperature is indefinitely reduced. Since specific heat varies with temperature, the mean and the actual thermal capacities per unit of mass have not in general the same value.

The total quantity of heat which is concerned in a temperature-change, therefore, is a function not only of the mass and of the extent of the temperature-change, but also of the particular substance employed in the experiment. Hence if we represent the mass by m, the temperature-change by t, and the mean specific heat by c, the amount of heat required to change the temperature of m units of mass of a substance whose specific heat is c, through t degrees, is mct units of heat.

268. Calorimetrical Methods.—Three general methods of calorimetry are made use of in practice. These are known as (1) the method of cooling, (2) the method of melting ice, and (3) the method of mixtures.

I. **The method of cooling** depends upon the fact that the rate at which a hot body loses heat by radiation is a function of its temperature and of the nature of its surface only. If, for example, a thin copper vessel be filled to the same point in successive experiments with different liquids, all at the same temperature (either above or below that of surrounding objects), the quantity of heat passing to or from this vessel during the same interval of time will be precisely the same in all the experiments. But since the amount of heat thus gained or lost per unit of time is the same for all at the same temperature, it is evident that those liquids which have the highest specific heat, i.e., those which give up or take in most heat for a given temperature-change, will cool or heat least rapidly. In practice the copper vessel is suspended by silk threads in a second and larger one which is immersed in water and is kept at a constant temperature. As a preliminary experiment the inner vessel is filled with hot water, say at 60° or 70°. Suppose that it holds 100 grams. A thermometer is placed in the water, and for each degree that it falls in temperature, 100 units of *heat (water-gram-degrees)* have of course

been lost by the water. Note the time required for each degree of temperature-fall. The inner vessel is now filled to the same level with the liquid whose specific heat we desire, already heated to the same temperature as the water in the last experiment. It is then allowed to cool in the same way, being constantly stirred; and the successive intervals of time required for it to cool through each successive degree of temperature are accurately noted. If T and T' be these time-intervals for equivalent degrees in the case of the water and of the liquid, then $1/T$ and $1/T'$ will be the fall of temperature per unit of time; i.e., the rate at which heat escapes in the two cases. But this rate of cooling at the same temperature is constant for both liquids under these conditions; and hence $1/T = 1/T'$. If v be the volume of liquid employed and c_v its specific heat per unit of volume, the heat-change corresponding to this fall of 1° in temperature is vc_v; and hence since the volume was the same in the two experiments, and c in the case of water is unity, $1/T = c_v/T'$, or $c_v = T'/T$; i.e., the specific heat per unit volume is proportional to the times of cooling. Since $vc_v = mc_m$, both representing the thermal capacity of the given body, $c_m = vc_v/m$ or $c_m = c_v/\delta$. Whence the specific heat per unit mass is obtained by dividing the specific heat per unit volume by the density. Evidently it may also be determined directly.

EXAMPLE.—Let the liquid be alcohol of density 0.8, and suppose that the 100 cubic centimeters of it cool through 5° in 2·4 minutes, the same volume of water requiring five minutes to cool 5°. The total heat lost by the water per degree of cooling is the product of its volume by its specific heat, or vc_v: that lost by the alcohol being $v'c_v'$. Since these heat-losses are proportional to the times of cooling, the temperature-change being the same, we have $vc_v : v'c_v' :: 5 : 2·4$. But the volumes are equal and c_v is unity; whence $c_v' = 2·4/5$ or 0·48, the thermal capacity per unit volume of alcohol. Dividing this by the density 0·8, we have 0·6 for the thermal capacity per unit mass. Again, since the mass of the 100 c.c. of alcohol is 80 grams, we have by the above reasoning $mc_m : m'c_m' :: 5 : 2·4$; whence $400c_m' = 240$ and $c_m' = 0·6$, the thermal capacity of unit mass, or the specific heat of the alcohol, as before.

Certain corrections are necessary, as will be seen presently. The method is readily applicable to liquids and to solids which can be placed in liquids for the experiment. The liquid should be kept well agitated and should be protected from evaporation.

II. **The method of melting ice** was proposed by Wilke. In principle it depends on the fact that 80·025 heat-units are required to convert one unit of mass of ice at 0° into water at 0° (Bunsen). If, therefore, a hot body be made to give up its heat to ice at its melting point, and the amount of ice melted be measured, the heat lost by the body may be ascertained. In his original experiments, Wilke used a block of ice hollowed out to receive the hot body, and closed with an ice lid. The body was heated to a known temperature, placed in the cavity and allowed to remain until its temperature fell to zero. By a tared sponge the mass of the water melted was found; suppose it to be m grams. If M be the mass of the body, c_m its thermal capacity per unit mass, and t the temperature-change—it having been cooled from t to zero—then the heat lost by the body is Mc_mt; and this has melted m grams of ice, each requiring 80·025 heat-units for this purpose. But since the gain and loss must be equal, we have $Mc_mt = 80{\cdot}025m$. Hence $c_m = 80{\cdot}025m/Mt$, the specific heat per unit mass.

Since it is not easy always to obtain a single block of ice suitable for this method, Lavoisier and Laplace devised a modified form of ice-calorimeter, with which they obtained good results. It consists of an outer cylindrical vessel containing ice in order to protect the calorimeter proper from the access of heat from without. The calorimeter itself is also a cylinder filled with ice, within which is a smaller vessel to contain the substance used. This substance is heated to a known temperature, and placed in the calorimeter. The ice melted by the heat which it gives out in cooling is determined by weighing the water drawn off from the calorimeter by a special tap. The calculation is made in the same way as in the last case.

A valuable modification of the ice-calorimeter was made by Bunsen in 1871. The instrument is of moderate size and is made entirely of glass (Fig. 99). A tube *a* open at top serves to receive the material to be experimented upon. This tube is enclosed by a second and larger tube *b* entirely filled below with mercury and above with recently boiled water, and provided with a lateral tube *c* coming up from the bottom and terminating in a capillary tube used for the purpose of measuring, by the movement of the mercury column within it, the change of volume which takes place in the instrument. After the water in the calorimeter has been frozen and the entire instrument has been brought to zero by immersion in melting ice, the heated substance is dropped into the open tube, which of course contains water at 0°. The heat given up by the substance in cooling melts some of the surrounding ice in the body of the instrument; and since water occupies less volume than the ice, the contraction becomes evident by the motion of the mercury in the capillary tube. By proper calibration, the actual diminution of volume thus resulting is determined, and thence the amount of ice melted. More recently the capillary tube *d*, instead of being horizontal, is vertical as shown in the figure. The instrument is brought to zero with the end of this capillary tube dipping beneath mercury in a small dish, and the dish is weighed and replaced. After the substance has been dropped in, and the whole is again in equilibrium, the dish is again weighed. The difference divided by the density of mercury represents the contraction in volume. With this apparatus, Bunsen has determined the specific heats of many of the rarer elements.

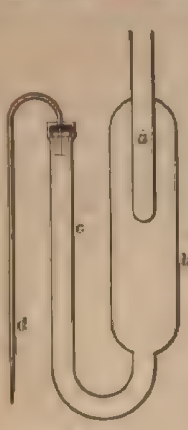


FIG. 99

111. **The method of mixtures** is perhaps more employed

than any other for measuring the amount of heat concerned in temperature-changes. Its principle is, broadly, that when two or more bodies are placed in contact or are mixed together, an exchange of heat goes on between them which results in the production of a final common temperature, depending alone upon the temperatures, masses, and specific heats of the bodies employed. If, for example, 500 grams of water at 100° be mixed with 500 grams of water at 0°, there will evidently be 1000 grams of water at 50°. If, however, 20 grams at 50° be mixed with 100 grams at 10°, calculation shows that the resulting 120 grams of water will have a temperature of nearly 17°. And further, if 100 grams of water at 0° be mixed with 100 grams of mercury at 100°, there will result 200 grams of the mixture at a temperature of 3.2°. In general, if m grams of one substance, of specific heat c, be mixed with m' grams of another substance, of specific heat c', then mc and $m'c'$ will represent the thermal capacities of the two; i.e., the number of heat-units required to raise these masses one degree. As this product represents also the number of grams of water which would be heated one degree by this number of heat-units, it is often called the **water-equivalent** of the mass in question. Suppose now the first mass be heated to $T°$, and be then mixed with the second mass at $t°$. The former will be cooled and the latter will be heated, till some common temperature, θ say, is reached. The hot body will have been cooled through $(T - \theta)$ degrees, the cold body will have been heated through $(\theta - t)$ degrees; and the total heat-change will be in the case of the former $mc(T - \theta)$ units, and in the case of the latter $m'c'(\theta - t)$ units. But since the loss on the one side is obviously equal to the gain on the other, provided, of course, that no heat has entered the apparatus or has escaped from it, we may equate these values; and we have

$$mc(T - \theta) = m'c'(\theta - t); \text{ whence } \theta = \frac{mcT + m'c't}{mc + m'c'}, \quad [40]$$

by which formula the resulting temperature may always be calculated. Evidently c and c' represent the mean specific heats of the substances between the limits of temperature stated.

To employ the method of mixtures for the measurement of specific heats, a known mass of the substance whose specific heat is required is heated to a known temperature—say to 100°, in the vapor of boiling water—and is then placed in a known mass of water, say at 10°. The water is contained in a suitable calorimeter—a thin copper vessel, preferably polished exteriorly, suspended by non-conducting supports in a second vessel polished on its interior surface. The temperature of the water is equalized by means of a stirrer and is noted by a thermometer as soon as it ceases to rise. From the equation just given we get $c = \frac{m'c'(\theta - t)}{m(T - \theta)}$, which is true whatever the substances employed. In the above case, water is used and c' is unity. The specific heat of the substance is obtained therefore simply by dividing the product of the water-mass into its temperature-change, by the product of the substance-mass into its temperature-change.

A portion of the heat lost by the hot body, however, has been expended in raising the temperature of the containing vessel, of the thermometer, and of the stirrer; and this must be allowed for. If m'' be the mass and c'' the specific heat of the vessel, m''' and c''' the same quantities for the thermometer taken as a whole, and m'''' and c'''' the mass and specific heat of the stirrer, then as the total heat lost is divided among these bodies, it must evidently be the sum of the losses to each; and hence we have

$$mc(T - \theta) = (m'c' + m''c'' + m'''c''' + m''''c'''' + \text{etc})(\theta - t). \qquad [41]$$

The sum in brackets is simply the total water-equivalent of the calorimeter. In case the substance experimented on is a liquid, so that a containing vessel is made use

of, the first member of the equation will require an additional term to represent the heat lost by this vessel.

C.—SPECIFIC HEATS.

269. Table of Specific Heats.—By some one of the methods now described the mean specific heats given in the following table have been obtained. The values represent the fractions of a heat-unit required to raise the temperature of unit mass of the substance through one degree between the limits of temperature given in the second column.

1. SPECIFIC HEATS OF SOLIDS.

Substance.	Temperature.	Specific Heat.
Aluminum	15° to 97°	0·2122
Bromine	− 78° to − 20°	0·0843
Bismuth	9° to 102°	0·0298
Brass	15° to 98°	0·0858
Carbon (wood-charcoal)	0° to 90°	0·1935
Copper	15° to 100°	0·0933
Ice	0° to − 20°	0·5040
Iron	50°	0·1124
Lead	19° to 48°	0·0315
Mercury	− 78° to − 40°	0·0319
Platinum	0° to 100°	0·0323
Silver	0° to 100°	0·0559
Zinc	0° to 100°	0·0935

2. SPECIFIC HEATS OF LIQUIDS.

Substance.	Temperature.	Specific Heat.
Alcohol	0° to 40°	0·5977
Bromine	13° to 45°	0·1071
Chloroform	0° to 30°	0·2339
Carbon disulphide	0° to 30°	0·2376
Ethyl ether	0° to 30°	0·5379
Lead	350° to 450°	0·0402
Mercury	17° to 48°	0·0335
Turpentine	0° to 40°	0·4322

As an example of the use of these tables, it may be observed that the amount of heat which would raise a mass

of water from 0° to 100° would raise the same mass of iron from 0° to 900°; or to a bright red heat. It should also be noted that the specific heat is less in the solid than in the liquid state, for the same substance. Water for example, has the specific heat 1·000, ice the specific heat 0·5040. Liquid bromine has the specific heat 0·1071, solid bromine 0·0843. The specific heat of liquid lead is 0·0402, and of solid lead 0·0315. That of liquid mercury is 0·0335, and that of solid mercury is 0·0319.

270. Variation of Specific Heat with Temperature.—While in general the mean specific heat of solids as well as of liquids increases with the temperature, the rate of increase is greater with liquids. The following tables show the character of this increase for platinum and for water:

PLATINUM (POUILLET). Temperature Limits.	Mean Specific. Heat.	WATER (REGNAULT). Temperature Limits.	Mean Specific Heat.
0° and 100°	0·0335	0° and 40°	1·0013
0° and 300°	0·0343	0° and 80°	1·0035
0° and 500°	0·0352	0° and 120°	1·0067
0° and 700°	0·0360	0° and 160°	1·0109
0° and 1000°	0·0373	0° and 200°	1·0160
0° and 1200°	0·0382	0° and 230°	1·0204

This relation may be put in another form. According to Regnault, the number of units of heat H required to raise a given mass of water from 0° to t° is represented by the equation

$$H = t + 0{\cdot}00002t^2 + 0{\cdot}0000003t^3. \qquad [42]$$

Hence the mean thermal capacity of this mass of water between 0° and t°, being the quantity of heat divided by the temperature-change, is

$$H/t = 1 + 0{\cdot}00002t + 0{\cdot}0000003t^2.$$

And the actual thermal capacity at $t°$, which is the limiting value of the mean thermal capacity as the temperature-change is made smaller, is

$$D_t H = 1 + 0{\cdot}00004t + 0{\cdot}0000009t^2. \qquad [43]$$

By means of this formula the actual specific heat of water at any temperature may be calculated.

Rowland, however, has shown that, beginning at about 5°, the specific heat of water decreases slowly as the temperature rises, reaching a minimum at about 29° and then slowly increasing again.

271. Use of the Calorimeter in measuring Temperatures.—By transposing the specific heat formula already given we may obtain from it the value of T, thus:

$$T = \frac{m'(\theta - t)}{mc} + \theta. \qquad [44]$$

Suppose a platinum cylinder whose mass is 200 grams to be heated in a furnace and to be placed in a calorimeter containing 1000 grams of water at 13°, and suppose further that the temperature of the water rises to 20°. Assuming 0·0373 for the specific heat of platinum, and substituting numerical values in the above formula, we have 958° as the temperature of the furnace. The calculation may be simplified by taking a mass of metal such that the ratio of m' to mc may be a round number, say 100; i.e., 268·1 grams of platinum.

272. Atomic and Molecular Heats.—In 1819 Dulong and Petit measured the specific heat of thirty of the elements and observed that when these specific heats are multiplied by the atomic masses of these elements, the products, called the **atomic heats**, are very nearly constant and are approximately 6·4. It follows, therefore, that the specific heat of an element is inversely as its atomic mass. But, as we have seen, the product of an atomic mass by the specific heat is the thermal capacity of the atom. Hence the law of Dulong and Petit

expresses simply the fact that the thermal capacity of all elementary atoms—or perhaps better, elementary molecules—is the same. The observed variations from the law are such, for the most part, as can readily be accounted for by the differences in the condition of the elements used, or of the temperatures at which the specific heats are measured.

Neumann has enunciated a similar law for compounds. He finds that for compound bodies having a similar chemical composition and an analogous constitution, the same constant represents the product of the specific heat and the molecular mass. Subsequently Woestyn showed that the elements retain their specific heats unaltered when they enter into combination, and that consequently the molecular heat of a compound body is the sum of the atomic heats of its constituent elements.

273. Specific Heat of Gases.—The quantity of heat required to raise a given mass of gas through a given temperature is greater, for reasons to be discussed later, when the gas is allowed to expand than when its volume is preserved constant. Hence gases have two specific heats: one when the pressure is constant and the volume varies, the other when the volume is constant and the pressure varies. They are called the **specific heat at constant pressure** and the **specific heat at constant volume**, respectively; the former being the greater. The specific heat of air at constant pressure was determined with great care by Regnault, by passing the air first through a coil placed in hot oil, to heat it to a known temperature, and then through another coil contained within the calorimeter. Knowing the mass of air employed and its initial temperature, the specific heat is readily calculated from the ordinary calorimetrical formula. With the same apparatus the specific heats of thirty-five other gases and vapors were determined by the same careful experimenter. He concludes: 1st, that, for approximately perfect gases, the specific heat does not vary with the temperature; 2d, that the thermal capacity of unit mass is independent of

the pressure or density of the gas, and hence that the thermal capacity per unit volume varies as its density; 3d, that the thermal capacities of equal volumes are equal for the simple and more difficultly condensible gases, and also for compound gases formed without condensation; and 4th, that for easily condensible gases these laws do not hold; the specific heat, for example, increasing with the temperature.

SPECIFIC HEATS OF GASES AND VAPORS

Substance.	Equal Volume.	Equal Mass.
Air	0·2375	0·2375
Oxygen	0·2405	0·2175
Nitrogen	0·2368	0·2438
Hydrogen	0·2359	3·4090
Chlorine	0·2964	0·1210
Carbon dioxide	0·3307	0·2163
Marsh gas	0·3277	0·5929
Steam	0·2989	0·4805
Alcohol	0·7171	0·4534
Ethyl ether	1·2266	0·4796
Chloroform	0·6461	0·1566
Carbon disulphide	0·4122	0·1569
Benzene	1·0114	0·3754
Turpentine	2·3776	0·5061

In the above table the values headed "equal mass" represent the specific heat proper; i.e., the thermal capacity of unit mass. In the other column, the values were obtained by multiplying the specific heats as given, by the specific gravity of the gas or vapor referred to air as unity.

SECTION III.—EFFECTS OF HEAT.

A.—EXPANSION.

274. Action of Heat upon Matter.—Since heat is energy, the effect of expending heat upon a body is clearly to increase the total energy of this body. This increase may be either of potential energy or of kinetic

energy or of both. The first and most obvious effect of expending heat-energy upon a body is to raise its temperature; and this increase of temperature, as we have seen, is only an increase in the molecular kinetic energy of the body. The second effect is to increase the potential energy of the mass heated, the work being done against cohesion; and the result is a change in the volume, or in the elasticity, or in the viscosity, etc. Again, heat-energy may be expended in doing work within the molecule itself, either in producing atomic motion therein or in effecting atomic rearrangements against chemism. In all these cases the work is done within the body heated, and is called, therefore, internal work. Besides this, however, the expenditure of heat-energy upon a body may enable it to do external work, as when by heating a bar of iron under stress, it expands and overcomes resistance through distance.

275. Expansion of Solids.—Rise of temperature in a body is usually accompanied by an increase in its size. Indeed the expansion produced by heat is generally regarded as some function of the temperature-change. In considering the dilatation of solids by heat, it is convenient to distinguish between length-expansion and volume-expansion.

EXAMPLES.—An ordinary clock runs faster in winter than in summer, since its pendulum contracts by cold and is therefore shorter. Spaces must be left between the ends of rails and of the sections of tubular bridges in order to allow for their expansion and contraction by changes of temperature. It is calculated that a bar of iron when heated eight or nine degrees will produce in expanding a pressure of 140 kilograms upon every square centimeter of its section. The tire of a wheel is placed on it after being heated; so that on cooling it shall contract and bind the parts of the wheel firmly together. The walls of buildings after having bulged outward have been drawn together by the contraction of heated iron rods. Volume-expansion is illustrated in the rise of the mercury column in thermometers, and in the increase in bulk of alcohol and other liquids in summer.

276. Expansion-coefficients.—When heat is applied to a body, it is found that the expansion which takes

place in it is proportional not only to the size of the body and to the number of degrees through which it is heated, but also to a quantity depending upon the nature of the body itself and called its **expansion-coefficient.** Suppose a rod of length l_0 at 0° to be heated to $t°$, so that its length becomes l_t. The difference of length $l_t - l_0 = e$, which is evidently the expansion. But, as has just been stated, $e = \alpha t l_0$; in which α represents the expansion-coefficient, t the temperature-change, and l_0 the initial length. Consequently since, from this expression, $\alpha = e/tl_0$, we may define a linear-expansion coefficient as the amount of the linear expansion which unit length of the body undergoes for one degree change in its temperature. The volume-coefficient is similarly defined.

If, in the first equation above given, we substitute for e its value, we have $l_t = l_0 + \alpha t l_0$, or

$$l_t = l_0(1 + \alpha t). \qquad [45]$$

This expression $1 + \alpha t$ is called the expansion-factor. It is evidently the ratio of the final to the initial length. Dividing both sides by this expansion-factor we get also $l_0 = l_t/(1 + \alpha t)$. Hence to obtain the length of a solid at a temperature $t°$, having its length at zero we have only to multiply its length at zero by the expansion-factor. Conversely, by dividing the length at $t°$ by the expansion-factor the length at zero is obtained. If the length at $t°$ is given and the length at $t'°$ is required, then since $l_t = l_0(1 + \alpha t)$ and $l_{t'} = l_0(1 + \alpha t')$, we have by division

$$l_{t'} = l_t\frac{1 + \alpha t'}{1 + \alpha t}; \text{ or approximately } l_{t'} = l_t(1 + \alpha(t' - t)). \qquad [46]$$

If a cube whose side is unity be heated one degree, its volume will be $(1 + \alpha)^3$ or $1 + 3\alpha + 3\alpha^2 + \alpha^3$. Since, however, the linear coefficient α is a very small quantity, the powers higher than the first may in general be neg-

lected and the volume-coefficient γ may be taken as three times the linear α.

277. Measurement of Expansion-coefficients. — From the formulas just given, it is evident that to determine an expansion-coefficient, we must know (1) the total expansion, (2) the temperature-change, and (3) the initial length or volume. The experimental determination may be conducted as follows: The rod whose linear coefficient is desired has two marks made near its ends, and is placed within a somewhat wide glass tube, held in the two corks which close its ends. Through each of these corks passes a thermometer and a piece of glass tube. By means of micrometer-microscopes and a scale, the distance between the marks upon the rod is first measured, and this is taken as the initial length of the rod. The initial temperature is also noted. Steam is then blown through the apparatus by means of the glass tubes in the corks; and when the temperature of the whole is in equilibrium, the length of the rod is again measured, and the temperature noted. The difference between the final and the initial lengths is of course the total expansion. That between the final and initial temperatures is the temperature-change. Dividing the total expansion by the temperature-change, we have the expansion for one degree. And the quotient of this divided by the initial length is, by the definition, the expansion-coefficient.

Matthiessen determined the volume-coefficient of a large number of solids, by weighing them carefully in water at different temperatures. Since the volume of a given mass is inversely as its density Δ, and since $V_t = V_0(1 + \gamma t)$, we may write

$$\Delta_0 = \Delta_t(1 + \gamma t); \quad \text{whence} \quad \gamma = (\Delta_0 - \Delta_t)/\Delta_t t. \qquad [47]$$

By determining the density of an alloy at 0° and at $t°$, therefore, the volume-expansion-coefficient γ can be calculated.

TABLE OF EXPANSION-COEFFICIENTS.

Substance.	Mean coefficient between 0° and 100°. Linear.	Volume.
Glass	$8{\cdot}37 \times 10^{-6}$	$25{\cdot}4 \times 10^{-6}$
Copper	$17{\cdot}16 \times 10^{-6}$	$51{\cdot}3 \times 10^{-6}$
Lead	$28{\cdot}82 \times 10^{-6}$	$88{\cdot}0 \times 10^{-6}$
Tin	$19{\cdot}59 \times 10^{-6}$	$69{\cdot}0 \times 10^{-6}$
Zinc	$29{\cdot}76 \times 10^{-6}$	$89{\cdot}0 \times 10^{-6}$
Iron	$12{\cdot}04 \times 10^{-6}$	$35{\cdot}5 \times 10^{-6}$
Platinum	$8{\cdot}57 \times 10^{-6}$	$26{\cdot}6 \times 10^{-6}$

278. Non-isotropic Expansion.—In what has been said above, we have assumed the substances to be isotropic, i.e., to have identical properties in all directions. Crystalline bodies, however, are non-isotropic and have therefore different expansions along different axes. Of course there are, in consequence, in crystals having three unequal axes, three linear expansion-coefficients in these directions. The volume-coefficient is the sum of the three linear coefficients. The case of Iceland spar is specially interesting. This substance when heated expands parallel to its principal axis, but contracts at the same time equally in all perpendicular directions.

279. Variation of the Expansion-coefficient with Temperature.—It has been experimentally proved that the expansion-coefficients of solid bodies increase slowly with the temperature. Thus the volume-coefficient of glass, which is $25{\cdot}4 \times 10^{-6}$ between 0° and 100°, becomes $30{\cdot}4 \times 10^{-6}$ between 0° and 300°. The coefficient of copper similarly increases from $51{\cdot}3 \times 10^{-6}$ to $56{\cdot}5 \times 10^{-6}$; and that of iron from $35{\cdot}5 \times 10^{-6}$ to $44{\cdot}1 \times 10^{-6}$. Hence the coefficients given in the above table are mean coefficients between the temperatures stated. The actual coefficient at any temperature is the limiting value of the expansion per unit length or unit volume as the temperature-change is indefinitely diminished.

280. Expansion-coefficients of Liquids.—Since liquids require a containing vessel, their expansion is somewhat masked by that of the vessel itself. It is

common, therefore, to speak of liquids as having two expansion-coefficients: a real coefficient, which is that of the liquid itself; and an apparent coefficient, which represents the difference between the real expansion of the liquid and the volume-expansion of the vessel. Thus if a large thermometer filled with water be suddenly heated, the column will at first fall, and then begin to rise; the glass envelope in expanding increasing the interior volume of the bulb.

The real expansion of mercury has been directly determined with great care by Regnault. The principle of his method is as follows: A U-tube is filled with this metal and one of its branches is surrounded with melting ice while the other is kept at a temperature t°. The two columns balance each other hydrostatically, but since that on the hot side has expanded, it is higher than the other. If H_0 and H_t are the heights and Δ_0, Δ_t the densities of the columns, we have $H_0\Delta_0 = H_t\Delta_t$, since the heights are inversely as the densities. But (277) $\Delta_0 = \Delta_t(1 + \gamma_r t)$; whence substituting and reducing $\gamma_r = (H_t - H_0)/H_0 t$. Hence by dividing the difference in the heights of the columns by the height of the column at 0° and by the temperature-difference, the mean real coefficient of expansion of mercury γ_r is obtained.

The apparent expansion of mercury is directly determined by filling with it at 0° a large thermometer having a recurved stem, and then heating the whole to t°. The mercury and glass both expand, but the greater expansion of the mercury forces some of the metal out of the tube. This is collected and weighed. If its mass be m, that of the mercury originally filling the instrument being M_0, that left in the bulb will be $M_0 - m$; and we have $m/(M_0 - m)t$ as the apparent expansion-coefficient per unit of mass; or, since volumes of the same liquid are proportional to their masses, $v/(V_0 - v)t$ is the coefficient of expansion per unit of volume. Knowing the value of γ_a, the coefficient of apparent expansion, we may use this apparatus as a weight-thermometer. For the formula gives us $t^\circ = m/(M_0 - m)\gamma_a$; so that by

heating the instrument, after filling at 0°, to an unknown temperature $t°$, we may determine this temperature by dividing the mass of the mercury which has escaped by that of the mercury left in the bulb, multiplied by the expansion-coefficient.

Since the coefficient of real expansion of a liquid is the sum of the coefficients of apparent expansion of this liquid and that of the glass envelope in which it is contained, either of these three coefficients may be determined if the other two be known. If, for example, the coefficient of real expansion of mercury be $181{\cdot}5 \times 10^{-6}$ and the coefficient of apparent expansion in a given glass bulb be $154{\cdot}3 \times 10^{-6}$, then the difference of these values, or $27{\cdot}2 \times 10^{-6}$, represents the coefficient of volume-expansion of the glass.

281. Anomalous Expansion of Water.—When water at 0° is heated it contracts instead of expanding; and this exceptional result continues until its temperature reaches 4° (Fig. 100). Then expansion commences and at about 8° the liquid has the same volume as at 0°. The temperature of 4°, therefore, which is the point of minimum volume, is also the point of maximum density of water. By means of a water-thermometer with a large bulb this relation between the volume and the temperature of water near the zero point may be easily obtained. If the degrees of temperature thus obtained be plotted as abscissas and the heights of the column in the tube as ordinates (Fig. 100), the curve of variation may be drawn. In the figure the curve *abc* shows the results of experiment and therefore represents also the variation due to the glass of the

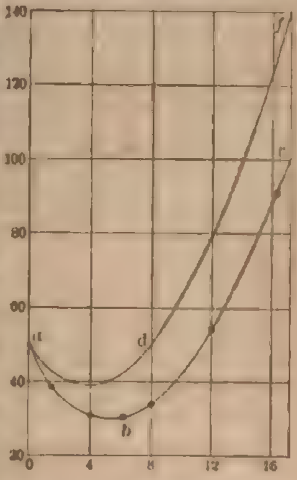


FIG. 100.

thermometer. Correcting for this the curve *adf* is obtained which shows the maximum density of water to be at 4°.

EXPERIMENT.—This fact is experimentally illustrated usually by means of a cylinder of glass filled with water and having two thermometers inserted horizontally into it, one near the top, the other near the bottom. On placing a freezing mixture in a metal jacket which surrounds its middle portion, the water, at first becoming denser as it cools, sinks to the bottom and the lower thermometer falls, until it reaches 4°. Then, on further cooling, the water expands and rises to the top; and the upper thermometer, which has thus far been stationary, begins to fall, continuing to do so until the water upon the surface is frozen. (Hope.)

The expansion-coefficient of water was accurately determined by Matthiessen by weighing in it at different temperatures a block of glass whose coefficient was known.

282. Increase of the Coefficient with Temperature.—With most liquids, the expansion-coefficient steadily increases as the temperature rises. This is shown in the following table for several well-known liquids (Pierre)

Liquid.	Volume at 0°.	Volume at 10°.	Volume at 40°
Water	1	1·000146	1·007492
Alcohol	1	1·010661	1·044882
Carbon disulphide	1	1·011554	1·049006
Wood-spirit	1	1·012020	1·050509
Ethyl ether	1	1·015408	1·066863

For some liquids the expansion may be represented by a formula containing but two constants; but for those which are most expansible three constants are required. Thus for the volume of mercury at $t°$, the formula adopted by the International Committee of Weights and Measures is

$$V_t = 1 + \cdot0001817 92t + \cdot000000000175t^2 + \cdot000000000035116t^3. \quad [48]$$

For water between 4° and 3°, Matthiessen found the volume to be represented by the formula

$$V_t = 1 - \cdot00000253(t-4) + \cdot0000008389(t-4)^2 + \cdot00000007173(t-4)^3. \quad [49]$$

r ether, at t°, Pierre gives the formula

$$V_t = 1 + \cdot 0015132t + \cdot 0000023592t^2 + \cdot 000000040051t^3. \quad [50]$$

quefied gases are remarkable for their great expansiility with heat. Thilorier has shown that liquid carbon oxide is four times as expansible as air, and Drion has nfirmed the general statement for liquid sulphurous ide and ethyl chloride. According to Wroblewsky, e expansion-coefficient of liquid oxygen at $-139°$ is 01706; and that of liquid nitrogen at $-153 \cdot 7°$ is 0·0311.

283. Expansion of Gases.—The effect of heat upon gas may be measured either by noting the change in s volume when the pressure upon it is constant, or the hange in its pressure when its volume is preserved unhanged. This change, either of pressure or of volume, like the corresponding temperaturechange, is due simply to a change in the molecular kinetic energy; and onsequently the coefficient of volme-change in a perfect gas, as meastred by the temperature-change, hould be numerically equal to the oefficient of pressure-change. Since, owever, no actual gas is a perfect as, these coefficients have values which differ from each other to an xtent depending upon the compressiility of the gas.

284. Measurement of Coefficients.—We owe to Regnault the ost accurate determination of these as-coefficients. The apparatus which s used is essentially an air-thermoter (Fig. 101). The volume-coefficent was obtained by placing the ulb *a* first in melting ice and then

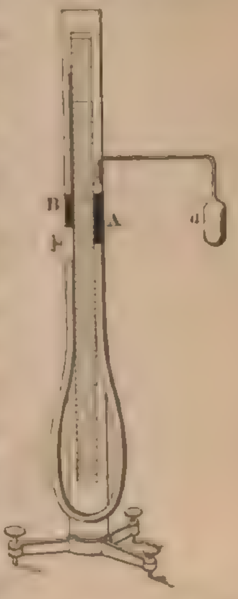


FIG. 101.

the vapor of boiling water, and noting the difference the volume of the gas as measured by the height of e mercury column in the tube *A* in the two cases;

the difference in height between A and B being made zero in both experiments. Knowing the volume of the bulb and of the tube, it is easy to calculate the ratio of the increase of volume under constant pressure to the original volume, for 100°. In ascertaining the pressure-coefficient, the tube B was raised until the mercury in A stood at the same point when the bulb was at 100° as when it was at 0°. The volume being constant, the increased height of the mercury column in the second experiment gave the increased pressure for 100°. The ratio of this increase of pressure to the original pressure is the pressure-coefficient.

285. Law of Charles.—The results of Regnault's investigations fully confirm a law first stated by Charles in 1787, and subsequently, with more accuracy, by Gay Lussac. This law asserts that not only are the volume- and pressure-coefficients the same for any one gas, but they have approximately the same value for all gases between 0° and 100°. Gay Lussac obtained the value 0·00375 for the mean volume-coefficient for 1°, within this range. But Rudberg by drying the air more carefully obtained 0·003665, a value comparable with that of Regnault; as appears from the following table, giving both coefficients for different gases:

VOLUME- AND PRESSURE-COEFFICIENTS.

Gas.	Pressure constant	Volume constant
Hydrogen	0·003661	0·003667
Atmospheric air	0·003670	0·003665
Nitrogen	——	0·003668
Carbon monoxide	0·003669	0·003667
Carbon dioxide	0·003710	0·003688
Nitrogen monoxide	0·003719	0·003676
Sulphur dioxide	0·003903	0·003845
Cyanogen	0.003877	0·003829

286. Combination of the Law of Charles with that of Boyle.—By the law of Boyle, the product of the pressure and the volume for a given mass of gas is con-

stant; i.e., $pv = C$. But since by the law of Charles the changes of pressure and of volume are proportional to the expansion-factors, we have $pv/(1 + \gamma t) = C$; or $pv = C(1 + \gamma t)$, where γ is the coefficient 0·003665. If we assume this equation to hold good at all temperatures, an important conclusion follows. For if t be made equal to $- 1/\gamma$, then $pv = 0$, and of course, either the gas will no longer have any volume, or it will no longer exert pressure upon the containing vessel. Since γ is 0·003665, the temperature at which $pv = 0$ is that at which $t = - 1/0·003665$; i.e., $- 273°$ very nearly. This is therefore called the **absolute zero of temperature**, since there is reason to believe that at this temperature all bodies are entirely deprived of heat. Assuming now this new zero point, we may write the above equation thus: $pv/\theta = K$; in which θ is now the absolute temperature, and K is a constant. From this it follows that at constant pressure $v : v' :: \theta : \theta'$; and at constant volume $p : p' :: \theta : \theta'$. Or in other words, the volume of a gas at constant pressure, or the pressure of a gas at constant volume, is directly proportional to the absolute temperature.

287. Applications of the Laws of Expansion.—Numerous applications of the use of expansion-coefficients in practice might be mentioned. The primary standard of length of the metric system, adopted in 1799, is a bar of platinum formerly deposited at the Palais des Archives in Paris and hence known as the "mètre des Archives." This bar is rectangular in section, 25 mm. wide and 3·5 mm. thick, the distance between the planes perpendicular to its axis, which constitute its ends, being the true length of the meter, at the temperature of melting ice. For comparison at other temperatures, the maker of this bar, Lenoir, determined its coefficient of expansion for 1° to be 3·1 microns per meter; that for certain provisional brass meter bars being 9·2μ per meter and that for certain iron meters made at the same time being 6·3μ. Rogers's indirect comparisons of this platinum bar with the Imperial yard have shown that the "mètre des

Archives" is 3·37012 inches longer than the Imperial yard, both being at 0°. Moreover, he finds for bronze (Baily's metal) the coefficient 17·17μ for one meter; for steel (Jessup's) 10·246μ; and for plate glass (Chance & Sons) 7·428μ; these substances being used by the British Board of Trade for constructing standards. A pendulum may be compensated or made independent of temperature, by making its shaft of two different metals, so arranged that their directions of expansion are opposite, as in the gridiron and mercurial pendulums. Since the expansion for 1° is al for the one metal and $a'l'$ for the other, and the length of the pendulum will remain invariable if these two expansions are equal and opposite, $al - a'l' = 0$, or $al = a'l'$; whence $l : l' :: a' : a$. Or the lengths of the metallic rods must be inversely as their expansion-coefficients. Metallic thermometers are constructed containing a curved compound bar made of two strips, one of brass and the other of iron. Since the brass expands more than the iron, the bar will increase in curvature with a rise in temperature, if the brass is upon the outside; and so by moving an index, indicate the temperature upon a suitable scale. The balance-wheels of watches are made compensating, upon the same principle. In weighing gases, since their coefficient is so considerable, it is important to make suitable corrections for temperature.

B.—CHANGE OF STATE.

(a) *Liquefaction.*

288. Fusion.—Besides change of temperature and of volume, heat produces change of state. When a solid body is exposed to heat, not only does its temperature rise and its volume increase, but it liquefies; provided of course that it is not previously decomposed. Moreover when a liquid body is thus heated it is vaporized, and becomes a gas or vapor. The former process is called **liquefaction** or fusion; the latter **vaporization**.

289. Laws of Fusion.—Certain solids, when subjected to heat, change their state abruptly; as in the case of ice, sulphur, tin, etc. Others again pass gradually into the liquid state, becoming viscous or pasty during the process. Such solids are sealing-wax, glass, and wrought-iron. For solids of the first class the following laws of fusion have been found to hold:

I. Every substance begins to melt at a perfectly definite point of temperature, called its fusing point, which is always the same for the same substance under the same pressure.

II. After fusion commences, the temperature of the mass remains constant at the fusion point, in whatever way the heat may be applied, until the liquefaction process is complete.

TABLE OF FUSING POINTS.

Substance.	Temperature.	Substance.	Temperature.
Alcohol..........	— 130·5°	Camphor..........	175°
Sulphurous oxide.	— 78·9°	Tin................	235°
Mercury.........	— 39·5°	Antimony..........	440°
Water...........	0°	Silver.............	954°
Gallium.........	30°	Palladium..........	1500°
Phosphorus......	44·2°	Platinum..........	1775°
Sodium..........	96°	Iridium............	1950°

The last four of the above values are given on the authority of Violle.

Example.—If a piece of ice cooled to —10° be gradually heated, its temperature will rise until it reaches zero, the fusing point. Then it will begin to melt, and so long as any ice remains the thermometer will remain constant at zero. When all the ice has been liquefied, the thermometer will again begin to rise.

290. Change of Volume on Fusion.—In general, solids expand on melting; so that the volume of the liquid is greater and its density less than that of the solid. There are, however, certain notable exceptions to this rule. Water, bismuth, type-metal, and cast-iron,

for example, contract during liquefaction, and in consequence the lighter solid floats on the denser liquid. Moreover, as already stated, the change in volume may be sudden or gradual. In the accompanying figures the character of this change is shown graphically.

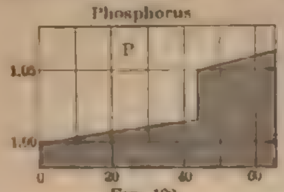


FIG. 102.

Phosphorus (Fig. 102) suddenly expands when heated to 44·2°, and water (Fig. 103) suddenly contracts when

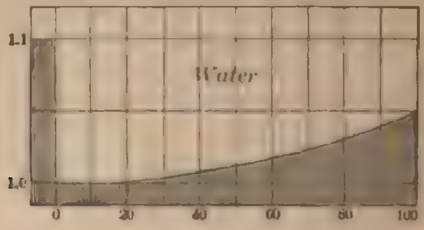


FIG. 103.

cooled to 0°; their respective melting points. While wax (Fig. 104), beginning to soften below 20°, gradually ex-

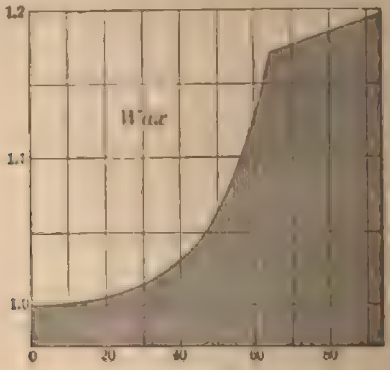


FIG. 104.

pands and does not reach complete liquefaction until the temperature rises to 64°. The practical importance of this gradual change lies in the fact that only such bodies

pass through this pasty condition on fusion are capa- of being welded. Wrought-iron and platinum, for ample, are readily welded, while copper, which passes ddenly from the solid to the liquid condition, cannot welded.

291. Effect of Pressure on the Fusing Point.—Since, en a solid expands on melting, external work is done the heat in addition to that required to change the te of the body, it follows that by increasing the press- e upon the solid, the external work, and hence the at required to do it, will be increased also. In conse- ence the fusing point of such a solid is raised by sub- ting the solid to pressure. On the other hand, a solid e ice, which contracts on melting, requires less heat to lt it under pressure; since then the pressure acts in e same direction as the heat. Thus the melting point paraffin is raised 3·5° by a pressure of 100 atmospheres; d that of wax 10° by a pressure of 500 atmospheres. e fusing point of ice, however, is lowered 0·0075° for ch atmosphere of pressure (Thomson). So that to duce the melting point of ice one degree requires a essure of 140 kilograms upon each square centimeter. ousson liquefied ice at — 20° by means of a pressure 13000 atmospheres.

EXPERIMENT.—Boil water in a flask and, after the air is expelled, e the flask tightly with a cork. When cold place it in a freezing xture until a portion of the water is frozen. If now, after the ice de is partly melted again, the flask be placed in water containing gments of ice, the water inside will be seen to freeze at the tem- ature at which the ice outside is melting. (Von Helmholtz.)

292. Regelation.—When two pieces of ice at 0° are essed together, they unite at the point of contact. is phenomenon, to which Faraday gave the name **gelation**, is a simple consequence of the effect of press- e in lowering the fusing point. The pressure lique- s the ice at the plane of contact; but as the water oduced is below the fusing point, it freezes again as on as it is relieved from pressure, and joins the whole gether.

EXPERIMENT.—Support a block of ice horizontally, and place across it a loop of wire to the ends of which considerable weights are attached. The wire will be observed gradually to sink into the ice and eventually to cut its way through it and fall upon the floor (Bottomley). But the ice itself will not be divided, and will remain as solid as at first. The pressure of the wire liquefies the ice immediately beneath it, and the resulting water flows round the wire to its upper side; there, being relieved from the pressure, it re-solidifies and unites again the separated portions.

Regelation can take place only when the temperature is at the melting point. Faraday floated a number of pieces of ice in water and by causing them to come into contact successively, formed them into a train. By taking hold of the end piece the entire train may be lifted from the water. The effect is more striking when the experiment is made in hot water. The same principle is illustrated in the making of a snowball. It is only when the weather is mild and the snow melting that snowballs can be successfully made. The pressure of the hands in molding them liquefies a minute portion of the snow, the water freezing again as soon as the pressure disappears; until finally the ball becomes a mass of solid ice. Intermittent pressure on a snow-covered pavement or roadway soon converts the snow into ice. And by placing crushed ice in a mold and submitting it to hydrostatic pressure, clear transparent ice-masses may be obtained of any form desired. This process of regelation has been applied to explain the phenomena of glacier motion, at first supposed to be due solely to viscosity. Under the superincumbent pressure the glacier ice liquefies, the mass moves and the pressure is relieved. Instantly the water freezes again and the glacier is as solid as before. So too if we suppose the materials composing the earth's interior to expand on fusion, it is evident that the enormous pressure above them—about 500 atmospheres per kilometer—may be quite sufficient to prevent their fusion even under the very high temperature which exists there. And thus even though this temperature be above that of the ordinary fusing point

f such materials, the earth may still be, as Sir Wm. Thomson supposes, nearly as rigid as a globe of steel.

293. Solution.—"Solutions," says Ostwald, "are homogeneous mixtures—mixtures which allow no separation of their components by mechanical means." Gases may unite with each other in all proportions to form such mixtures, the pressure exerted by the mixture being the sum of the partial pressures of the constituent gases. The solution of gases in liquids is well-nigh general; but the amount dissolved depends not only upon the nature of the substances and upon the temperature, but is also proportional to the pressure. So that, calling the quantity of the gas contained in unit volume, both of the liquid and of the space above it, the **concentration**, the law of Henry (1803) simply states that the ratio of the two concentrations is constant and independent of the pressure. This ratio is called the **coefficient of solubility**. Evidently at first, more molecules of the gas enter the liquid than leave it; but the ratio decreases as the liquid becomes saturated, and equilibrium is reached when this ratio is unity. The solution of a solid in a liquid is also dependent upon the nature of the substances and upon the temperature; the process continuing until a definite concentration is reached, determined by these conditions. Since osmotic pressure (192) corresponds to vapor-pressure, a definite solution-pressure must be exerted by every solid with respect to a given solvent at a given temperature; and Nernst has shown that solution like vaporization will go on until the opposing pressure has become equal to the solution-pressure. Thus, to saturate 100 grams of water, 35·7 grams of sodium chloride are required at 0°, 36 grams at 20°, and 39·7 grams at 100°; while of potassium nitrate, 13·3 grams are required at 0°, 31·7 grams at 20°, and 246 grams at 100°. Solubilities are best represented graphically.

Solution, however, has a chemical side as well as a physical one. While 17 grams of ammonia-gas evolves only 4400 heat-units in assuming the liquid state, it

evolves 7535 heat-units in dissolving in 18 grams of water; so that the formation of the solution NH_4B_4O evolves 3135 heat-units more than is due simply to change of state. To fuse 124 grams sodium thiosulphate 9700 heat-units are required; while only 5700 heat-units are absorbed when it dissolves in water; showing that about 4000 heat-units must be evolved in the act of the chemical combination of the salt with water, notwithstanding the cooling effect due to the solution itself. Hence Mendeléeff regards solutions as "fluid, unstable, definite chemical compounds in a state of dissociation."

294. Solidification.—Just as a solid may be melted by the addition of heat, so conversely, a liquid may be solidified by its subtraction. Hence for every liquid there is a definite solidifying point, the temperature of which is constant so long as the solidifying process is in progress. Moreover, this solidifying point is identical with the fusing point of the solid substance. Substances like bismuth and antimony, which expand on solidifying, give sharp castings; while gold, silver, and copper, which contract, cannot be cast into coins but must be stamped in a die. In the case of water under pressure, solidification takes place only at the instant when the pressure is relieved. Hagenbach submitted closed bomb-shells containing water to severe cold and observed that the ice which protruded from the openings after the shell had burst had the forms of solidified water-jets. Williams filled a 12-inch shell with water, closed it with a wooden stopper, and exposed it to a temperature of $-28°$. When the water froze, the stopper was projected to a distance of more than 270 meters and a solid cylinder of ice 20 centimeters long protruded from the opening.

When solidification takes place very slowly, either from fusion or from solution, the resulting solid may take a regular geometrical form, called a crystal. Under these conditions the molecules are free to move, and therefore may arrange themselves in accordance with the law of symmetry.

295. Surfusion.—Supersaturation.—Although under rmal conditions, solids cannot be heated above their sing points without melting, liquids as such can be oled below their solidifying points. This phenomenon called **surfusion**; or **supersaturation**, when the liquid is solution. If some water previously freed from air by iling be placed in a closed cylinder provided with a ermometer, and if the cylinder be placed in a freezing ixture, the water may be cooled several degrees below ro without solidifying. On agitating the vessel, howver, the water will at once freeze and the thermometer ill rise to zero. Despretz cooled water contained in pillary tubes to $-20°$ without freezing; and Dufour btained the same result with globules of water floating in mixture of almond-oil and chloroform of the same denity as the water. Phosphorus, which melts at 44·2°, has this way been cooled to $-5°$ without becoming solid. nd Gernez has cooled melted sulphur, whose fusing oint is 115°, to the ordinary temperature in a liquid of e same density, without solidification. Upon dropping to such a liquid, either surfused or supersaturated, a agment of the solid, solidification begins at once. And ernez has observed that surfused sulphur, obtained by eating the fused mass to 170° and then cooling to etween 90° and 100°, crystallizes in the octahedral rm on introducing a fragment of octahedral sulphur, the prismatic form if a fragment of prismatic sulphur introduced, and in a third crystalline form if the ottom of the vessel be rubbed with an iron rod. The henomenon of supersaturation is well shown by hot turated solutions either of sodium sulphate or sodium etate. After cooling, the excess of the salt may be ade to crystallize by adding a solid fragment of the roper salt to the supersaturated solution.

296. Heat-changes on Fusion and Solidification. —By the second law of fusion above given, change of ate is not accompanied by any temperature-change. onsequently since the heat-energy which enters a ody in order to melt it does not increase its kinetic

energy, it must be stored up in the body as potential energy, this heat being all expended in doing internal work. When re-solidified, this potential energy reappears as heat and causes a rise in temperature. Since a liquid contains more energy than the corresponding solid, the process of liquefaction must evidently be a cooling operation, and that of solidification a warming one.

EXPERIMENT.—The phenomenon of surfusion is well shown by melting forty or fifty grams of sodium thiosulphate in a flask, corking the flask, and then allowing the whole to cool to the ordinary temperature. The salt will remain in the liquid condition, in the surfused state. But upon dropping in a fragment of thiosulphate, the whole will become solid, evolving a surprising amount of heat. An analogous experiment made with sodium sulphate or acetate in saturated or nearly saturated solution in much the same way, will show the production of heat in connection with the phenomenon of supersaturation.

297. Heat of Liquefaction.—The heat of liquefaction of a solid substance is the amount of heat, measured in heat-units, which is required to change unit mass of that substance into a liquid under the atmospheric pressure without alteration of its temperature. This constant has its highest value in the case of water; and therefore careful experiments have been made to determine it accurately. According to Bunsen, the heat of liquefaction of ice is 80·025 heat-units; i.e., one gram of ice at 0°, in changing to water at 0°, absorbs sufficient heat to raise 80·025 grams of water one degree between 0° and 4°.

EXPERIMENT.—Mix 500 grams of ice at 0° with 500 grams of water at 80°. When the ice has all melted, the temperature of the liquid will be still zero, provided that no heat be lost. In this case 500 grams of water has cooled through 80°, losing 40000 heat-units, by which 500 grams ice has been melted. Hence 80 heat-units has been required to melt one gram of ice without altering its temperature. This is, therefore, the heat of liquefaction of ice.

Liquefaction by solution is also attended with absorption of heat. If ammonium nitrate be dissolved in an equal mass of water, both at 10°, the temperature falls

to $-15°$. If three parts of sodium sulphate at 10° be dissolved in two parts of dilute nitric acid also at 10°, a temperature of $-19°$ is obtained. If six parts sodium sulphate, five parts ammonium nitrate, and four parts dilute nitric acid, all at 10°, be mixed, a temperature of $-26°$ results. Nine parts sodium phosphate and one of dilute nitric acid mixed at 10° produce a cold of $-29°$. Hence such solutions have been frequently used as freezing mixtures.

298. Fusing Point of Mixtures.—Mixtures of two or more solid substances have fusing points which are frequently much below that of either of their constituents. Thus an alloy of five parts of tin and one of lead melts at 194°. An alloy of one part of cadmium, one of tin, two of lead, and four of bismuth melts at 70°. A mixture of potassium and of sodium chlorides fuses at a lower temperature than either salt alone. The use of a mixture of potassium and sodium carbonates as a flux in mineral analysis depends upon this principle. A mixture of ice and salt fuses at $-22°$; and hence liquefies with great absorption of heat at all temperatures above this; whence its use as a freezing mixture. Guthrie has shown that for every salt there is a minimum temperature below which no aqueous solution of that salt can exist. And he calls that particular strength of solution which requires this lowest temperature for its solidification a **cryohydrate**. Sodium chloride cryohydrate, the body formed in a mixture of ice and salt, solidifies as a whole at $-22°$.

HEATS OF LIQUEFACTION.

Substance.	Heat-units.	Substance.	Heat-units.
Water	80·025	Bismuth	12·6
Sodium nitrate	63·0	Iodine	11·7
Potassium nitrate	47·4	Sulphur	9·4
Zinc	28·1	Lead	5·4
Silver	24·7	Phosphorus	5·0
Tin	14·25	Mercury	2·82

299. Molecular Depression of the Freezing Point.—Although Blagden observed in 1788 a relation between the strength of a solution and its solidifying temperature, it was not until 1882 that Raoult discovered the general law that all equimolecular solutions have the same point of solidification. If n molecular masses of any substance be dissolved in w grams of a given solvent, and if Δ be the depression of the freezing point of the solvent thence resulting, we have $\Delta = rn/w$; where r is a constant depending only on the solvent employed. If p grams of a substance be thus dissolved, then $p = nm$, in which m is the molecular mass; and $\Delta = rp/mw$ or $m = rp/\Delta w$. So that since p and w are known, the molecular mass of the substance may be obtained by determining Δ and r. A thermometer graduated to 0·01° serves to ascertain the freezing point of the solution. And by observing the depression produced by a substance of known molecular mass, we obtain r from the equation $r = \Delta w/n$.

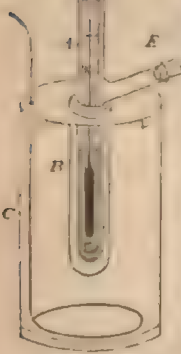


FIG. 105.

Beckmann's form of apparatus for determining the freezing point is shown in the figure (Fig. 105). A hard glass tube A two or three centimeters in diameter is provided with a lateral branch E, which can be closed with a cork. In this tube is placed the thermometer D, and a wire of platinum for a stirrer. The lower half of this tube is enclosed in a wider tube B, and is held in its place by a cork. The whole is placed in a beaker C containing the freezing mixture. About 15 or 20 grams of the solvent is placed in the tube A, and its freezing point is ascertained. A known mass of the substance is then added and a second determination is made. The difference is the depression of the freezing point due to the known mass dissolved.

(*b*) *Vaporization.*

300. Vaporization of Solids.—Sublimation.—Whenever the boiling point of a substance is below its fusing point, **it sublimes**; that is to say, it passes directly into the condition of vapor without previously becoming a liquid.

The pressure at which, for any substance, the boiling point of its liquid and the melting point of its solid coincide, is called the **fusing-point pressure** for that substance. Since a liquid can exist at any temperature only when the pressure upon it is greater than the pressure of its vapor at that temperature, it is clear that a fusible solid under a less pressure than the vapor pressure at the fusing point cannot be melted, but on heating will sublime; i.e., will pass directly into the state of vapor. So that a solid body will evaporate without fusing whenever the pressure upon it is less than the vapor pressure at the fusing point. If the fusing-point pressure of a substance is greater than the atmospheric pressure, it sublimes on heating; and in order to fuse it the pressure upon it must be increased. Arsenic, arsenous oxide, and carbon dioxide are examples. The vapor pressure of carbon dioxide at its fusing point (– 65°) is three atmospheres; and hence under any less pressure it sublimes. Conversely, if the fusing-point pressure is below the atmosphere pressure, non-fusibility is secured only by reducing the pressure. Thus the fusing-point pressure of mercuric chloride is 420 mm., that of iodine is 90 mm., that of benzene is 35·6 mm., and that of ice is 4·6 mm. Ice, therefore, cannot be melted if the pressure upon it is less than 4·6 mm. of mercury.

301. Vaporization of Liquids.—The process of converting a liquid into a vapor is also called **vaporization**. The production of vapor may take place solely from the surface of the liquid, or it may take place throughout its mass. In the former case the vaporization is effected by **evaporation**; in the latter, by **ebullition**. Liquids which evaporate readily are called **volatile** liquids; in distinc-

tion from those which do not, which are said to be fixed. Alcohol, for example, is a volatile liquid, olive-oil is a fixed one.

When a volatile liquid is placed in a vessel devoid of air, at ordinary temperatures, the production of vapor begins at once. The space above the liquid is rapidly filled with the rectilinearly moving molecules, the total kinetic energy of which causes a pressure upon the walls of the vessel and so partially destroys the vacuum. Equilibrium will of course be reached when the number of molecules which pass out of the liquid in a given time is the same as the number which pass into it. The pressure above the liquid thus produced by the evaporation depends only upon the volatility of the liquid used and upon the temperature at which the experiment is performed. Thus if water be the liquid and 20° the temperature, the pressure exerted by the vapor would be about 23000 dynes upon each square centimeter, and would support a mercury column 17·4 millimeters high.

EXPERIMENT.—Fill a barometer-tube with mercury, invert it in its reservoir, and by means of a curved pipette pass a few drops of ether up into the vacuum above the mercury. The ether will at once evaporate, its vapor will produce pressure, and this pressure will depress the column of mercury. If the ether be in excess and be pure, and if no air be present, this depression at 20° will be about 432 millimeters.

By preparing four such tubes, and by passing ether, alcohol, and water into three of them respectively, leaving the fourth for comparison, the differences in the vapor-pressures of these liquids, as measured by the depressions produced, even at the same temperature, may be clearly noted.

302. Point of Ebullition.—A vapor thus continually in contact with its liquid is called a **saturated** vapor. As the temperature is increased, the pressure of a saturated vapor increases until finally it attains the pressure of the atmosphere. The point of temperature at which the saturated vapor of a liquid exerts a pressure equal to that of the atmosphere is the **normal boiling point**, or point of ebullition of that liquid. Since at this temper-

ature the elasticity of the vapor is the same as that of the atmosphere, the production of vapor can go on throughout the entire mass, and thus give rise to the active escape of bubbles characteristic of ebullition.

303. Pressure of Aqueous Vapor.—On account of its importance water-vapor has generally been taken as the typical vapor, and has been carefully studied. The most accurate investigations upon the pressure of aqueous vapor have been made by Regnault, who studied separately the value of this pressure (1) at temperatures between $-32°$ to $+50°$ and (2) between $+50°$ and $230°$. For the low temperatures, he employed a couple of barometer-tubes enclosed in a jacket by means of which they might be heated. One of these had the usual vacuum in its upper part, the other contained water above the mercury. So that by noting the depression of the column and the temperature, the pressure was at once obtained. For high temperatures Regnault made use of the principle that the vapor-pressure of a liquid at its boiling point is equal to the air-pressure to which the liquid is subjected. A cylindrical boiler was connected, by means of a long neck kept cool by water, with a spherical reservoir into which air could be compressed. The temperature of the water in the boiler was measured by thermometers and the pressure in the balloon by a manometer. The results were carefully corrected for latitude and for height above the level of the sea, and were then plotted graphically. They are usually given in tabular form expressing the pressure in millimeters of mercury or in dynes per square centimeter at the latitude of Paris for each degree of temperature between the extremes above noted.

304 Pressure of other Vapors.—Regnault experimented with other vapors also, and the results, expressed in dynes per square centimeter, in the latitude of Paris, are given in the following table, for alcohol, ether, carbon disulphide, and chloroform; that of water being added for comparison.

VAPOR-PRESSURE OF VOLATILE LIQUIDS

Temperature.	Water.	Alcohol.	Ether.	Carbon disulphide.	Chloroform
− 20	1236	4455	$9{\cdot}19 \times 10^4$	$6{\cdot}31 \times 10^4$	—
− 10	2790	8630	$1{\cdot}53 \times 10^5$	$1{\cdot}058 \times 10^5$	—
0	6133	16940	$2{\cdot}46 \times 10^5$	$1{\cdot}706 \times 10^5$	—
10	12220	32310	$3{\cdot}826 \times 10^5$	$2{\cdot}648 \times 10^5$	—
20	23190	59310	$5{\cdot}772 \times 10^5$	$3{\cdot}975 \times 10^5$	$2{\cdot}141 \times 10^5$
30	42050	$1{\cdot}048 \times 10^5$	$8{\cdot}648 \times 10^5$	$5{\cdot}799 \times 10^5$	$3{\cdot}301 \times 10^5$
40	73200	$1{\cdot}783 \times 10^5$	$1{\cdot}210 \times 10^6$	$8{\cdot}240 \times 10^5$	$4{\cdot}927 \times 10^5$
50	$1{\cdot}226 \times 10^5$	$2{\cdot}932 \times 10^5$	$1{\cdot}687 \times 10^6$	$1{\cdot}144 \times 10^6$	$7{\cdot}14 \times 10^5$
60	$1{\cdot}985 \times 10^5$	$4{\cdot}671 \times 10^5$	$2{\cdot}301 \times 10^6$	$1{\cdot}554 \times 10^6$	$1{\cdot}007 \times 10^6$
80	$4{\cdot}729 \times 10^5$	$1{\cdot}084 \times 10^6$	$4{\cdot}031 \times 10^6$	$2{\cdot}711 \times 10^6$	$1{\cdot}878 \times 10^6$
100	$1{\cdot}014 \times 10^6$	$2{\cdot}265 \times 10^6$	$6{\cdot}608 \times 10^6$	$4{\cdot}435 \times 10^6$	$3{\cdot}24 \times 10^6$
120	$1{\cdot}988 \times 10^6$	$4{\cdot}31 \times 10^6$	$1{\cdot}029 \times 10^7$	$6{\cdot}87 \times 10^6$	$5{\cdot}24 \times 10^6$

305. Vapor-pressure of Solutions.—The vapor-pressure of a solution is less than that of the pure solvent. Babo (1848) showed that this lowering of vapor-pressure is proportional to the amount of the dissolved substance; and further that for a given solution and a given temperature, the depression is always the same fraction of the vapor-pressure of the solvent at this temperature. Raoult (1887) further proved that the relative depression varies inversely as the molecular mass. Whence he deduced the following laws: (1) The molecular lowering of vapor pressure produced by all substances in the same solvent is constant; and (2) The relative lowering of vapor-pressure of any solution is equal to the ratio of the number of molecules of the dissolved substance to the total number of molecules in the solution. (Ostwald.)

306. Evaporation into Air.—Dalton's Law.—Experiments made by Dalton led him to conclude that the amount of evaporation is the same whether the space above the liquid contains air or is void of it; and hence that the vapor pressure is the same in a space filled with air as in a vacuum, at the same temperature. This statement, which is known as Dalton's law, has been modified by Regnault, who has shown the joint pressure to be slightly less than the sum of the pressures, owing to the attraction between the vapor and the air.

EXPERIMENT.—Introduce into a closed space filled with dry air a drops of a volatile liquid. The liquid will evaporate and the ssure in the space will be nearly the sum of the pressures of the and the vapor. Thus if, for example, the air-pressure is 76 centiers, and water be introduced at the temperature of 20°, its or pressure being then 1·74 centimeters, the total pressure hin the space will be 77·74 centimeters nearly.

The same principle applies to the mixture of two pors provided that they have no mutual action upon ch other. Thus a mixture of benzene and water or of rbon disulphide and water gives a vapor whose essure is the sum of the vapor-pressures of the conituents; but a mixture of ether and water gives a por-pressure scarcely greater than that of ether one.

307. Saturated and Unsaturated Vapors.—A vapor contact with its liquid is called a **saturated** vapor, cause under these circumstances evaporation goes on til the maximum pressure for that temperature is ached. If, however, the liquid be not in excess, it ay wholly evaporate without producing sufficient apor to exert the maximum pressure within the space occupies. Such a vapor is called an **unsaturated** apor; or if the pressure which it exerts is but a small rt of the maximum pressure for that temperature, it called a **gas**. A vapor may therefore be defined as a s, existing at a temperature at which the maximum essure is but slightly greater than the actual essure upon it. A gas may be defined as a vapor, isting at a temperature such that the actual essure upon it is very much less than the maxium pressure. Thus we call sulphur dioxide a gas cause the pressure under which we ordinarily see it, hich is the atmospheric pressure, is greatly less than maximum pressure for the same temperature were aturated; i.e., about four and two-thirds atmospheres 20°. We call steam a vapor because the maximum essure at ordinary temperatures, 0·023 of one atmosere, is much less than the atmospheric pressure. idently if an unsaturated vapor be compressed, the

pressure exerted by it will continually increase until the maximum pressure is reached. Subsequent compression will diminish the volume only by reducing a corresponding amount of vapor to the state of liquid. Moreover, an unsaturated vapor may be converted into a saturated one by a change of temperature as well as by a change of pressure. Since the vapor-pressure is a function of the temperature, we may cool the unsaturated vapor or gas down to a temperature at which the pressure upon it is equal to or exceeds the maximum pressure. Thus the maximum pressure for ammonia at 20° is nearly nine atmospheres, while at 5° it is only about five and a half atmospheres. A mass of this gas under a pressure of seven atmospheres would be unsaturated at 20°, would be saturated at about 12°, and if cooled to 5 would deposit as liquid a quantity of vapor represented by the excess of one and a half atmospheres (321).

308. Continuity of the Gaseous and Liquid States—Critical Temperature.—Andrews has thrown much light upon the relation existing between the liquid and the gaseous states of matter by showing that there is for every vapor a definite point of temperature above which no pressure however great can convert it into a liquid. This point is called the **critical temperature**. For example, the critical temperature for carbon dioxide is 30·92°. And Andrews found that while no liquefaction was possible at 31·1°, the gas liquefied at 21·5° under a pressure of sixty atmospheres, and at 13·1° under forty-nine atmospheres. Moreover, at the temperature of 30·92° and under a pressure of 73 atmospheres the gas is said to be in the **critical state**. Heated a little it is certainly gaseous; cooled a little it is as certainly liquid, since it is far less compressible. But if the pressure be maintained, the transition from one to the other is not recognizable. There is a perfect continuity between the liquid and gaseous states at all temperatures above the critical point. Vapors above the critical temperature are, therefore, **permanent gases**.

99. Density of Gases and Vapors.—The density of or a vapor is defined as the amount of matter is contained in the unit of volume. But since and vapors are highly susceptible to heat and it is important in stating the density to fix accu- the temperature and pressure. For gases the mass ams of one cubic centimeter at 0° and under a mega- pressure is called the **absolute density.** Relative ity is the ratio of the absolute density of a gas or va- that of air or of hydrogen. The most accurate de- inations of gas-density are those of Regnault. The o be examined was contained in a glass globe of n capacity, suspended to one arm of a sensitive bal- To the other arm was hung a similar globe having ame displacement and serving as a tare. The former was exhausted to a known point, then filled with as and weighed. By dividing the known mass of as by its known volume the mass of one unit of vol- i.e., the absolute density, was obtained. Obviously uotient of the absolute density of any gas divided at of air or of hydrogen, is the relative density of gas referred to the air or to the hydrogen standard.

DENSITY OF GASES.

Name of Gas.	Absolute Density	Relative Density. Air.	Hydrogen.
....	·0012759	1	14·44
en............	·0014107	1·1057	15·96
gen	·0012393	0·9713	14·02
rogen...........	·00008837	0·0693	1
on dioxide.......	·0019509	1·5291	22·08
on monoxide.....	·0012179	0·9545	13·78
h-gas.........	·0007173	0·5622	8·117
rine............	·0030909	2·4225	34.98
gen monoxide....	·0019433	1·5231	21.99
" dioxide......	·0013254	1·0388	14·99
hurous oxide....	·0026990	2·115	30·54
ogen...........	·0022990	1·8019	26·01
ant gas.......	·0012520	0·9819	14·17
onia....	·0007594	0·5952	8.59

It will be observed that the ratio of the above densities, referred to hydrogen, is also the ratio of the molecular masses of the gases mentioned; in accordance with Avogadro's law.

Relative vapor-densities are of importance for fixing molecular masses, since in the condition of vapor the molecular mass is twice the density. The experimental method consists in determining either the volume occupied by a known mass of the vapor, or the mass of the vapor which occupies a known volume. In Hofmann's modification of Gay Lussac's method a known mass of the liquid, contained in a small stoppered bulb, is passed up a barometer-tube into the vacuum above the mercury (Fig. 106). The upper portion of the tube is surrounded with a jacket into which the vapor of a liquid of definite boiling point may be blown, so as to maintain the tube at a sufficiently high temperature. The liquid in the bulb vaporizes and depresses the mercurial column, the volume occupied by the vapor being read off on the graduated tube. The temperature and the atmospheric pressure being noted, the relative density is easily calculated, being the inverse ratio of the volume of the vapor to that of the same mass of air or of hydrogen at the same temperature and pressure.

FIG. 106

In Victor Meyer's method, the volume of vapor produced from a known mass of the substance is determined from that of the air which this vapor displaces. His apparatus consists of a cylindrical bulb *a* (Fig. 107) having a long stem *c*, provided with a lateral delivery-tube and closed at top by a rubber stopper *d*. This bulb is placed in the outer vessel *b*, containing the vapor of a liquid of definite

boiling point. After equilibrium of temperature is attained and air-bubbles cease to escape from the delivery-tube, the stopper is removed and a known mass of the substance is dropped in. The vapor produced expels an equal volume of air at its own temperature; and this cooled to that of the outer air is collected in the graduated tube *e*. The quotient of the mass of the substance divided by the volume of the air expelled, reduced to normal pressure and temperature, is the mass of unit volume; i.e., is the density.

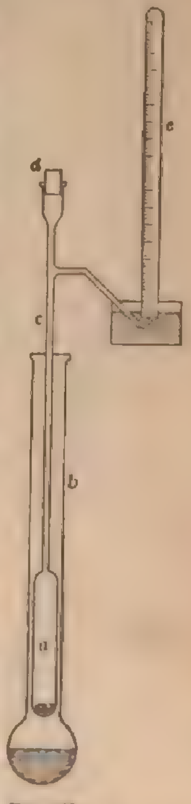


Fig. 107.

For high temperatures the method of Dumas is usually employed. A glass globe of known volume whose neck is drawn out to a fine point, is warmed, and the point immersed in the liquid to be vaporized. The liquid which is thus drawn into the globe on cooling is converted into vapor by placing the whole in a suitably heated bath. When vapor ceases to issue from the globe, the point of the neck is sealed, the temperature and pressure being noted. The increase in the weight of the globe gives the mass of the vapor it contains; and the ratio of this mass to that of the same volume of air at the given temperature and pressure is the relative vapor-density.

RELATIVE VAPOR-DENSITIES.

Substance.	To Air.	To Hydrogen.
Water	0·6225	8·989
Alcohol	1·6138	23·302
Acetic acid	2·0800	30·035
Ether	2·5860	37·342
Benzene	2·7290	39·436

RELATIVE VAPOR-DENSITIES—*Continued.*

Substance.	To Air.	To Hydrogen.
Carbon disulphide..........	2·6447	38·190
Phosphorus................	4·35	62·814
Sulphur....................	2·23	32·201
Mercury	6·86	99·059
Iodine....	8·72	125·92

310. On the Law of Watt.—Distillation.—When two vessels containing the same liquid at different temperatures are connected together, the vapor-pressure—which is identical in both of them—is the pressure corresponding to the lower temperature. This fact, which is known as the law of Watt, is capable of important applications. If one of the vessels be exposed to a source of heat, continual evaporation will take place within it, since the vapor-pressure will never reach its normal maximum. If the other vessel be kept at a low temperature, the pressure within it will be less, vapor will flow into it from the first vessel to equalize this pressure, and will be condensed within it. In this way by evaporation on the one side at a higher temperature and condensation on the other at a lower, the whole of a volatile liquid may be transferred from one vessel to another. This process is called **distillation** and it is generally made use of for the purpose of separating a volatile substance either from a non-volatile or from a less volatile one. Since at the same temperature the vapor-pressure of a liquid is proportional to its volatility, a mixture of two or more vapors when condensed will yield a liquid richer in the more volatile constituent.

FIG. 108.

311. Liquefaction of Gases in General.—Since a gas is only a vapor at a temperature higher or at a pressure lower than the critical point, it is evident that by increasing the pressure or by diminishing the temperature, or both, a gas may be converted into a saturated vapor; which of course, by a continuation of the process, will become a liquid. The pressure required to liquefy a gas may be obtained either by generating it in a confined space, as in the methods of Faraday and Thilorier, or by compressing it by means of a condensing pump, as in that of Natterer (Fig. 108).

CRITICAL TEMPERATURES AND PRESSURES.

Name of Gas	Critical Temperature.	Critical Pressure
Sulphur dioxide..........	155·4°	78·9 atm.
Ammonia	130°	115·0 "
Cyanogen...............	124°	61·7 "
Carbon dioxide..........	30·92°	73 "
Marsh-gas	− 73·5°	56·8 "
Nitrogen........	− 146·0°	35·0 "
Oxygen.................	− 140	320 "
Hydrogen (calculated)....	− 240	13·3 "

312. Liquefaction of Oxygen, Nitrogen, and Hydrogen.—Until oxygen, nitrogen, and hydrogen gases had been cooled to the critical temperature, no amount of pressure could liquefy them. Hence they were called permanent gases. Air, for example, had been compressed until its density far exceeded that of water, without a trace of liquefaction. Faraday expressed the opinion that the critical temperature for air, oxygen, hydrogen, nitrogen, carbon monoxide, and marsh-gas must be below − 166°. In 1879, Cailletet in Paris and Pictet in Geneva succeeded in reaching the critical temperature for several of these gases, and hence in liquefying them. In Cailletet's apparatus the gas was contained in a glass tube surrounded with a freezing mixture and cooled to − 29°, and was compressed by means of mercury forced

in by hydraulic pressure at three hundred atmospheres, so that when this great pressure was suddenly relieved by opening a tap, the rapid expansion caused such a depression of temperature that condensation into liquid drops took place, the gas becoming opake like a fog. Pictet, however, operated on a larger scale and employed three series of compression-pumps, the first of which compressed sulphur dioxide to a liquid at three atmospheres when cooled by water, the second compressed carbon dioxide to a liquid at four to six atmospheres when cooled in the boiling sulphur dioxide to $-60°$, and the third compressed the gas experimented on to two hundred or more atmospheres when cooled to $-140°$ by immersion in the boiling dioxide. By surrounding the gas under experiment with concentric tubes, the inner one containing liquid oxygen boiling at $-181°$ under atmospheric pressure, and the outer one liquid ethylene, temperatures were obtained low enough to solidify nitrogen, carbon monoxide, marsh-gas, and nitrogen dioxide. (Olzewski.) Solid nitrogen evaporating under a pressure of 4 mm. gives a temperature of $-225°$. Hydrogen does not liquefy at $-200°$ under a pressure of 200 atmospheres; although sudden expansion under these conditions produces a dense fog.

313. Laws of Ebullition.—Ebullition, or boiling, takes place when vaporization goes on throughout the entire mass of liquid. This happens at a temperature at which the vapor-pressure is equal to the air-pressure which the liquid supports; so that the boiling point is a function of the air-pressure upon the liquid. Whenever a liquid is heated in an open vessel, more rapid evaporation goes on from its surface as the temperature rises. Soon vapor is also formed on the walls of the vessel beneath the surface; but as soon as the bubbles begin to rise they are at once condensed by the cooler liquid, and thus the noise called "singing" is produced. Finally the liquid becomes so hot that the bubbles reach the surface without condensing and then the liquid boils. It is found that for a given pressure, such

for instance as that of the atmosphere, (1) every liquid has a definite boiling point, and (2) this point remains constant after ebullition commences, until all the liquid has been vaporized. These are the two laws of ebullition.

NORMAL BOILING POINTS OF VARIOUS LIQUIDS.

Liquid.	Boiling Point.
Sulphur dioxide	− 8·00°
Ethyl chloride	11·00°
Ethyl oxide (ether)	34·89°
Carbon disulphide	48·05°
Bromine	63·00°
Wood-spirit	65·50°
Alcohol	78·39°
Benzene	80·44°
Water	100·00°
Acetic acid	117·28°
Silicon bromide	153·33°
Phosphorus tribromide	175·28°
Sulphuric acid	337·77°
Mercury	350·00°

These boiling points are correct for an atmospheric pressure of 760 millimeters of mercury.

314. Causes affecting the Boiling Point.—Since the boiling point is a direct function of the pressure, it is evident that the boiling point of a liquid may be raised by increasing the pressure and lowered by diminishing it.

EXPERIMENTS.—(1) Place a tall beaker-glass containing ether under the bell-glass of an air pump and exhaust the air. The ether will boil at the ordinary temperature.

(2) Place a similar beaker-glass containing water at 30° in the exhausted bell-glass. The water will boil actively, under a pressure of 38 mm.

(3) Boil water in a long-necked flask until the air is expelled from the flask and its place is taken by steam. Cork the flask

tightly while the water is boiling and invert it with the mouth beneath water in a suitable jar, as shown in the figure (Fig. 109). As the apparatus cools the boiling will continue. By pouring cold water on the flask, the boiling is made very active; by pouring hot water upon it, the boiling ceases. The apparent fact that water is thus made to boil by cold and to cease to boil by heat has caused the name "culinary paradox" to be given to this experiment.

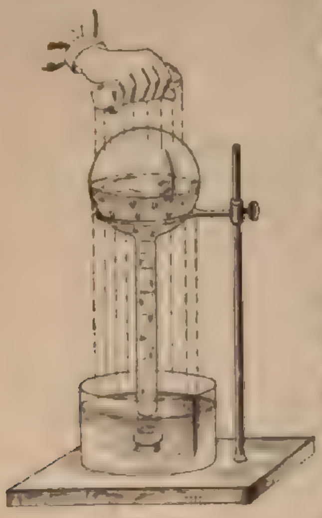


FIG. 109

The explanation of this phenomenon is very simple. As the flask cools, the vapor-pressure, which is a function of the temperature, diminishes; and of course the boiling point, which is a function of the pressure, also diminishes. The liquid, therefore, continues to boil at the continually decreasing temperature. If now the vapor-pressure be markedly decreased by pouring on cold water, active ebullition takes place to restore the initial pressure; while if the vapor-pressure within the flask be increased by pouring on hot water, the temperature of the water inside is not sufficient to cause ebullition at this pressure. On the top of Mont Blanc, for example, water boils at 85° and at Quito at 90°; so that many culinary operations can be successfully carried on there only under pressure. Another application of this principle is found in the so-called vacuum-pan, in which sugar-sirup is concentrated to the crystallizing point at a temperature below that at which it begins to char and become discolored.

An increase of pressure, on the other hand, raises the boiling point. Under a pressure of two atmospheres water boils at 120·6°, and of ten atmospheres, at 180·3°. Use is made of this fact in the arts in the apparatus

known as a digester. This is essentially a strong boiler, in which, along with water, substances are placed which are to be subjected to the action of steam at high temperatures; such as bones for the extraction of gelatin.

EXPERIMENT.—Franklin's "pulse-glass" consists of a glass tube bent twice at right angles, and having a bulb on each end. It is partly filled with a colored liquid, say alcohol, which is then boiled until the vapor expels the air, when the apparatus is sealed. When cold, if one of the bulbs be held in the hand, the vapor in that bulb is expanded and drives the liquid into the other bulb, afterward passing through it in bubbles, as if it were boiling. Of course the phenomenon is entirely independent of the pulse.

315. Use of Boiling Point to measure Heights.—Hypsometry.—It is evident that by noting the temperature of ebullition of a liquid whose normal boiling point is known we may determine the pressure to which it is subjected. The measurement of heights by means of the boiling point is also called **hypsometry**, and a **hypsometer** is simply a compact and easily transportable apparatus in which water may be boiled. It is furnished with a delicate thermometer—reading, say, to 0·01°—to indicate the temperature, the range of the thermometer being small, say 10° or 20° only. The change of temperature is roughly one degree for every 293 meters of vertical ascent above the sea level.

316. Effect of Cohesion.—The boiling point of a liquid is affected also by cohesion. The nature of the vessel in which the liquid is contained and the character of the substances which it holds in solution have a marked influence. Water which boils in a copper vessel at 100° was found to have a boiling point of 101° in a glass one; and if the glass vessel had been carefully cleansed with acid, the boiling point rose even to 105° or 106°. A saturated solution of sodium chloride boils at 102°, one of potassium nitrate at 116°, one of potassium carbonate at 135°, and one of calcium chloride at 179°; the effect of salts being in general to raise the boiling-point. This result is obviously due to the consumption of additional energy in order to overcome the cohesive

attractions involved in the above solutions. In all cases, however, the vapor from a saline solution has the same temperature as if evolved from pure water at the same pressure.

The effect of dissolved air upon the boiling point is very remarkable. Donny has ascertained that water when deprived of air by prolonged boiling can be heated in a closed glass vessel to 135° without boiling; and Grove has expressed the opinion that since water still contains traces of nitrogen even after the most prolonged boiling, no one has yet seen the phenomenon of pure water boiling. By placing water in a liquid of the same density with which it does not mix, but which has a higher boiling point, Dufour was able to heat it to high temperatures. In a mixture of oil of cloves and linseed oil, for example, globules of water 10 mm. in diameter were heated to 120° to 130°; and smaller globules 1 to 3 mm. reached a temperature of 175°, a condition under which the vapor-pressure on a free surface is 8 or 9 atmospheres. Contact with a solid body or the evolution of gas in the hot liquid caused an almost explosive ebullition, with a hissing sound like that produced by a hot iron.

When one liquid is mixed with another of lower boiling point, the boiling point of the former is generally lowered; and *vice versa*. A mixture of two parts of alcohol and one of water boils at 83°, a mixture of two parts of carbon disulphide and one of ether at 38°. In some cases, however, the boiling point of the mixture is below that of either constituent. Thus a mixture of water and carbon disulphide boils at 43°. Hence if these two liquids are mixed at 45°, the mixture at once begins to boil.

317. Effect of Chemical Composition on Boiling Point.—Homologous Series.—Not only has every liquid a definite boiling point, depending upon its chemical character, but a series of liquids alike in composition though progressively differing in molecular mass has a series of boiling points also progressively differing.

ns, for example, Kopp has shown that in the series of mal alcohols, of acids, and of compound ethers of the y class, the members of which differ progressively by , in composition, there is a progressive difference of in their boiling points for each CH_2 added. Such a ies of compounds is called a **homologous series.**

318. Thermal Changes accompanying Vaporiza-n.—Since a liquid and its vapor may have the same nperature, that portion of the heat-energy which is parted to a liquid to vaporize it is expended in doing ernal work and is stored up in the vapor. A vapor nsequently possesses more potential energy than its uid. We owe to Regnault the most accurate determi-tion of the heat absorbed in vaporizing water; or at is the same thing, the heat which is set free when am is condensed. He found that one gram of satu-ed steam at 0° evolves 606·5 calories in condensing to ter at 0°. If the temperature of the steam is not 0° t $t°$, then the heat evolved is $606{\cdot}5 + 0{\cdot}305t$. Thus, example, if the steam be at 100°, the heat set free l be 606·5 + 30·5 or 637 calories or water-gram-de-es, in passing to water at 0°. If the water be at 100°, heat set free will evidently be 100 calories less, or 7. The following table gives the results obtained by drews for several of the more common liquids:

HEAT OF VAPORIZATION.

Liquid	Calories.	Liquid.	Calories.
ater	535·9	Ether	90·45
thyl alcohol	263·7	Carbon disulphide	86·67
hyl alcohol	202·4	Bromine	45·60
hyl acetate	92·68	Stannous chloride	30·53

this table, the temperature of both liquid and vapor upposed to be that of the boiling point of the liquid.

EXPERIMENT.—Place in a beaker-glass a weighed quantity of er at 0° and conduct into it a jet of steam from a boiler. The am will condense and the water will rise in temperature until the ling point is reached. If the operation be then stopped and the ker be again weighed it will be observed that the water which

it contained has been raised from 0° to 100° by the condensation in it of less than one fifth of its mass of steam.

319. Production of Cold from Change of State.—Whenever a solid changes into a liquid, or a liquid changes into a vapor, the heat required to produce this change of state must be supplied from surrounding sources. When ice is exposed at any temperature above 0° it melts, takes heat from all surrounding bodies, and thus becomes a source of cold. When a volatile liquid like ether is allowed to evaporate, the heat necessary to convert it into vapor is taken from the objects with which it is in contact and they are thereby cooled. Upon facts such as these depend the various methods in practical use for the production of cold artificially. A mixture of ice and salt, as we have seen, fuses at −22°. Hence if exposed to any temperature above this, it melts and produces cold. A glance at the tables already given, however, will show that the heat absorbed in vaporization is far greater than that required for liquefaction: one gram of ice requiring only 80 calories to melt it, while one gram of water requires 537 calories to vaporize it. Freezing-machines therefore are generally constructed to make use of volatile liquids, such as ether, carbon disulphide, or ammonia. The ammonia ice-machine of F. Carré is perhaps the best known. One form of it consists (Fig. 110) of a strong wrought-iron cylinder containing a saturated solution of ammonia in water. Connected with this is a wrought-iron annular condenser conical in form. Upon placing the condenser in cold water and the cylinder in a charcoal-furnace, the ammonia-gas is expelled from the water; and being in a closed space, is condensed by its own pressure until the saturation-point is

FIG. 110.

hed for that temperature. Then liquefaction begins continues until most of the ammonia-gas has been lled from the water. The apparatus is now reversed, cylinder being immersed in the cold water. The monia-gas is re-absorbed by the water in this cylinder the liquefied ammonia evaporates to maintain the ration-pressure, of course absorbing heat in the pro-. If a vessel containing water be placed in the cylin-al space in the condenser, and the whole be wrapped non-conducting jacket, this heat is abstracted from water, which is soon frozen; the apparatus reverting in to its original condition.

Water is cooled by its own evaporation in hot cli-tes by placing it in vessels of porous earthenware, er conditions favoring poration from the sur-. In some parts of lia ice is said to be pro-ed by exposing water ing the night in shal-vessels of porous ware ed upon rice-straw. lie froze water by its evaporation by plac-it under the receiver of r-pump, with some sul-ric acid to absorb the or. And E. Carré has ed this experiment

FIG. 111

he production of domestic ice-machines (Fig. 111). h a good exhaustion, these machines exhibit the ap-nt paradox that water will boil even under a surface e.

t should here be noted that while the amount of abstracted by the evaporation of a given quantity liquid is constant, being always the product of its s by its heat of vaporization, yet that the tempera-produced is in general lower, in proportion as the oration is the more rapid; since under these cir-

cumstances more heat is removed from the bodies concerned than is supplied to them. Hence the greater cold produced by the evaporation of more volatile liquids, such as ether and alcohol, which evaporate faster than water. And hence also the advantage of hastening the evaporation by blowing upon the liquid surface or by exhausting the air above it. By a mixture of solid carbon dioxide and ether in vacuo, Faraday obtained a cold of $-110°$; and by a similar mixture of liquid nitrogen monoxide and carbon disulphide, Natterer produced a temperature of $-140°$. By evaporating liquid air, Olzewski produced a temperature of $-210°$. The cold produced by the evaporation of liquid carbon dioxide in the air, when it is relieved from pressure, is sufficient to freeze the greater part of it, producing a solid mass like snow, which evaporates slowly producing a temperature of $-90°$.

320. Spheroidal State.—When a drop of water is placed on a highly heated surface, it does not come in contact with the surface but rolls about upon it, oscillating and gradually evaporating. It is said to be in the **spheroidal state**. This phenomenon has been studied by Leidenfrost and by Boutigny, who have shown (1) that the temperature at which it is produced is higher the more elevated the boiling point of the liquid used, and (2) that the temperature of the liquid spheroid is always below that of its point of ebullition. Thus for water the heated surface must not be below 200°, for alcohol 134°, and for ether 61°. When in the spheroidal state, water has a temperature of 95°, alcohol of 75°, ether of 34°, and sulphur dioxide of $-11°$. This last fact enabled Boutigny to perform a curious experiment. Since sulphur dioxide in the spheroidal state has a temperature below zero, he found that by placing this liquid in a red-hot platinum dish it assumed the spheroidal condition and slowly evaporated. On introducing now some water into the mass, it was immediately frozen by the abstraction of its heat to the sulphur dioxide; and a piece of ice was thus produced in and thrown from the

red-hot vessel. In the same way Faraday, by using a mixture of solid carbon dioxide and ether, in the spheroidal state, succeeded in freezing mercury in a red-hot crucible of platinum.

In all these cases the liquid spheroid is supported upon a layer or cushion of its own vapor which, being a poor conductor of heat, allows only a very slow transfer of heat from the heated plate; so that the evaporation easily keeps the temperature below the boiling point. The fact that there is no contact is easily observed if the surface be but very slightly convex. The light of a candle on the other side can be seen beneath the drop. The layer of vapor which has to support the drop is called a **Crookes layer**; i.e., a layer of molecules whose mean free path, in the language of the kinetic theory of gases, is greater than the distance between the solid and liquid surfaces (372). This active molecular bombardment it is, then, which supports the spheroidal mass above the surface.

321. Dew-point.—Hygrometry.—If air containing a given quantity of aqueous vapor be cooled, it will continually approach and finally reach saturation; since the lower the temperature, the less the maximum vapor-pressure. Hence if into such moist air a solid be introduced, whose temperature is below that of saturation for the given vapor-pressure, a condensation of the moisture will take place and it will be deposited upon the solid in the form of **dew**. If the solid is below zero, the dew freezes as it forms and is then called **frost**. The temperature to which air must be cooled in order that the amount of moisture actually present in it will be sufficient to saturate it, is called the **dew-point**. The ratio of the amount of moisture present to that which would saturate the air at the same temperature, expressed in percentages, is called the **relative humidity** of the air. For the purpose of these comparisons, either the actual amount of moisture in the air, expressed as grams per cubic meter, or, what is preferable, the vapor-

pressure of the moisture, expressed in dynes per square centimeter, may be taken.

EXAMPLES.—Suppose the vapor-pressure in a given mass of air at 20° to be 15000 dynes per square centimeter, and that a mass of iron maintained at 10° be introduced into it. The air in contact with the iron will be cooled to 10° and dew will be deposited upon it, since at 10° the saturation-pressure is only 12220 dynes. Since 23190 − 12220 = 10970; and since 15000 − 12220 = 2780, we have 10970 : 2780 : : 10 : 2·5; whence the dew-point under these circumstances is about 12·5° and the relative humidity [illegible] = 61·77 per cent.

Hygrometry is the science of measuring the state of the air as regards its moisture, and is of very considerable meteorological importance. Several methods of determining atmospheric moisture are in use. In the simplest or chemical method, a known volume of air is drawn over calcium chloride or other hygroscopic substance previously weighed carefully. The increase in weight gives of course the absolute amount of moisture in this volume of air, whence the absolute humidity or the number of grams in a cubic meter may be calculated. Daniell's hygrometer determines the dew-point; i.e., the temperature at which the amount of moisture actually present in the air would saturate it; whence the amount in absolute measure is also known. It consists of a glass tube bent twice at right angles, each end terminating in a bulb. The apparatus contains only ether and its vapor. One of the bulbs is of black glass and contains a delicate thermometer. The other is covered with muslin. If now ether be poured on the muslin, its evaporation cools the bulb and condenses the vapor within it. And as a result, the liquid in the dark bulb evaporates and cools this bulb. It is carefully watched and the temperature within it is noted the instant its surface is bedewed. The application of the ether is suspended, and the temperature again noted at the instant of the disappearance of the dew. The mean of these two temperatures is the dew-point. Modifications of this instrument have been made by several observers; notably by Regnault, Crova, and Alluard.

The form of hygrometer in use for meteorological purposes was devised by August and is known as a **psychrometer.** It is quite simple in its construction, consisting only of two accurate thermometers one of which has its bulb covered with cotton in communication with a reservoir of water. Obviously when there is no evaporation, i.e., when both bulbs are dry or when the air is saturated, both thermometers will read the same. And in proportion as the air is dry and evaporation consequently more active, the difference in the readings will be the greater. By means of the empirical formula

$$p = p' - 0{\cdot}00077(t - t_1)h, \qquad [51]$$

in which p' is the maximum vapor-pressure at $t_1°$, the temperature of the wet-bulb thermometer, $t°$ is the temperature of the dry-bulb thermometer, and h the barometer-reading, the actual vapor-pressure p of the aqueous vapor may be calculated, and the dew-point thus obtained.

322. Mass of Aqueous Vapor present in Air.—Having obtained the dew-point and therefore the actual vapor-pressure in the air, the absolute humidity may readily be computed. Since a cubic meter of dry air weighs 1275·9 grams at 0° and under a megadyne pressure, the mass M of this volume at a temperature $t°$ and a pressure p dynes per square centimeter will evidently be

$$M = 1275{\cdot}9 \times \frac{p}{10^6} \times \frac{1}{1 + {\cdot}00367t}.$$

And since aqueous vapor has a relative density of 0·6225 as compared with air, the mass M' of this volume of vapor is

$$M' = 0{\cdot}6225 \times 1275{\cdot}9 \times \frac{p}{10^6} \times \frac{1}{1 + {\cdot}00367t}. \qquad [52]$$

EXAMPLE.—Required the mass of aqueous vapor in a cubic meter of air, at the temperature of 15°, the barometer standing at 750 mm. and the dew-point being 10°. The vapor-pressure p in dynes, corresponding to 10°, is ·9165 × 980 × 13·596. Hence we have $M =$

$$0·6225 \times 1273·9 \times \frac{·9165 \times 980 \times 13·596}{10^6} \times \frac{1}{1 + (·00367 \times 15°)} = 9·19$$

grams.

SECTION IV.—TRANSFERENCE OF HEAT.

A.—CONDUCTION.

323. Definition of Heat-conductivity.—Since heat is molecular motion, it is evidently capable of ready transfer from one portion of matter to another. Precisely as mass motion is transferred when two bodies whose speeds are different come into contact, so molecular motion, or heat, is transferred when two bodies of different temperatures are made to touch. Heat always tends to establish equilibrium of temperature; and hence under these circumstances there is always a flow of heat either from the hotter to the colder body, or from the hotter to the colder portions of the same body. This transference of heat from molecule to molecule is called **conduction**. Those substances which allow a ready transfer of heat through them are called good conductors; such are copper, brass, iron, and the metals in general. Those substances which transmit heat imperfectly are called bad conductors; such are wood, glass, horn, and also liquids and gases. One end of a copper rod if placed in a flame becomes scarcely red before the other end is too hot to hold in the hand; while a much shorter glass rod may be melted at one end without becoming even warm at the other.

EXPERIMENT.—Place along a rod of copper and along a similar rod of iron a series of marbles fastened by wax. Support both rods horizontally with their ends in contact, and heat the junction with a gas-flame. The heat will be transferred along the bars by conduc-

tion and the marbles will successively fall. They will not only fall more rapidly on the copper rod, but the last one to fall on this rod will be at a greater distance from the source of heat; showing the better conductivity of the copper.

If a series of rods of different substances, say of silver, copper, iron, steel, brass, lead, glass, and wood, all of the same diameter and length, be passed through corks closing openings in the side of a rectangular metal trough, and if the portions outside the openings be covered with a thin layer of wax, then it will be found on filling the trough with hot water, that the gradual transfer of heat along the rods by conduction can be followed by the melting of the wax. If the temperature of the water be maintained constant, it will be observed that after a certain time all the heat transferred to the rods is lost by cooling; so that they cease to rise in temperature. On examining them now, it will be observed that the wax has melted to the greatest distance on the silver rod; and next in order on those of copper, brass, iron, steel, lead, glass, and wood. Hence their conductivities are in the order given.

In such experiments as the above, care should be taken to distinguish between thermometrical and calorimetrical conductivity; i.e., between a transference of temperature and a transference of heat. Obviously, the desired temperature will reach a given point on a rod the sooner, the lower the specific heat of the rod and the higher the conductivity. Indeed if one of two substances be actually a better conductor than the other, and yet the second have a proportionally smaller specific heat, it may happen that the time required for a given temperature to reach a given distance may actually be less in the poorer conductor. Thus Tyndall placed two cubes, one of bismuth and the other of iron, whose upper surfaces were coated with wax, upon the top of a heated plate. The wax melted soonest upon the bismuth cube, although the iron cube was much the better conductor of heat. Though a less amount of heat had entered the bismuth, it had raised its temperature higher, its specific heat being less.

Thermometrical conductivity therefore is simply the ratio of calorimetrical conductivity to the specific heat of unit volume. It is called **thermal diffusivity** by Thomson.

324. Coefficient of Calorimetrical Conductivity. We owe to Fourier the first satisfactory investigation of conductivity. It is found that the quantity of heat which is transferred through a solid cube depends directly (1) upon the difference of the temperature upon its two sides Δt, (2) upon its surface area A, (3) upon the time T, and (4) upon the material used, K and inversely upon the distance between its faces d. Whence we have

$$Q = K\frac{\Delta t}{d}A\,T. \qquad [53.$$

From which the value of K is obtained:

$$K = Qd/\Delta t\,A\,T.$$

If d, A, T, and Δt be made unity, K will equal Q. In other words, the coefficient of conductivity K may be defined as the quantity of heat which passes in a unit of time through unit of surface of an infinite layer of a substance, of unit thickness, when the difference of temperature between its sides is unity. In the C. G. S. system this coefficient represents the number of therms or calories passing per second between the two faces of a plate of any given substance, one square centimeter in area and one centimeter in thickness; these two faces being maintained at a difference of temperature of one degree. Thus, for example, Neumann obtained the following absolute conductivities: for copper 1·108, zinc 0·307, iron 0·163, german-silver 0·109, ice 0·0057. So that if the two faces of a plate of zinc one centimeter thick were kept at a difference of 1° in temperature there would be transferred through each square centimeter of its area in each second sufficient heat to raise 0·307 gram of water one degree in temperature.

5. Temperature-gradient.—If AC (Fig. 112) rep- the temperature on one side of an indefinite and BD the temperature on the side, AE will represent the nce of temperature, or Δt; and miting ratio of AE to EB, rep- ted by $\Delta t/d$ in the preceding la, which is the tangent of the of slope ABE, and which rep- ts the rate of temperature- ge with the thickness, is called **mperature-gradient.** If the time the area of the plate be both , the coefficient of conductivity $/\Delta t$; i.e., is the quotient of the of heat by the temperature- ent. Evidently, therefore, K may be ascertained easuring (1) the flow of heat across unit section of n bar in unit of time; and (2) the temperature- ent.

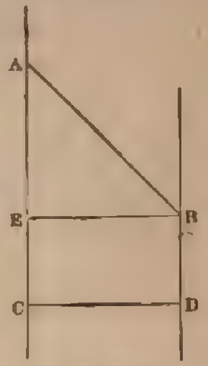


FIG. 112.

26. Method of measuring Conductivity.—The ap- us used by Forbes for measuring conductivity con- of a square iron bar about 2½ meters in length centimeters on a side, one end of which was main- at a constant high temperature by immersion in h of melting lead. Holes were drilled into the bar tervals, into which accurate thermometers were ted, a little mercury being poured in to secure good ct. After the lapse of six or eight hours a perma- state of temperature was reached, the reading of thermometer being constant. Under these condi- , the heat which flows into the bar by conduction gh any cross-section of it is dissipated by the ng of the bar beyond this point. So that the ometers will show a gradually decreasing temper- along the bar. Laying off these differences of erature between the points of the bar and the sur- ding air as ordinates, the distances of the thermom- from the source of heat being plotted as abscissas

a temperature-curve may be drawn (Fig. 113) and the temperature-gradient obtained. By means of an auxiliary and shorter bar, provided with a thermometer and originally heated to the highest temperature of the

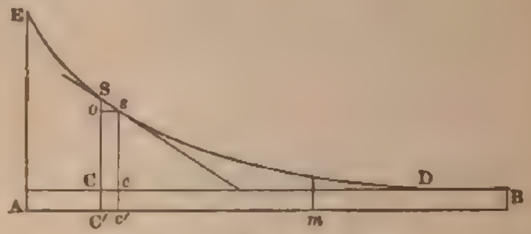


Fig. 113.

main bar and then allowed to cool in the air of the room, readings being taken every minute, the amount of heat lost by this shorter bar per unit of length per second may be calculated for any of the observed temperatures. And by summing these values, the whole heat lost by the main bar beyond a given cross-section, i.e., the flow of heat through this section, may be calculated.

327. Relative Conductivity.—By assuming a value for some given substance, the relative conductivity of other substances may be obtained in terms of this. Thus Wiedemann and Franz, using a thermo-electric pair to measure the temperature of the bars, obtained the following values for the relative conductivity of the metals given, assuming that of silver to be one hundred.

RELATIVE CONDUCTIVITIES.

Silver	100	Iron	11·[illegible]
Copper	73·6	Steel	11·[illegible]
Gold	53·2	Lead	8·[illegible]
Brass	23·1	Platinum	8·[illegible]
Zinc	19·0	Bismuth	1·[illegible]

328. Isothermals.—If heat be propagated from a point in an isotropic medium, the temperature will be

the same at the same distance in all directions. So that these points of equal temperature will all lie on a spherical surface, which for this reason is called an **isothermal** surface. The lines of flow of the heat are radii, and are of course perpendicular to the isothermal surface. Hence there is no lateral propagation of heat over such a surface. If, however, the medium be not isotropic, i.e., if it have a different conductivity in different directions, the isothermal surface will not be a sphere and a plane section of it will not be a circle. De Senarmont used this principle to show that crystals are not isotropic unless belonging to the isometric system (Fig. 114). If a plate of quartz be cut parallel to the principal axis, and its center be heated by means of a wire passed through it, a layer of wax spread upon the surface will be found to melt over an elliptical area; the maximum conductivity being along the principal axis. If a section be cut perpendicular to this axis, however, then the area of the melted wax will be circular; the conductivity along the lateral axes being the same for all. In a quartz crystal, then, the isothermal surface is a prolate spheroid. Wood is found to conduct heat less well in a radial direction than along the fibers; a fact which acts to preserve the interior of a tree from sudden changes of temperature.

FIG. 114.

329. Conductivity of Liquids and Gases.—The conductivity of liquids is exceedingly small. The following values have been obtained by Weber, for the absolute conductivity of the liquids given, at the temperature of 9° to 15°:

CONDUCTIVITY OF LIQUIDS.

Water	·001360	Chloroform	·000288
Alcohol	·000423	Glycerin	·000670
Ether	·000303	Carbon disulphide	·000343
Acetic acid	·000472	Sulphuric acid	·000765

In gases the conductivity is even less. Stefan found the value in hydrogen, which conducts seven times as well as air, to be only 0.0000558. This is only about one twenty-thousandth of the conductivity of copper and one three-thousandth of that of iron.

330. Applications.—Abundant application of the principles above stated can be made. Metal or stone feels colder to the hand than wood or cloth, even when they are shown by the thermometer to have the same temperature. Hot bodies are readily handled by interposing non-conductors between them and the hand. Water at 52° is too hot to be borne, while it is quite possible to remain in air having a temperature near that of boiling water. The extended use of porous substances as non-conductors depends upon the air which is enclosed in them. The use of furs for clothing, of sawdust for packing ice, and of asbestus for covering steam-pipes may be cited as examples. Silica in the form of rock-crystal is a better conductor than lead; but when finely divided it conducts only very slightly. Lava has been known to flow over a layer of ashes underneath which was a bed of ice, without melting it. Double windows are efficacious only because of the layer of air between them. The Davy safety-lamp (Fig. 115) depends upon the fact that flame is cooled and extinguished by the high conductivity of metal gauze, and therefore, although the explosive gaseous mixture may burn within the gauze, the flame cannot pass through it to the gas outside.

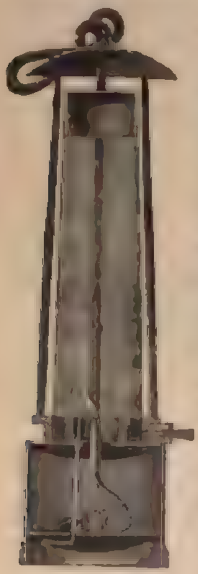


FIG. 115.

B.—CONVECTION.

331. Mass-transference of Heat.—While in solids, heat is transferred from one point to another mainly by

nduction, in fluids this transfer is effected chiefly by nvection. When, for example, the temperature of a ss of water is uniform throughout, all parts of it are equilibrium. But now if the lower portion be heated the upper cooled, the former will expand and become er, the latter will contract and become denser than rest of the liquid; thus destroying the equilibrium d causing a current. The rise of the warmer liquid rying with it its heat, effects a transference of this t from the warmer to the colder portions of the mass; circulation being maintained until all parts of the id are at the same temperature. This process of nsferring heat by transferring the matter possessing s called **convection**. The effect is the greater the more pansible the fluid employed; since in this case the ater is the difference in its density produced by a en temperature-change. For this reason the phe-menon of convection is much more decided in gases n it is in liquids.

EXAMPLES.—Convection-currents in heated air are well shown by ing a candle-flame in a cloud of smoke. The upward movement ntensified if a vertical tube be held just above the flame, thus strating the action of a chimney in producing a draft. In the e way, if water containing solid particles of about the same sity be heated by a small flame, the convection-currents which thus established will be rendered visible by the moving particles.

332. **Applications of Convection.**—This subject is e of great importance not only meteorologically, on broad scale of nature's operations, but also commer-lly, as in artificial ventilation. The air of the tropics, werfully heated by the sun, rises, while the surround-g air from the north and south temperate zones rushes to take its place. This motion, combined with that of earth's rotation, produces what are known as the de winds; blowing in the northern hemisphere from north-east, and in the southern from the south-east. eanwhile the heated air from the equator divides into northern and a southern portion moving toward the les, and producing, in consequence of the rotation of earth, what are known as the return trades; i.e.,

currents at high altitudes having a direction from the south-west in the northern and from the north-west in the southern hemisphere.

The ocean currents are also convection currents. But as to produce currents, the surface must be cooled, it is clear that the polar cold has to do with the ocean circulation much more than the tropical heat. The upper surface of the water, therefore, like that of the air, flows from the equator; and modified in its direction by the earth's rotation, produces the south-west ocean-currents of the northern hemisphere such as the Japan current and the Gulf Stream; and the north-west currents of the southern hemisphere.

The sea breeze at the shore is due to the movement of the sea air toward the land to supply the place of the heated air which has risen from the land previously heated by the sun. The land breeze results from the more rapid cooling of the earth producing an inverse convection current. The land breeze rises just before morning, as the sea breeze comes just before night.

Joule made use of convection currents for the purpose of fixing the temperature of maximum density of water. Two upright metal cylinders each 1·35 meters long and 12·5 cm. in diameter (Fig. 116), placed side by side, were connected at top by a shallow trough and at bottom by a tube provided with a tap. On filling the cylinders with water nearly to the top of the trough, and on placing a small floating glass bulb in this trough, the direction of flow of the current, on opening the tap, due to a difference of density, could be determined. If the temperature in one cylinder was as much above that of maximum density as that in the other was below it, the density would be the same in both cylinders and no current would flow. By fixing several such pairs of temperatures, Joule found the maximum density of water to be about at 3·94°.

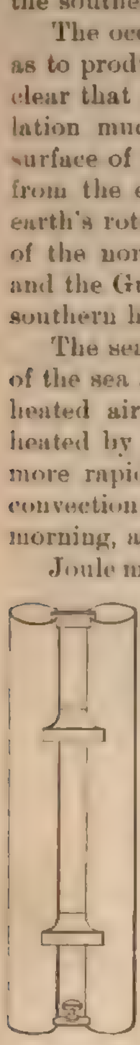


FIG. 116.

333. Cooling by Convection Currents.—The law according to which a heated body cools in consequence of convection has been studied by Dulong and Petit. According to these experimenters, the speed of cooling is (1) independent of the nature of the surface of the body; (2) proportional to the excess of temperature raised to the power 1·233; and (3) dependent not upon the density of the gas but upon its pressure. Hence the speed of cooling due solely to the contact of a gas may be represented by the expression

$$s = mp^a t^{1 \cdot 233}, \qquad [54]$$

in which m is a constant depending upon the size of the body and the nature of the gas, p is the pressure in millimeters, a is an index, determined by Dulong and Petit to be for hydrogen 0·38, for air 0·45, for carbon dioxide 0·517, and for ethylene 0·501, and t is the excess of temperature.

SECTION V.—TRANSFORMATIONS OF HEAT.

334. Conversion of Mechanical Energy into Heat.—Experience shows that heat, which is molecular kinetic energy, may be produced from mass kinetic energy. When a rifle-ball strikes a target, or two railway trains collide, the energy which they possess as mass motion entirely disappears. But since energy is indestructible, its form only is changed in these cases. The molecular motion of the impinging bodies is increased by the collision and they are heated. In general, whenever moving masses are brought to rest, either suddenly as in the case of impact, or slowly as in the case of friction, the kinetic energy of the moving masses is transformed into kinetic molecular energy and they are thereby heated.

335. Mechanical Equivalent of Heat.—The first accurate experiments made to determine the number of units of work which must be expended in order to produce one unit of heat were those of Joule, completed in 1849 In one form of his apparatus, a known weight by

falling through a known distance caused a paddle-wheel to revolve in water, and thus raised the temperature of the water by a known amount. Recently, Rowland has

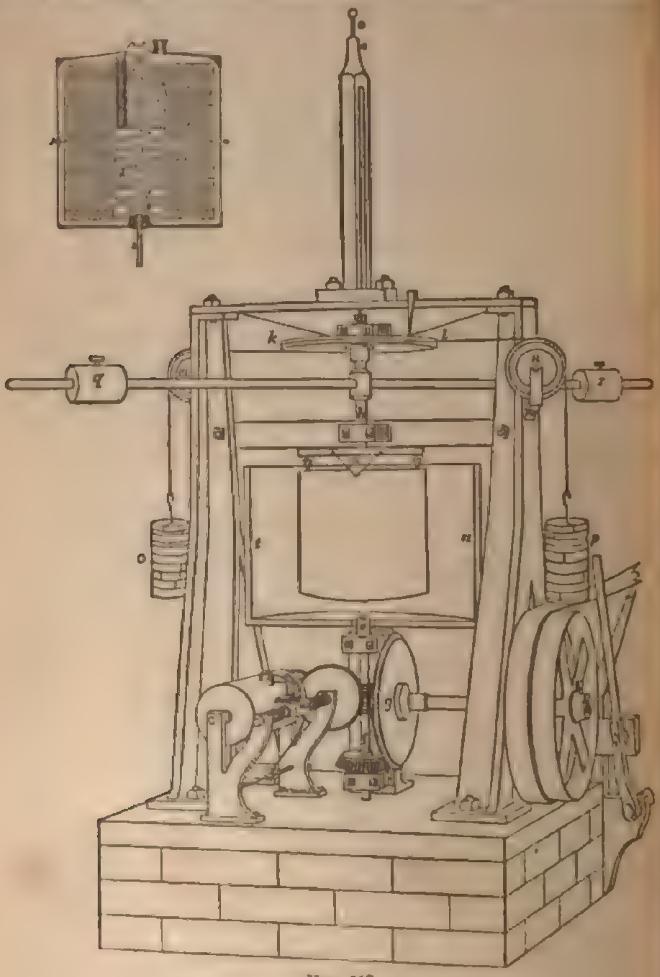


FIG. 117.

repeated this experiment on a more extended scal The vessel containing the water was suspended by wire (Fig. 117), and prevented from rotating by weigh attached to cords upon the circumference of a pull

the axis of the vessel. The paddle was engine. The work expended is evidently to the product of the moment of the couple ber of revolutions made in unit time. The r used multiplied by the rise of temperature at-units produced. And the ratio of the uct to the latter is the number of units of ed to produce one unit of heat; i.e., is the quivalent of heat.

method employed by Joule depends upon on of heat by the compression of a gas; or, upon the amount of work done by the ex- gas when heated through a known range of Suppose, for example, a rectangular ves-) one meter on a side and two meters in ing a piston placed midway. meter of air will thus be en- the piston, and the atmos- ure upon its surface will be megadynes. Let the temper- air be raised 273°; its volume led, the piston will be raised and so will do 10000 meter- (i.e., 10^{12} ergs) of work. Since eter of air has a mass of 1·2759 the temperature of 0° and the

FIG. 118.

one megadyne, and since the specific heat of ant pressure is 0·2375, the heat expended in emperature 273° is of course 1275·9 × ·2375 726 gram-degrees. This amount of heat lished two results; it has not only raised and thus performed 10000 meter-megadynes it has also raised the temperature of a cubic through 273°. The amount of heat re- he latter purpose is 1·2759 × ·1691 × 273 = degrees. The difference, 23826 gram-de- heat expended in doing 10000 meter-mega- 0^{12} ergs of work. Hence one unit of heat s) can do 10000/23826 or ·41971 meter-mega-

dynes of work; i.e., $4 \cdot 1971 \times 10^7$ ergs; which obviously represents the mechanical equivalent of heat.

MECHANICAL VALUE OF ONE HEAT-UNIT.

Authority.	Gram-meters.	Ergs.	Method.
Joule	424·7	$4 \cdot 1624 \times 10^7$	Water-friction.
"	435·4	$4 \cdot 267 \times 10^7$	Electric current.
Hirn	425·2	$4 \cdot 167 \times 10^7$	Compression of lead.
"	441·6	$4 \cdot 3277 \times 10^7$	Sp. heat of air.
Violle	435·2	$4 \cdot 265 \times 10^7$	Ind. curr. in copper.
"	437·4	$4 \cdot 286 \times 10^7$	" " " lead.
Regnault	437·0	$4 \cdot 282 \times 10^7$	Velocity of sound.
Rowland	426·4	$4 \cdot 179 \times 10^7$	Water-friction.

In Rowland's experiments, an expenditure of half a horse-power upon the calorimeter gave a rise of temperature of 35° per hour. This rise was recorded chronographically by noting the time required for a given increase of temperature. His results in some cases are the means of as many as 12000 distinct observations. They show a variation in the value of the mechanical equivalent with temperature. Thus at 5°, he obtained 429·8 gram-meters, and at 20°, 426·4 gram-meters for the latitude of Baltimore; or in absolute units $4 \cdot 212 \times 10^7$ and $4 \cdot 179 \times 10^7$ ergs respectively.

The value $4 \cdot 2 \times 10^7$ ergs, which is generally assumed as the mean value, in units of work, of one water-gram-degree is the true value at 10° according to Rowland. It is known as Joule's equivalent and is represented in mathematical expressions by J. Since a joule is 10^7 ergs, and a water-gram-degree is a therm, the above relation may be stated thus: One therm is equal to 4·2 joules.

336. Conversion of Heat into Work.—Since heat is the lowest form of energy, the conversion of other forms of energy into heat is much more readily and much more completely effected than the conversion of heat into other forms of energy. Indeed, while heat is a product in all energy-transformations, and sometimes the

sole product, the fraction of heat which it is possible to convert into any higher form of energy is exceedingly small. The principle of this conversion is quite simple. It is effected through the agency of some substance, like the steam in a steam-engine, which is called the working substance, and whose function is, while transferring most of the heat from a source at a higher temperature to a refrigerator at a lower one, to transform a fraction of it into mechanical energy or work.

337. Carnot's Cycle.—We owe to Sadi Carnot (1824) the conception which lies at the basis of all discussion concerning heat-engines. This conception is that of a hypothetical engine performing a cycle of operations, which is completely reversible in all respects. The advantage of a cycle in studying the operations of the working substance is found in the fact that at the end of the cycle this substance is found in its primitive condition, no internal work having been done either by or upon it during these operations.

The diagram of work has been already described (95). If a gas be compressed at constant temperature, the work done is represented by the area enclosed between the curve, the ordinates at its ends, and the axis of abscissas. Thus by Boyle's law, the product of the volume by the pressure is constant; i.e., $pv = C$. The curve representing the variations of volume and pressure is consequently an equilateral hyperbola. Thus if a gas under the pressure p_1 has the volume v (Fig. 119), the work done in compressing it until the pressure is p and the volume v_1, is represented by the area $vbav_1$. In the case

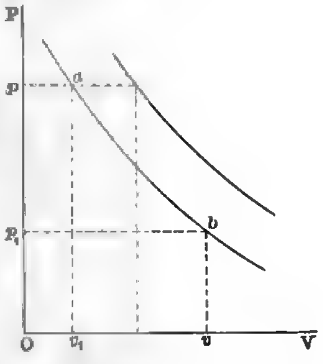


FIG. 119.

supposed ... been done upon the gas and the curve has been ... m b to a; i.e., in the negative direc-

tion. If the gas itself does work by expanding, the will be described from a to b and the work is positive.

Obviously for different temperatures the val the product pv will vary for the same mass of gas hence the curve will represent a different hyperb each temperature. Moreover, since along each h bola the temperature is constant, the line is call **isothermal** line. If the compression of a gas be ducted, therefore, that the heat produced is conti abstracted, its temperature will remain constant, a coördinates p, v, will constantly represent points same isothermal. If, however, the heat be pre from escaping during the compression, it will rai temperature of the ga volume will diminish rapidly than before f same increase of pr and the point p, v, w scribe a curve call **adiabatic** or **isentropic** (Fig. 120) more steep the isothermal curve.

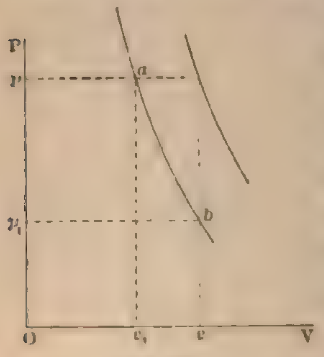


FIG. 120

We are now prepa consider Carnot's cycl consists of four d operations. In the fir working substance expands isothermally from A (Fig. 121). But since this expansion would cool i must be supplied from the source to maintain th perature constant. In the second operation the still allowed to expand, but no heat is permitted into or out of it. This expansion is therefore an tropic one along the line BC. During the third tion the gas is compressed isothermally, from C the heat thus generated escaping to the refrig Lastly, the compression is continued isentropically D to A, the working substance arriving finally same condition of temperature, pressure, and volu

The diagram has been described in the order ; i.e., more has been done working substance than has been upon it, and the of work is positive. In the first and operations work by the working ce; in the third urth, work is upon it. The done by it is

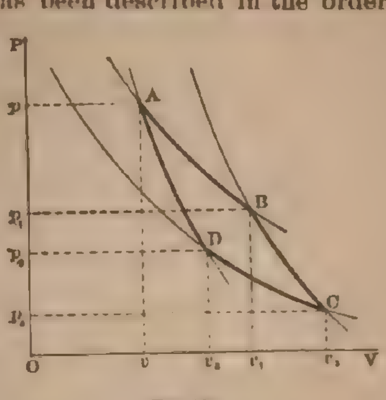


FIG. 121.

nted by the area $ABCv_3v$; that done upon it, by a $ADCv_3v$. The difference $ABCD$ represents, e, the excess of external work which has been uring the cycle by the working substance. The work done during the first operation, represented by the area ABv_1v, has been done at the expense eat supplied from the source. That during the represented by BCv_3v_1, has been done at the expense f the gas itself, no heat being supplied from The negative work done in the third operation, represented by CDv_2v_3, transferred heat to the reor corresponding in amount. And that done in rth operation appeared as heat in the working ce. Heat corresponding to the area ABv_1v has ken from the source during the cycle, and heat nding to the area CDv_2v_3 has been given to the ator. The difference, represented by the area is therefore the heat which has been converted rk.* The ratio

$$\text{rea } ABCD/\text{Area } ABv_1v, \text{ or } (H-H_1)/H, \qquad [55]$$

areas BCv_3v_1 and ADv_2v are evidently equal, since they represent nits of work the heat change in the working substance when ture is altered by the same amount; i.e., between the isother- the signs of these areas are opposite, their sum is zero.

represents the fraction of the heat taken from the source which the engine has converted into work during one revolution. This is called the **efficiency** of the engine.

The value of this ratio may be obtained from the proposition that the work done upon a gas in compressing it, and, conversely, the work done by the gas in expanding again to its original volume, the temperature remaining the same, is the continued product of the initial volume, the initial pressure, and the logarithm of the ratio of the final to the initial volume. That is, $W = v_0 p_0 \log v_1/v_0$. In the above diagram, if we call v and p the initial volume and pressure, and v_1 the final volume, the work represented by the area ABv_1v is $vp \log v_1/v$. That represented by the area CDv_2v_3 is $v_2p_2 \log v_3/v_2$. Since the heat received is $vp \log v_1/v$ and that given up is $v_2p_2 \log v_3/v_2$, the difference $vp \log v_1/v - v_2p_2 \log v_3/v_2$ represents that utilized; and the ratio of the heat utilized to the whole heat taken from the source, or

$$\frac{vp \log v_1/v - v_2p_2 \log v_3/v_2}{vp \log v_1/v}, \qquad [56]$$

represents the efficiency. Now if the area $ABCD$ be made indefinitely small, it can be shown that the above expression becomes $\dfrac{vp - v_2p_2}{vp}$, since then $\dfrac{v_3}{v_2} = \dfrac{v_1}{v}$. But $vp = R\theta$ and $v_2p_2 = R\theta_2$, where θ and θ_2 are the absolute temperatures of the isothermals. Whence $(R\theta - R\theta_2)/R\theta$, or $(\theta - \theta_2)/\theta$, represents the fraction of the energy utilized; or in other words, the efficiency of the engine. It will be seen that the function of the working substance is solely to receive the heat from the source and to divide it into two portions, one of which goes to the refrigerator, while the other is converted into work. The heat is received at the higher temperature θ and is given up at θ_2; and we have proved that the efficiency of such an engine, which is the fraction of the total heat received which is converted into work, is represented by $(\theta - \theta_2)/\theta$; or in other words, is a function only of the temperatures be-

tween which the engine is worked. Carnot's principle asserts, therefore, that the quantity of heat which can be converted into work is entirely independent of the nature of the working substance and is determined solely by the range of temperatures between which the conversion is effected.

This important generalization teaches us that under any conditions attainable in practice the efficiency of a heat-engine must necessarily be low. Thus suppose the boiler of a Carnot's engine to have a temperature of 130° (403° abs.) and the condenser one of 0° (273° abs.); then only a fraction of the whole heat-energy supplied, represented by (403 — 273)/403, or 130/403, can be converted into work. This is only about 32·25 per cent. Working even at six atmospheres pressure, the absolute temperature of the source would be only 429°. And since a lower temperature than about 316° is not attainable in the condenser, the efficiency of the engine would be only 26·34 per cent. Commercially, however, it is rare to attain half this efficiency. Moreover, the formula shows that complete conversion is unattainable, since $(\theta - \theta_2)/\theta$ is equal to unity only when θ_2 is zero; i.e., when the condenser is at the absolute zero of temperature; an impossible condition practically.

But this engine of Carnot's, since it is a completely reversible engine is a perfect engine. In other words, its efficiency is a maximum, and the amount of conversion which it effects is greater than that which can be produced by any other engine. For if not, suppose a reversible engine *A* geared to a non-reversible one *B*, both working between the same temperatures. Suppose the efficiency of *B* higher than that of *A*; then, since it is more perfect, the engine *B* will produce more work than *A* from the same quantity of heat; and the expenditure upon the reversible engine *A* of a part only of this work will suffice to restore this entire amount of heat to the source. If now *B* working forward be employed to drive *A* backward, then the work produced by *B* from a given quantity of heat will be sufficient not only to

drive A backward and thus restore all this heat to the source, but will in addition drive some form of machine. This excess of work has not come from heat yielded by the source; since the exact amount taken by engine B has been restored by engine A. It must therefore be derived from the heat which A takes from the refrigerator in excess of that given to it by B. If we assume the temperature of all surrounding bodies, except the refrigerator, to be the same as that of the source, we have here a result contrary to all experience; namely, the production of work by means of heat taken from a body colder than all surrounding bodies. Hence the hypothesis that there can be an engine having a higher efficiency than a reversible engine is incorrect. Moreover, it follows that all reversible engines have the same efficiency whatever the working substance employed, provided that they work between the same temperature-limits.

338. Thermodynamics.—The name thermodynamics is given to that department of physics which considers the relations of heat-energy to mechanical energy, and which treats of the conditions of their mutual convertibility. The first law of thermodynamics states the equivalence of heat and work. It is as follows:

"When equal quantities of mechanical effect are produced by any means whatever from purely thermal sources, or are lost in purely thermal effects, equal quantities of heat are put out of existence or are generated." (Thomson.)

The second law relates to the transformation of heat into work. It may be stated as follows:

"If an engine be such that when it is worked backwards the physical and mechanical agencies in every part of its motions are all reversed, it produces as much mechanical effect as can be produced by any thermodynamic engine, with the same temperatures of source and refrigerator, from a given quantity of heat." (Thomson.)

339. Absolute Zero of Temperature. — We have already seen that if in the expression $pv = C(1 + \gamma t)$, t be made $-1/\gamma$, pv will be equal to zero, and the kinetic energy of the gas molecules will also be zero. Since γ is 0·003665, this value of t is $-273°$; whence this point was called the absolute zero of temperature. Sir Wm. Thomson has pointed out the fact that the second law of thermodynamics furnishes still other data for constructing a definition of temperature which shall be absolute; i.e., independent of any particular substance and determined solely by the laws of energy-transformation. In the Carnot's cycle just referred to, we saw that the efficiency of the engine was represented by the

$$\frac{\text{Area } ABCD}{\text{Area } ABv_1v} = \frac{\text{Heat utilized}}{\text{Heat supplied}} = \frac{H - H_1}{H}.$$

and also by $(\theta - \theta_1)/\theta$. Whence $H_1/H = \theta_1/\theta$; or in other words, "the absolute values of two temperatures are to one another in the proportion of the heat taken in to the heat rejected in a perfect thermodynamic engine working with a source and refrigerator at the higher and lower of the temperatures respectively" (Thomson). Hence if $\theta - \theta_1$ represent the difference of the areas, θ and θ_1 will be proportional respectively to the areas representing the heat taken in and that rejected. The experiments of Joule and Thomson have shown that the ratio of these areas is as 1·365 to 1 when the heat is received at the temperature of boiling water and rejected at that of melting ice. Calling x the latter temperature absolute, $x + 100$ will represent the former temperature also absolute; and since $\theta : \theta_1 :: H : H_1$, we have $x + 100 : x ::$ 1·365 : 1; which gives 273·9 as the value of x. Hence the absolute temperature of melting ice is 273·9°; and the temperature of the absolute zero is $-273{\cdot}9°$; the degrees being of the same value as those of which one hundred measures the difference between the freezing and the boiling points of water.

SECTION VI.—SOURCES OF HEAT.

340. Terrestrial Sources.—Since heat is only a form of energy, and since all forms of energy are to a greater or less extent mutually convertible, it is plain that the terrestrial sources of heat must be as numerous as the forms of energy upon the earth's surface. Moreover, the energy which provides us with heat exists previously in both kinetic and potential forms. The energy of moving air and water, as in winds, tidal waves, and ocean currents, exists in the first form, that of stored water and of fuel exists in the second. By friction, by compression, by percussion, or by any process by which motion is gradually or suddenly arrested, heat is mechanically produced.

341. Chemical Action.—The most general method of heat-production upon the earth is combustion; by which is meant chemical combination in general. Chemical union is, in by far the greater number of cases, exothermic; that is, attended with the evolution of heat. Moreover, when a definite mass, say of carbon or of hydrogen, is burned, the quantity of heat produced is perfectly definite, also.

Extended calorimetric experiments on the heat produced by combustion have been made by Andrews, Favre and Silbermann, Berthelot, Thomsen, and others. The calorimeter employed by Andrews for gases consisted of a cylinder revolving on trunnions, containing a second cylinder closed at the ends, within which was a third cylinder filled with water. Inside of this was a thin copper vessel containing the gaseous mixture. The calorimeter was revolved for some minutes in order to bring the whole to a uniform temperature. By means of two wires leading in to the inner vessel a spark was passed and the gaseous mixture was fired. After a second rotation the temperature was again determined. Knowing the mass of the water, the water-equivalent of the calorimeter, and the rise of temperature, the total heat in water-gram-degrees is easily calculated. The

quotient of this value by the mass of the gas burned gives the calorific equivalent. By such a process as this or one not differing from it in principle, the following values have been obtained:

COMBUSTION-EQUIVALENTS.

Substance.	Compound formed.	Water-gram degrees.	Ergs.	Observer.
Hydrogen...	H_2O	34000	$1·43 \times 10^{12}$	A, F
Carbon.....	CO_2	8000	$3·36 \times 10^{11}$	A, F
Sulphur....	SO_2	2300	$9·66 \times 10^{10}$	A, F
Phosphorus.	P_2O_5	5747	$2·41 \times 10^{11}$	A
Zinc.	ZnO	1301	$5·46 \times 10^{10}$	A
Iron	Fe_3O_4	1576	$6·62 \times 10^{10}$	A
Carbon monoxide.....	CO_2	2420	$1·02 \times 10^{11}$	A
Marsh-gas...	CO_2 and H_2O	13100	$5·50 \times 10^{11}$	A, F
Olefiant gas.	CO_2 and H_2O	11900	$5·00 \times 10^{11}$	A, F
Alcohol.....	CO_2 and H_2O	6900	$2·90 \times 10^{11}$	A, F

For example, one gram of carbon in combining with oxygen to form carbon dioxide will set free 8000 water-gram-degrees of heat, equivalent to $1·43 \times 10^{12}$ ergs in mechanical value, according to Andrews and to Favre and Silbermann. A similar table can be constructed giving the units of heat evolved by the union of such a series of substances with any other electronegative element such as chlorine, sulphur, or phosphorus.

342. Calorific Intensity.—The temperature which is attained in combustion depends not alone upon the quantity of heat produced as measured in the calorimeter, but also upon the mass and the specific heat of the products of combustion. Thus, for example, one gram of hydrogen in combining with oxygen produces 34000 water-gram-degrees of heat, while one gram of carbon produces only 8000 such units. This one gram of carbon in burning yields 3·67 grams of carbon dioxide; and we may assume that the heat of combustion is all expended in raising the temperature of this combustion-product.

Since the specific heat of carbon dioxide is 0·2163, one unit of heat will raise the temperature of one gram 4·6°, and of 3·67 grams 1·26°; so that 8000 units would raise its temperature 10078°, were no heat lost. One gram of hydrogen in burning produces 9 grams of water-vapor. In the calorimeter this water-vapor is condensed; so that the above 34000 heat-units represents the heat of vaporization as well as that of combustion. As the former quantity has the value 536 heat-units for one gram of water-vapor, or 4824 units for nine grams, the latter quantity must be 34000 – 4824 or 29176 heat-units. Since the specific heat of water-vapor is 0·4805, one heat-unit will raise the temperature of nine grams of this vapor 0·2313° and 29176 such units will raise it 6747°, supposing that all the heat is expended in raising the temperature of the water-vapor produced. So that under these conditions the temperature of combustion, i.e., the calorific intensity, is higher in the case of carbon than in that of hydrogen, although the heat of combustion is greater in the latter case. In general the calorific intensity is obtained by dividing the heat of combustion by the product of the mass of the combustion-product into its specific heat.

When the combustion takes place in the air, however, the calorific intensity is less, since a part of the heat produced goes to raise the temperature of the nitrogen. As unit mass of hydrogen combines with eight units of mass of oxygen, and as in air 8 mass-units of oxygen are mixed with 26·8 mass-units of nitrogen, it follows that to raise this 26·8 grams of nitrogen 1°, since the specific heat of nitrogen is 0·2438, the heat required is 26·8 × 0·2438 or 6·534 water-gram-degrees. Inasmuch as 4·324 heat-units are required to raise the products of combustion 1°, it is evident that 10·858 heat-units will be required to raise both 1°. Whence the combustion-temperature under these circumstances is 29176/10·858 or 2687°. The temperature of the oxy-hydrogen flame is estimated to be from 2200° to 2400°; so that, owing to loss of heat by communication to surrounding bodies, by convection and

radiation, these theoretical temperatures are never tained in practice. A greatly increased calorific intenty, however, may be obtained by previously heating the used for combustion to a high temperature; as is en in the hot-blast iron-furnace, the Siemens-Martin generative furnace, and the Siemens regenerative gasmp.

343. **Heat-value of Fuels.**—Carbon unites with oxyn in two stages: in the first, one gram unites with $1\frac{1}{3}$ ams of oxygen to form carbon monoxide; and in the cond, this $2\frac{1}{3}$ grams of carbon monoxide unites with additional $1\frac{1}{3}$ grams of oxygen to form $3\frac{2}{3}$ grams of rbon dioxide. In the first operation, 2400 heat-units, ., water-gram-degrees, are produced; in the second, 00 heat-units. It is upon this high heat of combuson of carbon monoxide that its value in the blast-furce depends. Again, four grams of marsh-gas consists three grams of carbon and one of hydrogen; so that its at of combustion should be one fourth of $(8000 \times 3) +$ $4000 \times 1)$ or 14500 heat-units, if it is simply the sum of e combustion-heats of its constituents. Experimenlly, however, the heat of combustion of marsh-gas is und to be only 13100 heat-units. A moment's considation shows that before the carbon and the hydrogen n unite with oxygen, they must be separated from each her; and that this separation requires the expenditure energy in the form of heat. Hence of the 14500 heatits produced, 1400 are expended in separating the rbon and hydrogen; leaving 13100 available as the at of combustion. Evidently this 1400 heat-units repsents also the heat of combination when marsh-gas is rmed. From the analysis of a fuel, then, such as a al-gas, a wood, or a coal, the heat of combustion may readily calculated. If oxygen be present, the hydron necessary to form water with it must be subtracted om the total hydrogen. It is usual to calculate the st of the hydrogen as marsh-gas, taking the necessary antity of carbon; the rest of the carbon being rerded as free. In fact, however, the calculated heat of

combustion of a fuel varies considerably from that determined experimentally in the calorimeter; so that the latter method is generally preferred. In a general way the heat of combustion may be taken as 20000 heat-units for coal-gas, 10000 for petroleum, 8200 for bituminous coal, 6000 for anthracite coal, and 3600 for wood. By dividing these numbers by 536, the number of units of mass of water evaporated from 100° by the combustion of one unit of mass of the combustible is obtained. Evidently the efficiency of a boiler may be determined in this way by measuring the quantity of water evaporated per unit mass of a standard fuel.

344. Thermo-chemistry.—All changes in matter appear to involve corresponding energy-changes (7); and since the energy-change generally takes the form of heat, thermo-chemistry, which "treats of the measurement of chemical energy in thermal units," has become an important branch of chemical science. Although the fundamental law of thermo-chemical processes was first stated as early as 1840 by Hess, yet it is to the more recent researches of Thomsen and of Berthelot that we owe most of our present knowledge. In addition to expressing simply the mass-changes taking place, chemical equations are now required to express the heat-changes also. Thus the equation

$$Pb + I_2 = PbI_2 + 39800 \text{ heat-units}$$

expresses not only the fact that 206·9 units of mass of lead and 253·8 units of mass of iodine combine to form 460·7 mass-units of lead iodide, but also the fact that these masses of lead and iodine together contain 39800 heat-units of energy more than the mass of lead iodide resulting (Ostwald). The heat-changes which take place in chemical reactions have been classified by Berthelot as exothermic when there is an evolution of heat and endothermic when heat is absorbed. And he has stated the three underlying principles of thermo-chemistry as follows: (1) The amount of heat set free in

any chemical reaction is a measure of the total work accomplished in the reaction; (2) Whenever changes are produced in a system without causing an external mechanical effect, the evolution or absorption of heat thence resulting depends only on the initial and final states of the system; (3) Every chemical change effected in a system without the aid of outside energy tends to the production of that body or system of bodies the formation of which evolves the maximum heat.

345. Cosmical Sources.—The chief source of supply of energy to the earth, however, is the sun, whose radiation, when it reaches us, is for the most part converted into heat. The amount of heat-energy thus received by the earth was first measured in 1838 by Pouillet by means of his pyrheliometer, which consists of a hollow disk of steel containing mercury, in which is placed the bulb of a delicate thermometer, and which is so mounted on a stand that the face of the disk, which is covered with lampblack, can be placed normally to the sun's rays. The instrument is first exposed to clear sky for five minutes in the shade and the fall of temperature is noted. It is then exposed to the sun for five minutes and the rise of temperature is noted. A second exposure to the clear sky is made and the cooling observed. From the data thus obtained Pouillet calculated that the amount of heat which reaches each square centimeter

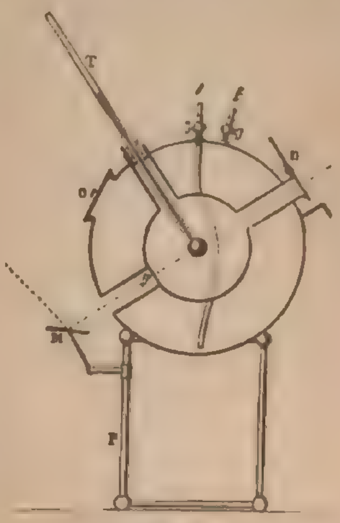


FIG. 122.

of the earth's surface in one minute, supposing the atmosphere absent, would raise the temperature of 1·76 grams of water one degree. In 1877, Violle made a much more accurate series of experiments with a globe actinometer (Fig. 122) devised for the purpose; from which he deduced 2·54 as the value of the solar constant. In 1881 Langley improved this instrument and used it in his researches made upon Mt. Whitney at an altitude of 14500 feet. He says: "My conclusion is that in view of the large limits of error, we can adopt **three calories** as the most probable value of the solar constant; by which I mean that at the earth's mean distance, in the absence of its absorbing atmosphere, the solar rays would raise one gram of water three degrees centigrade per minute for each normally exposed square centimeter of its surface." This, he points out, is "approximately 126 550 000 ergs per square centimeter per minute;" and it implies "a solar radiation capable of melting an ice-shell 54·45 meters deep annually over the whole surface of the earth." Measured as energy, every square meter of the earth's surface under a vertical sun receives 2·1 kilowatts continuously; this being reduced to about 1·6 kilowatts by the atmospheric absorption. Of this, 0·2 kilowatt approximately can be utilized practically. In Ericsson's solar engine, enough solar heat was collected by means of a reflector 3·35 meters by 4·87 meters, to work a two and a quarter kilowatt engine satisfactorily. On an average, the earth's surface receives annually upon each square meter an amount of energy which if it were all utilized would raise 1056·5 metric tons to the height of a kilometer (Young).

If we may assume that the radiation varies inversely as the square of the distance, the radiation at the solar surface for an equal area must be 46000 times that received by the earth; which is equal to about 1380 million calories per square meter per minute, or equal to more than 90000 kilowatts continuously in action. If the sun were frozen over to a depth of fifteen meters, this layer of ice would be melted in one minute. To produce

his heat by the combustion of anthracite coal would require the hourly consumption of eleven metric tons upon every square meter of the sun's surface, equivalent to a layer of such coal 5 or 6 meters thick over his entire area. At this rate the sun if made of coal would be entirely consumed in less than 6000 years (Young).

346. Solar Origin of Terrestrial Energy.—Moreover it is to the solar radiation, absorbed and transformed upon the earth's surface, that we are indebted not only for the phenomena called thermal, but also for all the other energy-phenomena which we witness. The water which falls upon and turns the water-wheel of the mill was raised by solar energy. The wood which we use as fuel and the grain which we consume as food are valuable to us only because of the energy which they have stored up from the sunlight in which they grew. Even the coal which is mined from the depths of the earth evolves on burning the energy of past ages gathered when the plants of the carboniferous epoch drank in the sunlight of those days. Both food and fuel owe their value to the potential energy which they contain; and this energy is all stored sunlight. Von Helmholtz has shown that the earth receives only one twenty-three-hundred-millionth part of the whole solar radiation. It is only this small fraction of the emitted solar energy which impinges upon the earth on its way into space. But this small fraction suffers many and wondrous transformations during its brief residence upon our planet. It turns the wheel of the mill, it drives the steamship, it moves the train. It lifts the blacksmith's hammer, it guides the farmer's plow, it transmits the merchant's commands. It appears in the song of the poet, the life-like creations of the sculptor, the symphony of the musician. And then having vivified and beautified our earth it speeds on its way again, to encounter new worlds possibly and to suffer in them also still other transformations.

347. Dissipation of Energy.—Since we cannot create energy and can only transform it, it is apparent that a

given quantity of one form of energy can be produced only at the expense of an equivalent quantity of some other form; whence it follows that a perpetual motion is impossible. Moreover, since no known natural process is completely reversible, and since heat is the form of energy which most readily escapes conversion, it follows that every transformation which is effected by an imperfect process must result in the simultaneous production of a certain quantity of heat; which heat tends to diffusion and to the consequent production of a uniform temperature. Inasmuch as heat is the lowest form of energy, there is in every energy-transformation a degradation of energy; i.e., a conversion of energy into a less available form, a form less capable than before of being transformed. As time goes on, therefore, there is a continual increase in the unavailable energy and a continual decrease in the available energy of the universe; albeit the total energy remains unchanged. Or as it is stated by Tait: "The entropy of the universe tends to a minimum"; using the term **entropy** to denote available energy. Ultimately, therefore, a uniform temperature will be reached, all the energy of the universe will become unavailable, and all motion and change of every kind will cease. This is Thomson's doctrine of the Dissipation of Energy.

348. Origin of the Solar Energy.—If it be granted that the radiation of energy from the sun is as enormous as has now been stated, it becomes an interesting question to ask by what agency this tremendous expenditure is maintained. That it cannot be due to combustion follows from Thomson's statement that were the sun composed entirely of those substances which evolve the maximum heat on burning, his entire mass, at the present rate of emission, would be consumed in about 5000 years. The meteoric theory, proposed by Waterston and at one time quite in favor, supposes the solar heat to be due to the impact of meteorites upon his surface. In support of this view Nordenskjold estimates that half a million tons of meteoric matter fall on

arth every year; and Thomson calculates that a of only 0·3 of a gram of matter per second upon square meter of the solar surface would furnish y sufficient to maintain his present radiation. over, Young computes that a quantity of meteoric r equal to 0·01 of the earth's mass striking the sun's ce annually with a speed of 600 kilometers per d would develop heat sufficient to cover his total tion. It would seem hardly possible, however, even one per cent of the sun's emitted radiation can counted for in this way. The theory now generaccepted is that of von Helmholtz. He considers the energy emitted by the sun is due to his contraca volume. Lane had shown, in 1870, that a gaseous e losing heat by radiation and contracting under n gravity, must rise in temperature and actually hotter, until it ceases to be a perfect gas; the c energy developed by the shrinkage of a gaseous being more than sufficient to replace the loss of which caused the shrinkage. Von Helmholtz has lated that a contraction of only 38 meters in the radius would account for the whole annual output of y. To reduce the sun's diameter by one second at ate would require 9000 years. If the sun were to act to one half his present diameter and therefore eighth of his present volume, his density would ly twice that of the earth, but the work done in ontraction would supply the energy of the solar tion for more than four million years.

PART FOURTH.

PHYSICS OF THE ÆTHER.

CHAPTER I.

...GY OF ÆTHER-VIBRATION.—RADIANT ENERGY.

SECTION I.—NATURE OF RADIATION.

... **Definition of Radiation.**—The name radiation ... to the transference of energy by means of peri... ...turbances in a special medium filling all space ...led the æther. The precise nature of these dis... ...es is unknown to us, although the laws according ...h they take place are those of wave-motion. ...e not due, however, to changes of position in the ...ecessarily, but may arise from changes in its ...n its electric condition, or in some other of its ...ies. "The essential character of the theory of ...ions," says Maxwell, "would remain the same if ... to substitute for ordinary motion to and fro any ...ccession of oppositely directed conditions."

... **Characteristics of the Æther.**—Like other ...f matter, the æther is known to us only by its ...ies. Since the vibrations which are assumed to ...te radiant energy are transverse vibrations, that ...vibrations perpendicular to the direction along ...he energy is transmitted, it follows that the me... ... which these waves are propagated must pos... ...idity; i.e., must in so far be a solid. Thomson ...gned to it the density $9{\cdot}36 \times 10^{-19}$; so that, in ...ords, the mass of one cubic centimeter is only ...000000 000000 936 gram. This density, although

about the same as that of the atmosphere at the height of 340 kilometers, is yet enormously great as compared with that which air would assume in interstellar space. The rigidity of the æther, according to the same authority, is approximately one thousand-millionth of that of steel; so that masses of ordinary matter can pass through it readily. In its structure the æther is assumed to be continuous and unlike that of ordinary matter, which is granular. As a whole, perhaps it may be regarded as "a perfectly continuous, subtle, incompressible substance pervading all space and penetrating between the molecules of all ordinary matter, which are embedded in it and connected with one another by its means" (Lodge). Through this æther are constantly passing in all directions the vibrations which we may assume to constitute radiation; these vibrations or waves being of all possible lengths and having all conceivable amplitudes.

351. Mechanism of Radiation.—The phenomenon of radiation involves two correlative processes, called respectively **emission** and **absorption**. The former term refers to the communication of disturbances to the æther, the latter to the reception of disturbances from it. Since the intermolecular spaces in all matter are filled with this æther, it is clear that the vibrating molecules must communicate their motion to it; and that thus periodic disturbances must be originated in it, which are propagated through it with a speed depending upon its density and its rigidity. Conversely, these æther-disturbances, on reaching a body, may communicate their energy to its molecules, and thus increase the total energy which the body possesses.

It should be observed, however, that it is to the energy of vibration of the molecules, rather than to their energy of translation, that the energy of the resulting æther-waves is due. Any increase in the total vibratory molecular energy of a body increases its total radiation; while any increase in the rapidity of the vibrations increases of course the number of disturbances produced

in a unit of time, and therefore affects the wave-frequency. There is a considerable analogy, consequently, in this respect, between the phenomenon of radiation and that of the propagation of sound in air.

352. Wave-length and Wave-frequency.—Theory, at present, can assign no limit to the possible length of ether-waves. The longest wave yet recognized is 30μ or 3000×10^{-6} centimeter (Langley); the shortest $0{\cdot}185\mu$ or $18{\cdot}5 \times 10^{-6}$ centimeter (Cornu). These values correspond to vibration-frequencies (210) of 10 million million and 1622 million million, respectively.

Radiant energy may or may not affect the eye and produce vision. Investigation teaches us that those waves which are competent to excite the optic nerve are comprised within somewhat narrow limits, the longest according to Angström being $7{\cdot}6 \times 10^{-5}$, and the shortest $3{\cdot}9 \times 10^{-5}$ centimeter; corresponding to vibration-frequencies of 392 million million and 757 million million, respectively. That portion of radiant energy whose vibration-frequencies lie between these limits is called light. Its phenomena differ in no respect from those of radiation in general, and hence need not be separately studied.

353. Color.—Evidently between the limits of vibration-frequency above mentioned, there must be an indefinitely large number of radiations competent to produce vision. Each of these separate vibration-frequencies represents a particular kind of light; and therefore the number of kinds of light must also be indefinitely great. The eye recognizes many of these separate vibration-frequencies as distinct. Each of them produces a sensation peculiar to itself which is called the sensation of **color.** When the eye recognizes the color of a flame containing salt as yellow, it does so simply because æther-vibrations impinge upon the retina having vibration-frequencies of about 508 and 510 million million, respectively. Color is defined objectively, therefore, simply by the vibration-frequency required to produce it. It is consequently analogous to pitch in music. Moreover, it is found that

waves of the minimum vibration-frequency produce the sensation of red, and those of the maximum frequency that of violet. So that if waves of intermediate frequency be made to enter the eye, in the inverse order of their vibration-frequencies, we shall perceive the colors red, orange-red, orange, orange-yellow, yellow, green, blue green, cyan-blue, blue, violet-blue, violet. Such a succession of colors, shading insensibly from one to the other, is called a **spectrum**. The following table gives the wave-lengths, in centimeters in air, and the vibration-frequencies, for the colors above mentioned. (Rood, Thomson.)

CONSTANTS OF LIGHT WAVES.

Color.	Vibration-frequency.			Wave-length.
Ultra red........	370	million	million	·00008100
Red	428	"	"	·00007000
Orange-red	483	"	"	·00006208
Orange..........	502	"	"	·00005972
Orange-yellow....	510	"	"	·00005879
Yellow	516	"	"	·00005808
Green......... .	569	"	"	·00005271
Blue-green..	590	"	"	·00005082
Cyan-blue........	604	"	"	·00004960
Blue.............	634	"	"	·00004732
Violet-blue.......	684	"	"	·00004383
Violet.	739	"	"	·00004059
Ultra-violet......	833	"	"	·00003600

If a given portion of light be made up of waves all of the same length, it is called simple, homogeneous, or monochromatic light. If a portion of light contains a mixture of waves differing in length, it is called compound light; and if the wave-lengths present are distributed with considerable uniformity through the spectrum, the compound light is white. Ordinary white light therefore is only a mixture of long and short æther-waves. Obviously, colored light which is simple and colored light which is compound may produce the

same color-sensation and therefore may not be distinguishable from each other by the eye.

Again, since white light contains a variety of colors these colors may conveniently be regarded as balanced in pairs. Thus if we assume that the colors making white are red, yellow, green, and blue, for example, we observe that if yellow be abstracted from white, the remaining light is blue. If red be removed, green light is left. Hence green and red, yellow and blue, are called **complementary** colors; since the two colors of each pair taken together make white. But white may also result from the simultaneous action of monochromatic pairs. Thus red and greenish blue, orange and cyan-blue, yellow and ultramarine blue, greenish yellow and violet, are in this sense complementary colors. So green and purple are complementary, though purple is a compound color. Sunlight, the type of white light, contains in 1000 parts, according to Rood, 54 of red, 140 of orange-red, 80 of orange, 114 of orange-yellow, 54 of yellow, 206 of greenish yellow, 121 of yellowish green, 134 of green and blue-green, 32 of cyan-blue, 40 of blue, 20 of ultramarine and blue-violet, and 5 of violet.

354. Change of Vibration-frequency.—Fluorescence.—Solutions of certain substances such as quinine and æsculin are observed to emit an opalescent blue light when placed in radiations of wave-frequency too great to excite vision. Stokes has shown that these substances are capable of emitting light of a less wave-frequency than corresponds to the energy which falls upon them, and hence have the property of rendering visible these invisible radiations. To this property Stokes has given the name **fluorescence**; a name derived from fluor spar, a substance possessing it to a marked degree. The organic substances fluorescein, eosin, and uranin are markedly fluorescent; as is also the hydrocarbon thallene, whose fluorescent spectrum has been described by Morton.

Becquerel, in studying the phenomena of phosphorescence, observed that in this case also the exciting

radiations are invisible. A phosphorescent su[illegible] is one which, after exposure to light, continues [illegible] in the dark. By means of his phosphoroscope, Becquerel measured with great accuracy the duration of the phosphorescence after insolation; and he found that in fluorescent substances the emission of light has a finite duration. Fluorescence and phosphorescence therefore must be regarded as identical in essence, differing from each other only in degree.

355. Form of Æther-motion.—Polarization.—If we imagine a single row of particles engaged in propagating a linear transverse wave (62), we may suppose that any one of these particles, while always vibrating perpendicularly to the direction of transmission, may describe any one of a variety of paths. In one case the particle may oscillate along a straight line parallel to the wave-front. In other cases it may oscillate in a circular or elliptic orbit whose plane is also parallel to the wave-front. If we suppose that all the particles of the above row, when concerned in propagating the wave, vibrate successively along lines lying in the same plane, the wave made up of such rows of particles vibrating in parallel planes is called a **plane-polarized wave** and the radiation is said to be **plane-polarized**. If, however, the particles all describe circles or ellipses whose planes are parallel to the wave-front, the radiation is said to be **circularly** or **elliptically polarized**. Moreover when the direction of motion of the particles is that of the hands of a watch, the polarization is said to be **left-handed**; and when it is in the inverse direction, the polarization is said to be **right-handed**.

We have discussed in Kinematics (58) the resolution and composition of simple harmonic motions. And from this discussion it is clear that if the path described by the vibrating particles be circular or elliptical in form, it may be resolved into two plane-polarized portions, the vibrations in these two portions being in planes perpendicular to each other, and the phase in one of these portions differing from that in the other by one quarter

of a period. So, conversely, two plane waves of equal period and amplitude, one a quarter of a vibration behind the other, may be compounded to produce a resultant circularly polarized radiation. If, however, the difference of phase be more or less than a quarter period, the radiation produced will be elliptically polarized. Moreover, in case the vibrations in the two planes be of variable amplitude and the periods be neither commensurable nor constant, the radiation is ordinary æther-energy; and this, in the case of those vibrations which affect our organs of vision, is called common light. Clearly, therefore, light-energy in the form in which we are most familiar with it is highly complex in its character. It is made up, not only of an indefinite number of waves of different lengths, but also of an indefinite number of modes of vibration assumed by these waves. The monochromatic light of a sodium flame plane-polarized constitutes one of the simplest forms of radiant energy. The white light emitted by an incandescent solid constitutes one of the most complex forms.

356. Energy of Radiation.—The kinetic energy of a moving mass is proportional to the square of its speed; and in the case of radiant energy as in that of sound, this energy is proportional to the square of the speed with which the vibrating particle passes its position of equilibrium. Further, since this speed determines the amplitude, it is also proportional to the square of the amplitude of the wave. Moreover, as in all motions of vibration, the energy is all kinetic at its middle point and all potential at the extreme points of the excursion. The mean or average energy of a vibrating particle, therefore, if a be the amplitude and $a\omega$ the maximum speed, is found to be $\frac{1}{4}ma^2\omega^2$; or since $\omega = 2\pi/T$, the energy is $m\pi^2a^2/T^2$, and the energy per unit time is $m\pi^2a^2/T^3$, to which the activity is proportional. Since $a^2\omega^2 = s^2$, $\frac{1}{4}ma^2\omega^2 = \frac{1}{4}ms^2$; or the mean kinetic energy of a vibrating particle is one half of its maximum kinetic energy.

SECTION II.—SOURCES OF RADIATION.

357. Matter-vibration.—Whatever can excite periodic disturbances in the æther may evidently be a source of radiation. A candle-flame, a heated stove, an electric spark, may originate such æther-vibrations, which being perceived by our senses are interpreted either as heat or light. Since the emission of radiant energy by a body is simply the production of æther-disturbances by its vibrating molecules or atoms which this æther surrounds, and since absorption of this energy is simply the production of these vibrations again by the æther-displacements, it is clear that in general the sources of radiation are to be found in the molecular or atomic vibrations of ordinary matter.

358. Effect of Temperature.—Let the temperature of a platinum wire be gradually raised by means of an electric current. At first, its radiations are capable of affecting the nerves of general sensation only; i.e., of producing heat. But at a temperature of about 525°, the eye perceives the wire as a dark red line, the radiations becoming now just visible and the wave-frequency having the minimum value necessary to this end. As the temperature continues to rise, the wire appears successively orange, yellow, and finally white, when it is said to be white hot. That is, wave-frequencies of higher and higher orders are continually added, the light emitted by the wire being the sum of them all; until finally, when all the wave-frequencies of the spectrum are emitted by the wire, the light reaches its maximum of complexity. In proportion, then, as a body is heated, do the wave-frequencies which it emits increase, both in number and in amplitude: shorter and shorter waves being successively added, while those previously emitted acquire an increased amplitude.

359. Theory of Exchanges.—Prevost's Law.—When a heated body is placed in the vicinity of another body of lower temperature, the former loses and the latter

ns heat by radiation until both have acquired the e temperature. But this is not a complete statet of the case. Prevost, about the close of the last tury, suggested that all bodies continually radiate , whatever their temperature, provided this temperabe above the absolute zero. According to this ry, the hotter body radiates to the colder one and colder body also radiates to the hotter one; but as former, owing to its higher temperature, radiates the latter more than it receives from it, its temperae falls. While, as the latter receives more energy n it radiates, its temperature rises. This theory (1) t at all temperatures above zero all bodies produce er-disturbances; and (2) that in consequence all lies are continually exchanging radiations, is known Prevost's **theory of exchanges**.

300. Radiating and Absorbing Power.—Suppose a mometer, having its bulb silvered, to be placed in an losure of any form, whose walls are covered with pblack on their interior surfaces, and which is mained at a temperature of 100°. The thermometer will cate 100° in any part of the enclosure; and by the ry of exchanges, since its temperature is constant ust radiate as much heat as it receives. Suppose her that of the entire radiation emitted by the lampk and received upon the thermometer-bulb 80 per is reflected. This leaves only 20 per cent to be abed in order to heat the bulb. But the bulb is not g in temperature; and hence it must radiate 20 per of the energy falling upon it; i.e., the radiation the absorption must be equal. Again, let the bulb covered with lampblack, a substance of minimum ecting power. Since the reflection is practically ing, the absorption must be 100 per cent. But to ntain the temperature constant, the radiation in this also must equal the absorption. We see then (1) absorption and emission of radiant energy go hand hand, good absorbers being good emitters and *vice a*; and (2) that good reflectors are bad radiators.

Experiments.—1. Take two plates of iron, one [illegible] and the other untinned, and solder to the center of each [illegible] a short piece of stout copper wire normal to the surface. [illegible] plates up ten or fifteen centimeters apart, and attach a [illegible] means of wax to the end of each rod. Place a highly [illegible] iron ball between the plates and observe that the marble [illegible] to the black plate drops off first, showing that this plate [illegible] better absorber.

2. Coat one bulb of a differential thermoscope with lampblack by smoking it, and cover the other with tinfoil. Place between the bulbs and equidistant from them a tin cube containing boiling water, one of whose faces is also smoked. If the smoked side of the cube face the metal-covered bulb of the thermoscope, the liquid column will remain stationary, the greater radiation from the former being compensated by the less absorption of the latter.

361. Quantity and Quality of Radiation.—By **quantity** of radiation is understood the total heating effect of the radiant energy as measured by the rise of temperature of a surface such as lampblack which absorbs it entirely. **Quality** of radiation refers to specific differences existing in it, such as wave-length and polarization, even when its total heating effect is the same. When two radiations are the same in quality, the mixture of wave-lengths in the one is the same as in the other, and the polarization is the same for both. The word **intensity**, as applied to radiation, is intended to apply to the ratio of quantity to surface; i.e., to the radiation per unit of surface.

Thus a beam of direct light may have the same intensity as one of reflected light; or one of pure green light the same as one of pure yellow light; as measured by their effects produced on absorption. But they differ obviously in quality since in the first case the reflected beam is polarized, at least in part, while the direct beam is not; and in the second case the radiation in [illegible] two beams differs in wave-length.

Now the experiments of Balfour Stewar[illegible] others have shown that the law of exchanges hold[illegible] for quantity and for quality of radiation. S[illegible], for instance, that the thermometer-bulb referr[illegible] above be covered with a layer having selective absorption; i.e.,

absorbing radiations of a particular wave-length only. Since under these conditions the thermometer-temperature remains constant, this experiment proves that its emissive power for this particular wave-frequency must be the same as its absorbing power for the same radiation.

EXAMPLES.—Thus, for example, a piece of green glass appears green because it absorbs the complementary color red, from the white light incident upon it. If, therefore, it be a good absorber of this particular quality of light, it should be a good radiator of the same quality, under suitable conditions. And it is found that a piece of green glass heated to a high temperature and viewed in the dark emits a light which is distinctly reddish. For the same reason a colored glass loses all its color when placed in a coal fire. For the color which it emits, being that which it absorbs, must be complementary to that which it transmits. And hence the sum of the emitted and the transmitted light must be white.

Again, a plate of tourmalin is known to absorb all radiations whose vibration-plane is perpendicular to the crystal-axis. Stokes has shown that if such a plate be placed in an iron bomb and heated to a high temperature, the light which it emits, when viewed through an aperture made for the purpose, is polarized in this principal plane; the quality of the radiation now emitted being the same as that of the radiation previously absorbed. It follows that if through an opening on the opposite side ordinary light be sent through the plate, the emergent light will be composed of a transmitted and of an emitted portion, both polarized, their polarization-planes being at right angles; consequently this emergent light will be itself unpolarized.

The truth of the law of exchanges as to quantity of radiation is readily shown by heating a piece of platinum foil having an ink-mark upon it, as suggested by Tait. The black ink-mark is a better absorber than the platinum at ordinary temperatures. But if the foil be heated till it emits light, the better absorber at the low temperature will become the better radiator at the high temperature and the mark will appear brighter than the surrounding platinum. So a chalk-mark on a poker becomes darker than the poker when highly heated. A black pattern on a white earthenware tile is reversed when the tile is highly heated, becoming a bright pattern on a darker ground, the quantity of emission being proportionate to the quantity of absorption.

362. Intensity of Radiation.—The intensity of radiation is the amount emitted or received per unit of

surface. It is therefore a function (1) of the distance which separates the surface from the source of radiation and (2) of the inclination of the surface to the radiant beam. The following laws express these relations:

1st. The intensity of the radiation which falls upon any surface varies inversely as the square of the distance between this surface and the source of the radiation.

2d. The intensity of the radiation received upon or emitted by any surface is proportional to the cosine of the angle which a normal to the receiving or emitting surface makes with the direction of the radiation.

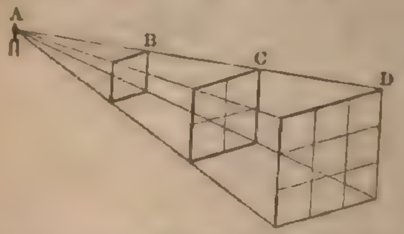


FIG. 123.

To illustrate the first law, let a candle be placed at *A* (Fig. 123) and let a screen with an opening a decimeter square be placed at *B*, one meter from it. If now a sheet of cardboard be placed at *C*, two meters from the candle, the section of the beam at this point will be two decimeters on a side, and its area will be four square decimeters. So, at *D*, the section of the beam will be nine square decimeters, evidently. The areas of the illuminated surfaces at distances 1, 2, and 3 meters from the candle are then 1, 4, and 9 square decimeters; and since the amount of light spread over these surfaces is the same, being the light from the candle, it follows that the intensity of this light, or the amount of light falling upon unit of surface in each case, will be in the ratio $1 : \frac{1}{4} : \frac{1}{9}$. Hence this experiment proves the fact that the intensity varies as the inverse square of the distance.

The same fact may be proved for invisible radiations by the thermopile (364), in the following way: Let a shallow tin box *BC* (Fig. 124), whose face is covered with lampblack, be filled with boiling water, and let a

thermopile A, provided at one end with a cone, be exposed to its radiations at one decimeter distance. Suppose the galvanometer-deflection to be noted. Let the thermopile be moved to A', a distance of two decimeters, and suppose the deflection to be again noted. It will be

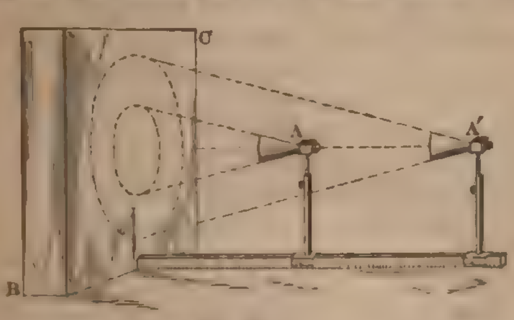


FIG. 124.

found to be the same as before. Obviously in the second case the radiating surface is the base of a cone of twice the height and hence of four times the area. Since, therefore, this fourfold radiation produces the same effect at twice the distance, it is evident that the intensity of this radiation must vary inversely as the square of the distance.

The second law is illustrated by the familiar fact that if an incandescent surface, such as a red-hot iron plate, be viewed through an aperture too small to allow the whole of it to be seen, it will appear equally bright what-

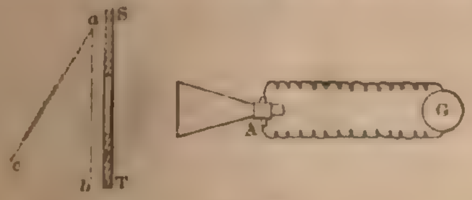


FIG. 125

ever the angle of inclination to the line of sight. To prove the law true for radiation in general, place the thermo-galvanometer AG (Fig. 125) in front of the screen

ST, behind the opening in which is a heated surface. Note the deflection of the galvanometer-needle when this surface is in the perpendicular position ab. Then move it into the inclined position ac and again note the deflection. It will be found to be the same as before. Since the surface of emission is increased as the plate is inclined, it is necessary in order that the total emission itself may not be increased that the radiation should diminish in proportion as the surface increases. Now the surface ac, as seen through the screen, is to ab, as unity is to the cosine of the angle of inclination. Since the surface varies inversely as the cosine of the inclination, the radiation must vary directly as this cosine. In other words, the emission from any surface or the radiation received by it, other things being equal, is proportional to the cosine of the angle between the initial and the final positions of the surface.

EXPERIMENT.—Provide a metal cube B (Fig. 126) one of whose faces is covered with lampblack. Fill it with boiling water and place it as shown, so as to radiate normally upon the face of the

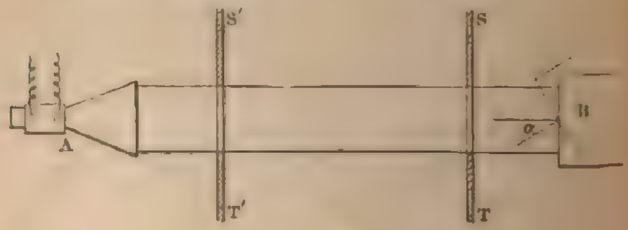


FIG. 126.

thermopile A, the size of the beam being regulated by the screens ST and $S'T'$. Note the galvanometer deflection and then rotate the cube about the center of the lampblacked face into the position shown by the dotted lines, the normal to the surface moving over an angle α. No change will be observed in the galvanometer deflection. But here again the radiating surface has been increased in the ratio of the cosine of the angle of rotation to unity, and hence the intensity of the emitted radiation must have been decreased in the same ratio, in order that their product should be constant, as the experiment shows it to be.

Since the intensity of radiation is simply the quantity itted or received per unit of surface, its value is tained by dividing the whole quantity of the radiation the whole surface. Or calling Q the quantity radiad, S the area of the emitting surface, and I the ensity, we have $I = Q/S$. Evidently if the surface reases so as to become $S/\cos\alpha$, the intensity will come $I' = Q\cos\alpha/S$; whence $I : I' :: 1 : \cos\alpha$. Or intensity is proportional to the cosine of the angle inclination. Obviously when $\alpha = 0$, cosine α will be ity; and the emission, being now normal to the surce, reaches its maximum. Evidently this law of the sine can be strictly true only for substances which e lampblack possess practically no reflecting power.

363. **Measurement of Radiation.**—The total energy radiation is measured by totally absorbing it and termining the total heating effect produced by it. Of the substances used for this purpose, lampblack has en found to be the most efficient.

364. **Thermogalvanometer.—Bolometer.**—In the lier experiments on radiation, the receiving instrument s simply a thermometer the bulb of which was coated th lampblack. Leslie (1804) used in his researches a ferential thermometer having one of its bulbs thus ated. But Nobili and Melloni (1833) showed that by mbining a thermopile with a galvanometer, an instrunt can be constructed far more delicate than any rmometer for measuring temperatures. The thermoe (632) consists of a bundle of metallic bars, generally bismuth and antimony, connected together alternately their ends, each pair being insulated from its fellows a slip of mica. When one end of this bundle has a her temperature than the other a difference of ential is developed between the first and last bars ich is proportional to the number of bars and to the perature-difference; so that on joining these ends by onductor an electric current is produced the strength which may be determined by means of a delicate vanometer. If the faces of the thermopile be

covered with lampblack, so as to absorb the energy radiated to it, the heat of the hand several feet away may be made to move the galvanometer-needle through twenty or thirty degrees. It was with this apparatus that Melloni made his classic researches upon the laws of radiation.

Boys (1887) increased greatly the sensitiveness of this apparatus by uniting in one instrument the thermopile and the galvanometer. His radio-micrometer consists of a square circuit one centimeter on a side, three of whose sides are made of fine copper wire and the fourth of a compound bar of bismuth and antimony, soldered end to end, each piece being five millimeters square and one sixth of a millimeter thick. This circuit is supported by a thin rod carrying a mirror, the whole being hung by a torsion fiber in the field of a strong magnet. When radiation falls on the bismuth-antimony junction a current is generated and the circuit is deflected. With a field of strength 100 in C. G. S. units, the instrument showed the radiation from a candle at 350 meters distance. With an improved construction he estimated that the instrument would indicate the one-hundred-millionth of a degree.

In 1881 Langley contrived another instrument of very great delicacy for measuring radiations, which he called a **bolometer**. Two strips of platinum or of iron 10 mm. long, from 0·01 to 0·001 mm. thick, and from 0·4 to 1 mm. wide are made two of the sides of a Wheatstone electric balance. So long as both are equally heated, the balance is in equilibrium; but when one of the strips is exposed to a source of radiation from which the other is protected the resistance of this strip increases, the balance is disturbed, and the galvanometer-needle is at once deflected. In this way 0·000001 of a degree may be indicated and 0·00001 of a degree may be measured with accuracy. These delicate strips are coated with lampblack and are enclosed in an ebonite cylinder. By using four such strips placed in the four arms of the Wheatstone balance, and by allowing the

radiation to fall only on the alternate ones simultaneously, von Helmholtz has considerably increased the delicacy of this ingenious instrument.

365. Measurement of Luminous Radiations.—Photometry.—Total radiant energy is measured either in calories (gram-degrees) or in ergs. Light, however, is measured only in terms of some secondary unit arbitrarily selected; such as a candle or a Carcel lamp. The standard candle in use in England and in this country is made of spermaceti, is cylindrical in form, $\frac{7}{8}$ inch in diameter, and is of such a length that six weigh a pound. It burns 120 grains (7·776 grams) per hour. The Carcel burner, which is used in France, burns 42 grams of colza oil per hour. The light of one carcel is equal to that of about 9·5 standard candles. The unit adopted by the International Congress of Electricians at its meeting in Paris in 1884 is the light emitted by a square centimeter of melted platinum at the temperature of solidification. According to Violle, this unit is equal to 2·08 carcels or 19·75 standard candles. In 1889 this Congress adopted one twentieth of this unit as a practical secondary standard.

For comparing a given light with the standard unit, various forms of **photometer** have been suggested, all depending on the fact that the eye, while not able to compare together accurately two illuminations of different intensities, can pronounce with considerable exactness upon equality of illumination. For such a comparison two portions of a screen are illuminated, the one by the standard source of light, the other by the source which is to be compared with it; and the brighter light is moved away until the two illuminations on the screen are equal. Since the intensity of illumination diminishes according to the square of the distance between the light and the screen, the amount of light emitted by the two sources will be proportional to the squares of these distances. Thus if one of the lights be a centimeters from the screen and the other b centimeters, when the illuminations are equal, the quantity

of light emitted in the first case is to that emitted in the second as $a^2 : b^2$. If the former be a standard unit, the quantity of light given by the latter is b^2/a^2 standard units, either carcels or candles.

One of the simplest photometers is that proposed by Count Rumford. It consists simply of a rod of wood or metal, and a white screen. The rod R (Fig. 127) is so

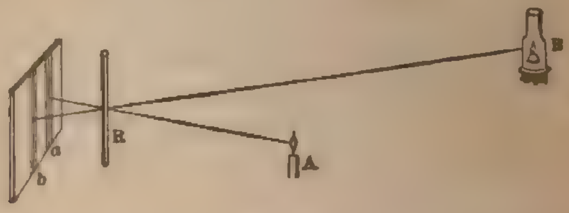


Fig. 127.

placed that its shadow b cast by the lamp B is near its shadow a cast by the standard A. By moving the brighter light away until the shadows are equal in intensity, and squaring the distances of the two lights from the screen, the value of the light emitted by the lamp is obtained as above. It will be noticed (Fig. 128) that the shadow b is illuminated only by light from A

Fig. 128.

and the shadow a only by light from B. By this device, we are able to compare together two portions of the screen each of which is illuminated by only a single source.

The photometer used in France was originally devised by Bouguer, but has been subseqnently modified by Foucault. A semi-transparent screen, of porcelain or ground glass, has an opaque partition placed at right angles to it, the lights to be compared being placed so as to illuminate each its own half of the screen. The partition may be moved to and from the screen so that the illuminated portions may be brought into contact and their intensities more accurately compared. When the brighter light has been removed until the illuminations are equal, the distances are measured. The observer is stationed on the other side of the screen from the lights and views the screen normally from a point opposite the line between the two illuminations.

The Bunsen form of photometer is the form more generally used in practice. Its important element is a disk of paper, one portion of which is made more translucent than another, originally by applying paraffin to it, but now more frequently by making it thinner. In this form the disk usually consists of three layers; the middle one, which is continuous, being of thin, and the two outer ones, which have a central star-shaped opening, being of thick paper. If such a disk be viewed by reflected light, the central spot will appear darker than the rest, since a portion of the incident light is transmitted by it, and so less light is reflected. If it be viewed by transmitted light the central spot will appear

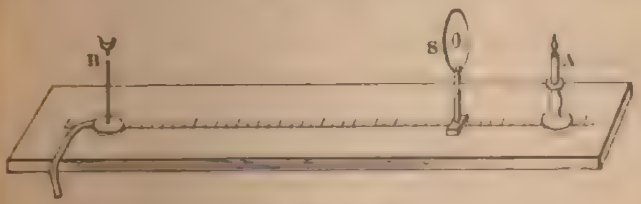


FIG. 129

brighter than the surrounding portions, since more light comes through it. If, therefore, a source of light B be placed on one side of the disk S (Fig. 129) and a

standard candle A on the other, their distances being adjusted until the central spot either disappears or appears alike on the two sides, the two lights are directly proportional in illuminating power to the squares of these distances. To facilitate simultaneous comparison of the two sides of the disk, two plane mirrors are placed behind it, so inclined that they reflect to the eye situated in front of its edge, these two sides at the same time.

366. Spectro-photometer.—A considerable difficulty arises when the lights to be compared are different in quality as well as quantity. If they differ in color, as when an electric arc light is to be compared with a standard candle, the most satisfactory results are obtained by limiting the comparison to two or more definite colors or wave-frequencies for each source. This may be done by interposing colored glasses between the source of light and the screen or disk. If a piece of red glass be thus interposed in each case, the result will be the relative illumination for this component of the original light. And if the experiment be repeated with green or blue glass, the relative illumination for these wave-frequencies may be ascertained.

A more accurate method, however, is to arrange the two beams of light to be compared so that they shall pass simultaneously through a prism; one of them through its upper and the other through its lower portion. Two closely superposed spectra will be thus obtained, one given by each source of light. And by limiting the field to any given color, by an adjustable slit, and then moving the brighter source away until the intensity is the same in the two portions, the data are obtained for a comparison of the lights for this particular color. Such an instrument is called a **spectro-photometer.**

367. Selective Radiation and Absorption.—Radiation, however, depends not only upon temperature, but also upon the nature of the radiating body itself and especially upon the condition of its surface. Leslie, in

his early experiments on radiation, made a hollow cube of tin plate, leaving one of its sides in its natural condition, roughening the second, and covering the third and fourth with lampblack and with white lead respectively. When filled with boiling water, he observed that the radiation from the roughened surface exceeded that from the polished metal, but was in its turn very far exceeded by that from the two other surfaces, which, though so different in color, had approximately the same radiating power.

Selective emission with rise of temperature is practically important, since for illuminating purposes it is desirable that the largest possible fraction of the energy expended shall be emitted in the form of visible radiations. Langley has shown that while only 2·4 per cent of luminous waves are contained in the radiation from a gas-flame, only 10 per cent in that of the electric arc, and only 35 per cent in that of the sun, the radiation of the fire-fly (*Pyrophorus noctilucus*) consists wholly of visible wave-frequencies. The distribution of the energy in the spectra of the three sources of radiation first named, according to Langley, is shown in Fig. 130. The total

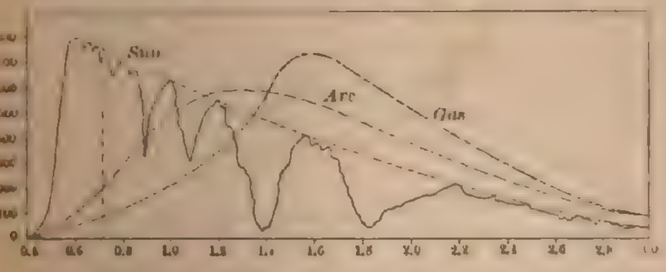


FIG. 130.

energy is the same for each of these curves, the portion which is luminous being represented by the area to the left of the vertical dotted line. The ordinates represent heat-energy and the abscissas wave-lengths in microns. Moreover, the researches of Nichols have proved that magnesium oxide and zinc oxide have a greater efficiency

as radiators than carbon; calling efficiency the ratio of the light-giving radiation to the total radiation. In the figure (Fig. 131) his radiation-curves are given for several sources of illumination, all for lights of the same candle-power, gas being taken as the standard of comparison. The ordinates represent multiples of the gas-value and the abscissas wave-lengths in terms of the spectrum. It will be noticed that while in the red (C) the flame of magnesium is but little more than half as strong as the gas-flame, in the blue (F to G) it is about ten times brighter; and further, that everywhere beyond the yellow except in the extreme violet, it surpasses even the electric arc. The radiant efficiency of magnesium has been recently determined by Rogers to be as high as 13·5 per cent.

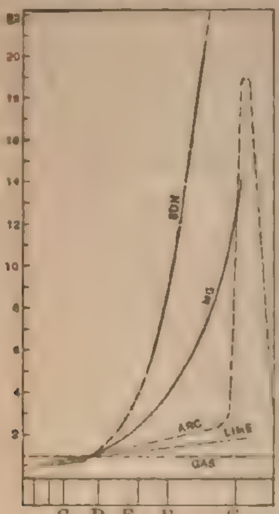


FIG. 131.

The absorptive power of lampblack is general and not special. The fact that it is black proves that it absorbs all luminous radiations alike. This also it appears to do for invisible radiations, although Langley has shown that it transmits certain radiations of great length. Hence by the law of exchanges lampblack should radiate waves of all lengths, when incandescent; and this it is found to do.

But there are other substances which exercise not a general but a special or selective absorption upon the radiations which fall upon them, reflecting or transmitting the rest. This is the cause of the color of pigments. Red lead, Prussian blue, chrome-yellow, Paris green exert selective absorption upon light, and reflect to the eye the color complementary to that absorbed. Red, green, yellow, and blue glass also absorb selectively; but here the complementary color is trans-

mitted, not reflected. In all cases, the sum of the reflected, the absorbed, and the transmitted portions must equal the initial incident radiation. Melloni observed that bodies act in the same way with reference to invisible radiations. Thus using four sources of heat, a Locatelli lamp, a platinum spiral, a copper plate at 400°, and a cube at 100°, rock salt was found to transmit the same proportion of the incident radiation, whatever the temperature of the source; thus acting toward invisible radiations as it acts toward visible ones, and exercising no selective absorption upon either. Plate glass, however, though equally transparent, transmitted only 39 per cent of the radiation from the lamp, 24 per cent of that from the heated spiral, 6 per cent from the copper plate at 400°, and none of that from the copper cube at 100°. Alum transmitted only 9 per cent of the radiation from the lamp, 2 per cent of that from the wire, and none from either of the other sources. Hence alum absorbs selectively the longer invisible radiations as green glass does the visible ones. Although black glass transmits only 26 per cent of the lamp-radiation, it transmits 55 per cent of that of the platinum wire.

368. Loss of Energy by Radiation.—Law of Cooling.—For a given body, the total emission is evidently a function of its temperature only. Suppose a thermometer whose bulb is covered with lampblack, to be enclosed in a spherical chamber exhausted of air, having its walls blackened interiorly. If the absolute temperature of the thermometer be T_t and that of the walls of the chamber be T_c, then since the cooling effect will be the difference between the radiation emitted and that received—both of which are functions of the respective temperatures—we have for this effect $f(T_t) - f(T_c)$. Newton supposed that a cooling body loses an amount of heat which is proportional simply to the difference of temperature between it and the surrounding medium. Dulong and Petit, however, have shown that an expression of the form $f(t) = Aa^t + B$ represents satisfactorily the results of their experiments, in which A and B are constants,

the former depending on the nature of the surface only, and in which a, which is practically the same for all bodies, has the value 1·0077. Hence for the cooling effect of the thermometer above

$$f(T_t) - f(T_c) = Aa^{T_t} - Aa^{T_c} \text{ or } Aa^{T_c}(a^{T_t - T_c} - 1). \quad [57$$

Substituting the value of a, we have $A(1{\cdot}0077)^{T_c}(1{\cdot}0077^{T_t - T_c} - 1)$. It therefore appears (1) that for a given temperature-difference, the cooling effect is proportional to $(1{\cdot}0077)^{T_c}$; and (2) that for a given temperature of the enclosure-walls, the cooling effect is proportional to $(1{\cdot}0077)^{\theta} - 1$, where θ is the temperature-difference. Hence the speed of cooling of a thermometer in vacuo for a constant excess of temperature increases in a geometrical progression when the temperature of the surrounding medium increases in an arithmetical progression, the ratio of this progression being the same whatever be the excess of temperature. Obviously these results can be approximate only since they ignore the composite nature of the radiations themselves. As to the absolute value of the constant A the experiments of Hopkins give as the value for glass about 0·126 gram-degrees per second per square decimeter of surface, whence it follows that the radiation from glass at 100°, the enclosure being at 0°, would be about 16·8 gram-degrees per second per square decimeter.

369. **Law of Stokes.**—According to the law of Prevost, the radiating power and the emitting power of any substance are precisely equivalent, both as to the quantity and the quality of the radiation. In 1817, Fraunhofer noticed that in the spectrum of a candle-flame a double yellow line occupied the position of a double dark line, which he called D, in the solar spectrum. In 1849 Foucault passed sunlight through the electric arc and observed the double line D to be darker than before. He then reflected the light from the carbon points back through the arc, and noticed that then the spectrum contained dark lines in the precise posi-

tion of the bright lines given by the arc itself. These phenomena were explained by Stokes upon considerations flowing from the law of Prevost, and illustrated, in the case of sound, by sympathetic resonance. It is well known that if a musical note be emitted near an open piano-forte, those strings only will vibrate whose periods are the same as the components of the note. Suppose a series of strings or of tuning-forks, all tuned to the same note. If this note be sounded on one side of them they all will be thrown into vibration; and hence a listener on the other side will receive less sound in consequence of this absorption. If a number of notes each of different pitch were sounded simultaneously on one side of these strings, an observer on the other would hear all the sounds with their normal intensities, save that one only which is in unison with the strings. The strings therefore, in consequence of their taking up the air-vibrations of the same period, would appear to be opake to such sounds.

When sodium, for example, is heated until its vapor becomes luminous, it emits yellow light of wave-frequencies 509 and 510 million million respectively. Since the vibrations which it emits are precisely those which it absorbs, it follows that one sodium flame must be opake to light from another similar flame. And this is found to be the fact. If a small non-luminous gas-flame colored yellow by sodium be placed in front of a larger flat flame similarly colored, the edges of the smaller flame will appear smoky, upon the background of the larger one; since they are cooler than it, and hence absorb more of that particular light than they transmit. This point is evidently essential; for if the temperature of both flames is the same, both will radiate equally; and the smaller one will emit precisely the same amount of light that it absorbs from the other and larger one. If the smaller one has the higher temperature, it will evidently radiate more than it absorbs; if the lower temperature, it will radiate less than it absorbs and will appear dark.

But it is when the spectrum is examined that the

most satisfactory proof is given of the equality of radiating and absorbing power. The spectrum of luminous sodium vapor at a moderately high temperature consists only of the two yellow lines of the wave-frequencies above given; the same that were noticed by Fraunhofer in the spectrum of a candle-flame. The spectrum of white light, since such light is made up of an uninterrupted number of wave-frequencies, is of course continuous; i.e., consists of an indefinite number of lines, so that there is no break in the colors. Suppose now white light from an electric lamp be passed through a sodium flame before forming a spectrum. The sodium flame will absorb the vibrations of the frequencies 509 and 510 million million from the electric light, and the spectrum of the white light will appear without these constituents; in other words, there will be two gaps in it, corresponding to these vibration-rates. Fraunhofer observed these gaps in the solar spectrum, and Foucault succeeded in producing them at will in the spectrum of the electric arc.

Stokes in 1850 recognized as true for radiation what had long been known for sound. Hence the law of equivalence for emitted and absorbed radiations is called the **law of Stokes.** It was developed for invisible radiations by Stewart in 1858, and for visible ones by Kirchhoff in 1859. Defining the emissive power of a body for a particular wave-length as the ratio of that part of its radiation to the corresponding part of the radiation of a black body at the same temperature; and defining the absorbing power as the fraction expressing the portion of incident radiation of that wave-length which is absorbed at a given temperature, Tait enunciates the law of Stokes as follows: The emissive power of a body for any radiation is equal to its absorptive power for the same radiation at any one temperature. It follows from this law that to produce a dark line in a continuous spectrum by absorption, the absorbing body must be at a temperature so low that a black body at that temperature will not emit the particular radiation concerned as

strongly as does the source. For if the radiation from the source be R, that from the black body R' at the temperature of the flame, and r the absorbing or emitting power of the flame, then the flame will absorb a portion Rr of the radiation and will itself emit $R'r$. Whence the total radiation passing the flame is $R - Rr + R'r$, or $R(1 - r) + R'r$. Putting this in the form $R + r(R' - R)$ it appears that the flame will increase or diminish the radiation from the source according as R' is greater or less than R; that is, according as the radiation from the source is below or above that from a black body radiating at the temperature of the flame. Stewart admirably illustrated this by placing a piece of red glass in a coal fire. When its temperature is the same as that of the coal behind it, it loses all color. But when it is cooler than the coal, it appears red, and when hotter than the coal, it appears of the complementary color, green. It emits radiations of precisely the same quality as those which it absorbs.

370. Radiation from Solids and Gases.—We have already seen that radiation increases with temperature, from the absolute zero upward, and that this increase for a black body is partly increased amplitude in existing wave-lengths, partly a progressive increase in the upper limit of the wave-lengths, so that shorter and shorter waves are continually added. The experiments of J. W. Draper led him to the conclusion that the temperature at which luminous radiations are emitted is the same for all solid bodies and is about 525°; these radiations containing continually shorter wave-lengths as the temperature increases. He found that at 1400° the light radiated from a strip of platinum is thirty-six times that radiated at 1000°; and that the total energy radiated at 1300° is 17·8 times that emitted at 525°. Weber has shown that these conclusions require some modification. He finds that the first visible radiations of an incandescent lamp-carbon are emitted at about 390°, and are of a spectral gray color; the spectrum increasing equally toward the red and toward the violet. More-

over, the first radiations perceptible to the eye come from the same part of the spectrum that emits the maximum energy at the highest temperature. He notes that an iron plate begins to send forth gray light at 378°, a platinum plate at 391°, and a gold plate at 417°. Evidently the question of visibility is one largely dependent on the sensitiveness of the observer's eye.

As to the quality of the radiation from incandescent solids, Stewart has shown that bodies which are not black obey the laws connecting quantity and quality of radiation with temperature for a black body, provided they are sufficiently thick. And hence that the radiation from all solids at a high temperature contains an indefinite number of wave-lengths, and affords a continuous spectrum (358). Such are the radiations from the incandescent carbon of the electric light, those from a coal fire and those of the lime-light.

In the case of gases, however, owing to their simpler constitution, the phenomena of radiation are much simpler. The general absorption of light by air is trifling; and Melloni has shown that heated air has a correspondingly low radiating power. Gases, however, including vapors at high temperatures, exert for the most part a strong selective absorption. Thus a column of air 1600 meters long produces absorption-bands in a lime-light spectrum, corresponding to the solar groups *A*, *a*, and *B* (418); these groups themselves being produced, as Egoroff has shown, by the absorption of the oxygen in the atmosphere. Keeler has rendered it probable that the great band *Ω* observed by Langley in the ultra-red invisible spectrum is due to an absorption exerted by the carbon dioxide existing in our atmosphere. But by the law of Stokes, the radiations absorbed are those which the same substance emits; and hence a cooler gas should be opake to radiations from a hotter one. Keeler's experiments show that a column of carbon dioxide 3·4 meters long absorbs 36 per cent of the radiation from a Bunsen flame. And Tyndall has proved that a thirtieth of an atmosphere of

this gas cuts off the entire radiation from a carbon monoxide flame.

Moreover, Tyndall's researches show that the absorbing power of gases and vapors at ordinary temperatures for invisible radiations is very variable. Using the radiation from a copper plate heated to 270°, he found that at a pressure of only one thirtieth of an atmosphere, the absorption produced by ammonia-gas was over five thousand times, that by olefiant gas over six thousand times, and that by sulphur dioxide six thousand four hundred and eighty times that of air. Ammonia in a tube a meter long, while perfectly transparent to light, is absolutely black to radiations from copper at 270°. The vapor of acetic ether, under a pressure of only one thirtieth of an atmosphere, produces the same absorption as ammonia-gas at one atmosphere. And the vapor of boric ether absorbs the same radiations 186000 times more powerfully than air at the same pressure. The odors of plants, too, are powerfully absorptive. Even the aqueous vapor in the atmosphere, as the same author has shown, produces an absorption from 60 to 70 times as great as that of the air itself. Langley concludes from his Mt. Whitney observations that the atmosphere as a whole exerts a powerful selective absorption. If all the solar radiation were of the wave-length 0·0020 mm. the earth would receive 90 per cent of it; but if these radiations were of wave-length 0·0004 the earth would receive but 40 per cent of it. "The temperature of the earth's surface," he says, "is not due principally to this direct radiation, but to the quality of selective absorption in our atmosphere without which the temperature of the soil in the tropics under a vertical sun would probably not rise above − 200°." "The temperature of this planet and with it the existence not only of the human race but of all organized life on the globe appears, in the light of the conclusions reached by the Mt. Whitney expedition, to depend far less on the direct solar heat than on the hitherto too little regarded quality of selective absorption in our atmosphere."

This selective absorption of the atmosphere appears in the terrestrial lines of the visible solar spectrum, which are directly traceable, as we have seen, to its carbon dioxide, its aqueous vapor, and its oxygen. But the selective absorption which renders the earth habitable has to do with radiations of far too great length to affect the eye; radiations such as those from the soil at temperatures between 0° and 100°, whose wave-lengths, as Langley himself has shown, are probably not far from 0·015 mm.; being therefore twenty times as long as the longest wave-length visible to the eye.

The æther-disturbances which are produced by geseous vibrations, however, are not due to molecular but to atomic motions. That is, they do not originate in the motions of molecular translation assumed by the kinetic theory of gases, but in the intra-molecular vibrations of the constituent atoms. This is proved by the fact that these æther-disturbances, these radiations, are characteristic only of the atoms concerned. Thus if hydrogen be raised by an electric spark to a sufficiently high temperature, it emits a crimson light composed of radiations whose wave-lengths are ·000656, ·000486, ·000434, and ·000410 mm. Together these four lines constitute the visible spectrum of hydrogen; and these four wave frequencies, whether found in radiations of terrestrial origin or received from the sun, the fixed stars, or the remotest nebula, indicate the presence at the source of the radiation of the element hydrogen. In order to determine the presence of a chemical element anywhere, we have only to examine the spectrum given by the radiations from that source. A bit of salt heated in a Bunsen flame is vaporized and its vapor is raised to the temperature at which it becomes luminous. Its spectrum then consists of the two bright lines already spoken of, of wave-lengths ·0005895 and ·0005889 millimeter respectively. But these lines constitute the spectrum of sodium, the chlorine not becoming luminous at this temperature. This method for detecting chemical elements by means of the spectra given by

their incandescent vapors is called **spectrum analysis**, and will be more fully considered later.

371. Chemical Effects of Radiation.—Photography.—Besides the luminous and heating effects produced when radiation is incident upon matter, there are also chemical effects. These effects may be of the nature of combination or of decomposition. Thus under the action of light, chlorine unites with hydrogen and the carbon dioxide of the air is decomposed in the leaves of plants, the energy of the radiation appearing as stored energy in the plant. Certain of these chemical changes are exceedingly prompt, as in the case of the darkening of the silver halides by light; a fact underlying the art of fixing the image produced by the camera, called photography. It is found that while a great variety of substances are sensitive to light, the radiations which effect the reduction of the salts of silver are of short wave-length, lying in or beyond the violet of the spectrum. Eder, however (1881), showed that the addition of a minute proportion of certain coloring matters, such as eosin, for example, renders silver bromiodide sensitive to a much greater range of the spectrum. Photographic plates thus prepared are called **orthochromatic** plates; and by their means Abney has obtained photographs even of ultra-red invisible radiations. Tyndall has shown the power of the electric light to decompose the vapors of organic liquids such as amyl nitrite and allyl and isopropyl iodides.

372. Radiometer.—Radiophone.—In 1873, Crookes observed that when radiation is allowed to fall on the blackened surface of a disk of pith suspended in an exhausted glass tube, there is attraction when the pressure is above 7 mm. and repulsion when below it; this critical pressure varying for different substances. In 1875, he devised the **radiometer** for showing this action of radiation. It consists of a globe of glass, suitably exhausted, carrying a steel needle, upon the point of which rests a cap to which are attached the four arms of a vane or fly made of aluminum, these arms terminating in four disks or squares of mica blackened on one side.

When radiation falls upon the instrument, it is absorbed more completely by the blackened surface than by the unblackened surface; and so raises its temperature to a higher point. The molecules of the enclosed gas coming in contact with the former surface become more heated and so move away from it more rapidly than from the latter surface; thus causing a greater pressure on the blackened side, which, since this surface is movable, acts by reaction to move it away from the radiant source. At ordinary atmospheric pressure, this increase of pressure causes a flow of the molecules round the edge to the other surface; thus producing equilibrium. But as the globe is exhausted the mean free path of the molecules increases, until finally the heated molecules, in place of losing their motion to other molecules very near the blackened surface, impinge upon the walls of the globe and produce a counter-pressure there. Then the maximum effect occurs, the excess of pressure upon the blackened surface, according to Crookes, being 11.5×10^{-6} gram per square centimeter. Evidently this pressure will cause rotation of the fly, as the blackened surfaces all face in the same direction. So that the radiometer is an actual heat-engine in which the vanes act as the heater and the glass as the refrigerator. That this explanation is the true one was proved by Schuster (1876) by suspending the globe bifilarly and showing that it tended to rotate in the opposite direction to that of the fly.

Since the rapidity of the rotation is proportional to the intensity of the radiation, it may be used to measure this intensity; whence the name radiometer. Crookes showed that with a candle 50 cm. distant, a certain radiometer made one revolution in 182 seconds; while at 25 cm. it rotated in 45 seconds and at 12·5 cm. in 11 seconds; thus following the law of the inverse squares. With a candle at 12·5 cm. distance behind green glass, one rotation was effected in 40 seconds; with blue glass in 38 seconds, with purple in 28, with orange in 26, with yellow in 21, and with light red in 20 seconds.

By balancing this molecular bombardment against gravity, Crookes showed that the radiation of a candle at 15 cm. distance exerts a pressure of one hundredth of a milligram upon each square centimeter; so that if sunlight is equivalent to 1000 candles at 30 centimeters distance, the Crookes pressure exerted by it is about 22 metric tons upon each square kilometer.

Every heated surface therefore is surrounded by a layer of molecules actively engaged in bombarding external molecules away from it; the thickness of this layer being dependent on the exhaustion. Water upon a hot surface of iron, for example, is supported upon such a **Crookes layer**; it does not wet the iron but assumes what is known as the spheroidal state (320).

In 1880–81, Bell and Tainter showed that sonorousness under the influence of intermittent radiation is a property common to all matter. Using a beam of sunlight rapidly interrupted by means of a rotating perforated disk, sounds were obtained when the light fell on various substances enclosed in a conical receiver provided with a hearing tube. Lampblack gave the best effect. The instrument is called the **radiophone**.

Section III.—Propagation of Radiation.

A.—Rectilinear Propagation.

373. Speed of Propagation.—The speed with which a disturbance is propagated through any medium is directly proportional to the square root of the elasticity of this medium and inversely proportional to the square root of its density (155); i.e., $S = \sqrt{E/\Delta}$. In the case of radiation, however, we are obliged to invert this formula and to determine the properties of the æther from the speed of radiation independently determined. The methods which have been used for ascertaining the speed of light are four in number, two of them astronomical in principle and two physical.

I. **Roemer's method.**—In 1675, Roemer, a Danish astronomer, then an observer in the Paris Observatory,

noticed that the intervals between the successive eclipses of the inner satellite of Jupiter were shorter than the true value when the earth was approaching the planet and longer than this when it was receding from it. Careful study proved that this discrepancy between theory and observation was due to the time taken by the light to cross the intervening spaces; the distances increasing in length between successive eclipses when the earth was receding from Jupiter and diminishing when it was approaching it. The extreme difference was observed between conjunction and opposition; and this difference was 16 minutes 36 seconds. Obviously, therefore, this was the time required by the light to pass over the maximum distance; i.e., the diameter of the earth's orbit. Calling this 299 270 000 kilometers, we have

$$\frac{299\,270\,000}{16'\ 36'' \times 60} = 300470 \text{ kilometers per second.}$$

II. **Bradley's method.**—In consequence of the earth's motion in its orbit, certain stars appear displaced in advance of their real positions; just as rain which falls vertically when viewed from a train at rest, appears to come from a point in advance of the train when it is in motion. The angle of displacement is the greater, the greater the speed of the train, and its tangent is always the ratio of the two speeds. In 1728, Bradley, the English Astronomer Royal, not only observed this displacement of the stars but measured it accurately, assigned the true cause of it, and gave it the name **aberration** of light. The angular displacement from the mean position observed by Bradley was 20·26″ of arc; and this is called the constant of aberration. The tangent of this angle is of course the ratio of the speed s of the earth in its orbit to the speed of light S; i.e., $\tan \alpha = s/S$. Whence the speed of light is equal to the speed of the earth in its orbit divided by the tangent of the angle of aberration. Since the tangent of 20·26″ is about 0·0001, and the speed of the earth is about 30 kilometers per second, we have $S = s/\tan \alpha = 30 \div \cdot 0001$ or 300000 kilometers per second as the speed of light.

III. **Fizeau's method.**—In 1849, the speed of light over terrestrial distances was measured for the first time. The space traversed was that from Suresnes to Montmartre, near Paris, the distance being 8663 meters. Since a speed is a distance divided by a time, it remained only to determine accurately the time required to pass over this distance and back again. The device employed by Fizeau for measuring the time consisted of a revolving toothed wheel, whose rate of rotation could be accurately observed and which was placed in the path of the light. When at rest, the light passed outward through a space between two teeth to the distant station, where it was reflected back over its path, passing through the same space on its return. If now the wheel be rotated with a gradually increasing speed, a time will come when, during the interval required for the light to go from the first station to the second and back, the wheel will have revolved so far that a tooth will have taken the place of a space, and on its return the light will be eclipsed. Since what is true for any one space is true for all, it follows that when this speed of rotation is reached the returning light will be entirely cut off. Knowing this speed of the wheel and the number of teeth, the time required for a tooth to move into the position of a space may be calculated. But this is the time which the light takes to pass over twice the distance between the stations. The wheel used by Fizeau had 720 teeth, and of course therefore the distance from a tooth to the next space was 1/1440 of the entire circumference. It was found that the light was eclipsed when the wheel was revolving 12·6 times a second. One circumference was described consequently in 1/12·6 of a second and 1/1440 of a circumference in $1/(1440 \times 12{\cdot}6)$ or 1/18144 of a second. If the light moved over 17·326 kilometers in 1/18144 of a second, its speed is evidently $17{\cdot}326 \times 18144$ or 314000 kilometers. In 1874, Cornu repeated this experiment and obtained a speed of 300400 kilometers.

IV. **Foucault's method.**—In 1851, Foucault proposed a

much more delicate method of measuring time, in consequence of which he was able to measure with remarkable accuracy the interval required for light to pass over a distance of only four meters. This method was based upon the use of a revolving mirror M (Fig. 132) upon which the light from the opening O fell, and by which it

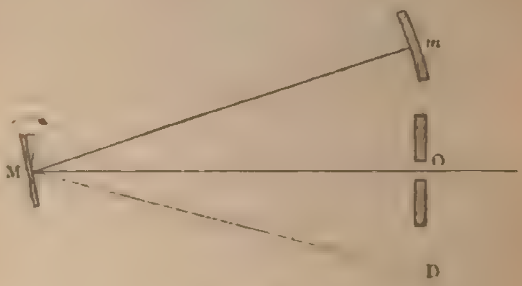


FIG. 132.

was reflected to a fixed concave mirror m. When the revolving mirror is in the proper position and at rest, the light returned from the fixed mirror is reflected to the starting point. But if it is in motion, so that during the time required by the light to go from M to m and back, it has moved into the position shown by the unshaded section, the light will be reflected not to O but to D. Knowing the speed of the mirror, the time required to produce the deviation OMD is easily calculable; and as this is the time during which the light moves over twice the distance mM, the speed of light may thus be ascertained. In 1862, Foucault made the experiment and obtained 298574 kilometers as the speed of light.

In 1878, Michelson improved the Foucault apparatus by employing a plane fixed mirror; thus permitting the use of any distance between the mirrors. The arrangement of his apparatus is shown in the figure (Fig. 133), in which S is a division of a scale ruled on glass, M a revolving plane mirror, L an achromatic lens, and S' a fixed plane mirror. The point S is so situated that its

age S' reflected in the mirror M is in one focus of the ns L, while the image of S' coincides with the mirror ″, which is placed at the conjugate focus. When M rns slowly, the light from S'' is reflected back through e lens, the image formed coinciding with S. When the irror rotates rapidly, however, the position of M will ave changed while the light travels from M to S'' and

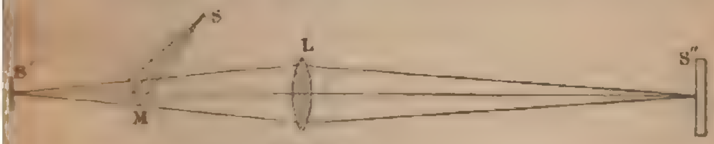


FIG. 133.

back again, and the image will be displaced in the direction of rotation of the mirror. As finally arranged the revolving mirror was 8·58 meters from the slit, and the fixed mirror 605 meters from the revolving one. With a speed of 257 rotations per second, the observed deviation was 113 millimeters. This gives 299992 kilometers for the speed of light in air and 299910 kilometers in vacuo. A second set of experiments made in 1882 gave for the speed of light in vacuo 299853 kilometers.

Newcomb in 1882 made a similar series of experiments, using as the revolving mirror a square prism of steel with polished faces. The distance between the two mirrors was 3720 meters. His result is 299860 kilometers, for the speed of light in vacuo.

SPEED OF VISIBLE RADIATION IN VACUO.

Experimenter.	Method.	Year	Place.	Speed.
Foucault.......	Foucault	1862	Paris	298574
Cornu..........	Fizeau	1872	Paris	298731
"	"	1874	"	300028
Listing.........	(Calculated from above)			299990
Michelson......	Foucault	1879	Annapolis	299910
"	"	1882	Cleveland	299853
Newcomb......	"	"	Washington	299810
"	(Using selected determinations)			299860

The latter number $2{\cdot}99860 \times 10^{10}$ centimeters per second may be accepted as the most probable value for the speed of light in vacuo.

These physical methods have been so much improved in recent years that their result may be accepted with entire confidence. And hence the speed of light thus obtained, when combined with an accurate determination of the time required for it to cross the earth's orbit, or with the aberration-constant, may be expected to furnish a more exact determination of the sun's distance than can be obtained directly by any purely astronomical measurement.

The speed in vacuo of all waves is the same and is therefore independent of the wave-length. Were this not the fact, then, inasmuch as the light of different wave-lengths is different in color, it is evident that one of the satellites of Jupiter, for example, on reappearing from behind the planet after an eclipse, would be seen first of the color which traveled fastest, the other colors subsequently appearing. Nothing of this sort being perceptible, the inference is that in space all waves have the same speed, whatever their length.

Lord Rayleigh has pointed out that in cases where the speed of wave-propagation depends upon the wave-length, there is a difference between the speed of a single wave and that of a group of waves. While the aberration method above described would give the actual wave-speed, the method founded on the observation of Jupiter's satellites and the method of Fizeau would give the group-speed; and the method of Foucault would give a value proportional to V^2/U, in which V represents the wave-speed and U the group-speed. As no such differences have been actually observed, the conclusion is that in air as in vacuo all waves have the same speed whatever their length.

374. Rectilinear Propagation.—Radiations in the same medium whether visible or invisible are propagated only in practically straight lines. But how is this accounted for on the wave-hypothesis? If radiant energy

nsist of the motion of material particles shot out from e radiating body, then obviously these particles should ove in straight lines. But if radiation is a wave-otion in an elastic medium, why does not this motion pread in all directions; just as sound does, which is a ave-motion? A sheet of paper intercepts the light from candle if placed between it and the eye; but it does ot perceptibly diminish the sound of a bell, when held etween it and the ear. The importance of this question ppears from the fact that Newton considered it fatal to he hypothesis of wave-motion, and Huyghens, the father f the wave hypothesis, was unable to explain it.

In the section on composition of wave-motions (60), it was shown that the disturbance at any point in an elastic medium is to be considered the resultant of all the wave-motions simultaneously acting there. Moreover, it has been shown that when two wave-fronts act simultaneously upon a particle, this resultant is dependent also upon the phases in which the waves reach this particle. If the waves are equal in amplitude, the resultant will be either zero or a wave of twice this amplitude, according as the two waves are in opposite phases or are in the same phase. Suppose, for example, the wave-front AB (Fig. 134) is advancing from left to right. We may in-

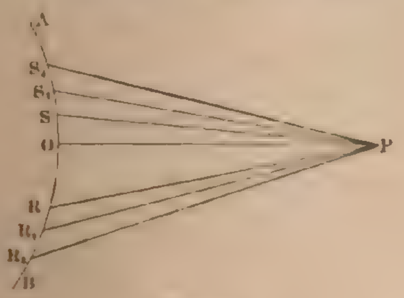


FIG. 134.

quire what portions of this wave will be effective at the point P. Draw the lines PO, PS, PS_1, PS_2, and also the lines PR, PR_1, PR_2, so that each of these lines differs

from the preceding one by half a wave-length. Then the radiation from S will be opposite in phase to that from O, that from S_1 will be similar in phase, that from S_2 opposite, and so on. And in general the radiation from the portion SO of the wave-front will be opposite in phase to that from the portion SS_1, and that from S_1S_2 similar to it. The same will be true for the portions RR_1 and R_1R_2. Since the effectiveness of the radiation from any portion of the wave-surface is proportionate jointly to the area of that surface and to the angle of emission, then as the elements of surface are made smaller, and as these elements are farther and farther from O, it is clear that ultimately the oblique radiations from the wave-front will mutually destroy each other; leaving only the portions near O to act upon the point P. In other words, the effective waves are those only which are propagated along the normal to the wave-front.

The construction which we have now given becomes possible only when the wave-length is very small in comparison with the wave-front. So that as in the case of sound, the greater wave-length results in the greater lateral divergence of the wave and sound-shadows are quite ill-defined.

375. Formation of Shadows.—One of the results of the rectilinear propagation of radiation is the production of shadows; a shadow being defined as the space behind an object which is itself incapable of transmitting the radiations. Such an object, with reference to luminous radiations, is called **opake.** If a disk be held opposite a candle-flame, and straight lines be drawn from the flame tangent to the edge of the disk, the conical space behind the disk which is un-illuminated is called the **shadow.** If the source of light be a point, the edges of a plane section of the shadow are sharp. If it have an appreciable surface, the edges of the shadow shade off gradually. Thus suppose three luminous points A, B, C to be taken (Fig. 135) and suppose MN to be the opake object; say a cardboard disk. Let OP be an intercepting screen. The portion of this

seen at a receives no light whatever from either of the ree sources. It is therefore absolutely dark and is lled the **umbra**. The portion $b'c'$ receives light from e two points B, C, and the portion bc from the two points , B; the portions ab and ab' receiving light only from

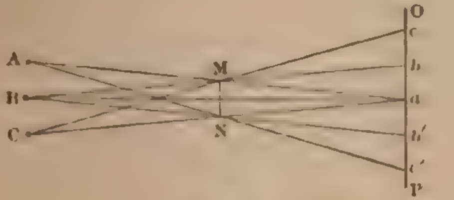


FIG. 185.

he points A and C respectively. The former two portions receive light from two of the three sources, the latter from only one. Hence the umbra is surrounded with a dark ring having one third of the light upon it, and with a second dark ring having two thirds the light upon it. This surrounding region through which the umbra gradually shades off is called the **penumbra**. It is always seen whenever the source of light has an appreciable size and the opake object is not too remote from it.

The **camera obscura** depends upon the fact that light is propagated in straight lines. It consists of a dark room having an opening in the window-shutter. On the wall opposite the opening is seen an image of outside objects in an inverted position. This image is sharper in proportion as the opening is smaller. But as this reduces the brightness of the image, it is necessary to make some sacrifice of sharpness in order to obtain a bright image. The camera obscura was invented by Baptista Porta in 1590. On the same principle Lord Rayleigh has constructed his pin-hole camera.

B.—REFLECTION.

376. Change of Direction with Change of Medium.—Rectilinear propagation of radiation is true only so

long as the medium traversed remains unchanged in character. When incident upon the surface of a second medium, the radiation is divided into two portions, one of which is turned back into the medium from which it comes, or is **reflected**; and the other enters the medium or is **refracted**; in general changing its direction in both cases. When reflected, it may be regularly reflected so as to form an image; or irregularly reflected or scattered. When refracted, that portion which enters the second medium is also divided into two parts, one of which is transmitted and the other absorbed. For convenience of discussion it is desirable to fix the attention not alone upon the wave-front, but also upon the normal to the wave-front, which is called a **ray**. Hence a ray is defined simply as the line along which the energy is propagated. A bundle of such rays when parallel is sometimes called a **beam**; and when convergent or divergent, a **pencil**. That part of the subject which discusses upon geometrical principles the change of direction which rays undergo in reflection and refraction is called **Geometrical Optics**.

377. Reflection of Radiation.—The laws of reflection are two in number: 1st. The angle of reflection is equal to the angle of incidence; and 2d, The plane which contains the incident ray and the normal contains also the reflected ray. Let CO (Fig. 136) be the normal to the reflecting surface RR', AO the incident and OB the reflected ray. Then the angle AOC is called the incident angle and BOC the angle of reflection. The first law, which asserts the equality of these angles, must be true kinetically; since if we resolve AO into two

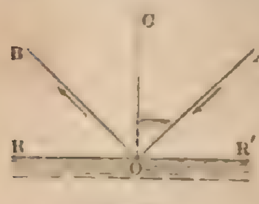


Fig. 136.

components, one parallel and the other perpendicular to the surface, we see that it is the latter only which is affected; and further that, for perfect elasticity in the medium, this is completely reversed; whence the re-

flected ray is the resultant of the original component along OR and of the reversed component along OC; and consequently the angle BOC is equal to AOC. Again, since the incident ray AO lies wholly in the plane containing the normal, i.e., the plane of the paper, it has no component in a perpendicular plane; hence the resultant OB will have no such component and will therefore lie in the same plane.

A similar proof may be given by considering the wave-front. Let a beam of radiant energy $AOO'A'$ (Fig. 137) be incident upon a reflecting surface RR';

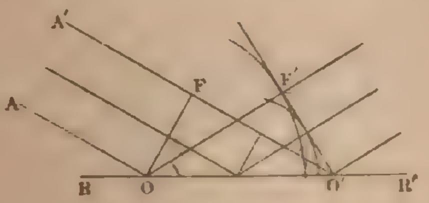


FIG. 137.

then FO will be the wave-front. After the ray A has reached the point O an interval of time will elapse before the ray A' reaches O'. During this time, the ray A reflected at O and traveling with the same speed as before, will have passed over a distance OF' equal to the distance FO'. So that if we draw a line $F'O'$, this will represent the new wave-front at the instant the ray A' reaches O'. Lines perpendicular to $F'O'$ drawn through O and O' represent the reflected beam. Since in the triangles OFO' and $OF'O'$ the side OO' is common, the angles at F and F' are right angles, and the side FO is equal to $F'O$, the triangles are themselves equal and the angle $FO'O$ is equal to $F'OO'$. Hence the angles of incidence and reflection, which are the complements of these angles, are also equal.

Experimentally the law of reflection may be proved thus: By means of a theodolite measure the angle SOC which the direct ray SO (Fig. 138) from a star makes

with the horizontal CO, as compared with the angle ROC which a parallel ray $S'R$ reflected from a surface of mercury at R makes with CO. These angles will be found to be equal. Hence their complements CRO and

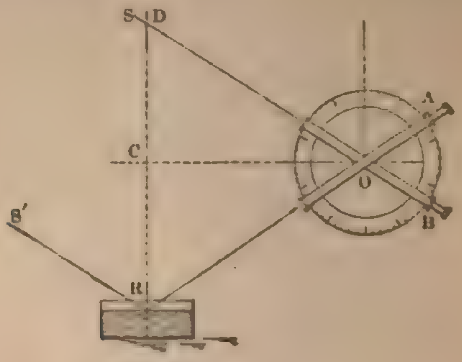


FIG. 138.

CDO are equal; and because $S'RD$ is alternate with RDO, and is equal to it, the angle of reflection ORD is equal to the angle of incidence $SR'D$. A mercury surface thus used is often called an **artificial horizon.**

378. Law of Fermat.—In passing directly from one point to another, light passes in a straight line and therefore pursues the shortest course. Ptolemy is said to have discovered the fact that in reflection from a plane surface the line formed by the incident and reflected rays is shorter than any other bent line having the same extremities and having its point of bending on the mirror; a fact which follows from the geometrical theorem that of all triangles having the same base and equal areas, that which is isosceles has the minimum perimeter. In 1667, Fermat enunciated the opinion, based on a cosmological necessity arising from the simplicity of the universe, that in all cases light takes the course which it can traverse in the shortest time. This statement, which is known as **Fermat's law**, is evidently true for reflection, since the shortest course is traversed in the least time. We shall see that it is true

also for refraction. As modified by Maupertuis, Euler, and Lagrange, it constitutes the celebrated **law of least action** which has played so important a part in all subsequent investigations in mathematical physics.

379. Reflection from Plane Surfaces.—Mirrors.— From these two laws, the conditions of formation of images in plane or curved mirrors can readily be deduced. Let AB (Fig. 139) be a plane reflecting sur-

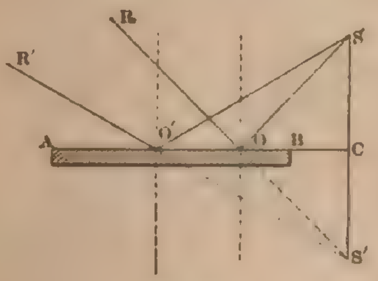


FIG. 139

face upon which a ray from a point at S is incident at O. By the first law it will be reflected to R, the rays RO and SO making equal angles with the normal at O. But the eye sees an object always in the direction of the ray which enters it; and will therefore see the object S at S', in the direction RO prolonged; i.e., as much behind the mirror as the object itself is in front of it and upon a perpendicular let fall from the object upon the mirror prolonged. The ray will therefore appear to come from a point S' as much behind the mirror as the object itself is in front of it. To prove this let a second ray $SO'R'$ be incident on the surface of the mirror at O'. The triangles SOO' and $S'OO'$ have the angles at O' equal, since they are both equal to $AO'R'$. They also have the angles at O equal, since each is the supplement of ROA. Moreover, the side OO' is common. Hence these triangles are equal and the side SO is equal to $S'O$. Since the triangles SOC and $S'OC$ have the

angles SOC and $S'OC$ respectively equal, both being equal to AOR, and since the side $SO = S'O$ and the side CO is common, they are equal and therefore the side CS' is equal to CS and the angle $OCS' = OCS = 90°$.

If instead of a point an object be taken such as an arrow, for example, the construction is the same, using the two extreme points as sufficient. Let fall a perpendicular upon the mirror RR' (Fig. 140) from each of

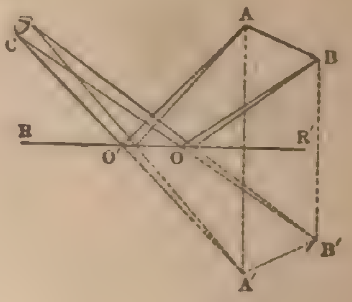


FIG. 140.

the points A and B, continuing them till the distance below the mirror is the same as that to the corresponding point above. The image $A'B'$ is obtained by connecting these points, as shown in the figure. From A and B let fall the rays AO' and BO, incident upon the mirror at O' and O, respectively. Draw through these points the lines CB' and CA'; they represent the rays entering the eye at C. In this case $A'B'$ is called the image of AB. It is of the same size as the object, is symmetrical with it, it is reversed in position though not inverted, and being behind the mirror is a virtual image.

380. Effect of Angular Motion of Mirror.—Let a ray Aa be incident perpendicularly upon a reflecting surface RR' (Fig. 141). The incident angle being zero, the reflecting angle is also zero and the ray returns

along its previous path, as shown in the upper figure. Let the mirror be now rotated forty-five degrees about an axis through the point of incidence. The incident ray will now be reflected to B through a right angle, as is shown in the lower figure; since BaN is equal to AaN,

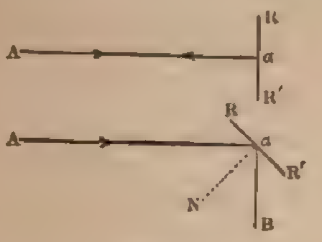


FIG. 141.

by the law of reflection. The mirror has turned through AaN or 45° by hypothesis; and the ray has turned through AaN plus BaN, i.e., through $2AaN$, or 90°. The angular motion of the ray is therefore double that of the mirror. This principle is illustrated in the sextant, in the goniometer, in the heliostat, and in reflecting instruments generally.

381. Multiple Mirrors.—Two cases are here to be considered, one where the mirrors are parallel, the other where they are inclined to one another. In the first case let a luminous point A (Fig. 142) send two rays to

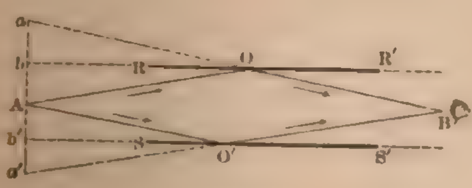


FIG. 142

the two parallel mirrors RR' and SS', incident on these mirrors at the points O and O'. To the eye placed at

B there will be two images a and a', the distance between them being twice the distance separating the mirrors. For Aa is twice Ab; and Aa' is twice Ab'. Hence $Aa + Aa'$ or aa' is twice $Ab + Ab'$, or bb', the distance between the mirrors. In Fig. 143 Aa is twice Ab and Aa' is twice Ab' as before. Hence $Aa' - Aa$, or aa', is twice $Ab' - Ab$, or bb'; the same result.

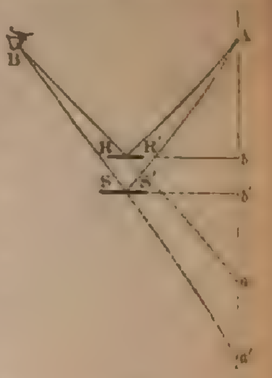


FIG. 143

If the mirrors are inclined to each other, similar constructions may be made. In the first place, suppose the angle between the mirrors to be α; required the change in the direction of the ray after reflection from both successively. Let a ray from a luminous point A (Fig. 144) be incident on the mirror SS' at O,

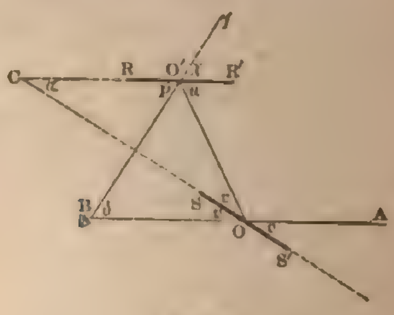


FIG. 144

and be reflected to O' on the mirror RR', and again reflected to the eye at B. Originally the object appeared in the direction BA; it now appears at f, its direction having changed through the angle ABf, which we may call δ. The angle u, exterior to the triangle $CO'O$, is evidently equal to $\alpha + v$; whence $\alpha = u - v$. So the angles $u + x$, exterior to the triangle $BO'O$, are

equal to $2v + \delta$; whence $\delta = u + x - 2v$. But $x = p$, being opposite; and $p = u$, by the law of reflection; whence $x = u$ and $\delta = 2u - 2v$. But $\alpha = u - v$; hence $\delta = 2\alpha$; or the deviation of the ray is twice the angle between the mirrors.

Again, suppose the mirrors to be at right angles and the source of light to be between them. Let RR' and SS' (Fig. 145) be the mirrors, and A the luminous point. By the method given above, the image A in RR' will be at a, and the image in SS' at a'. By a third reflection from both mirrors, a third image will be produced at a'', these three images and the object being symmetrically placed with reference to the mirrors. If the mirrors

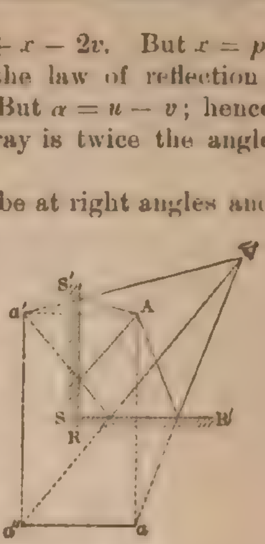


FIG. 145.

are inclined at an angle of 60° (Fig. 146) there will be five images; and in any case there will always be $n - 1$ images, where n is the number of times the angle between the mirrors is contained in the whole circumference. Upon this principle depend the kaleidoscope of Sir David Brewster and the debuscope of Debus.

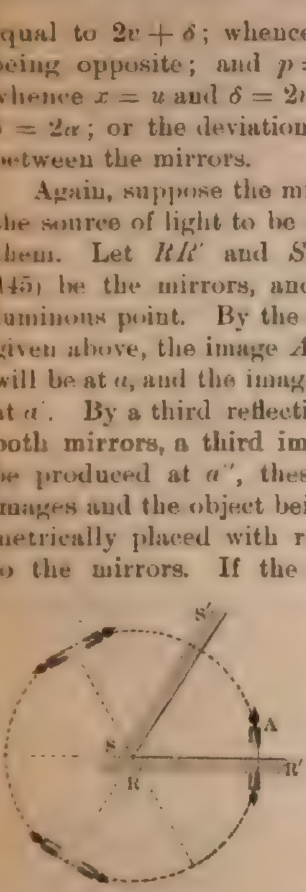


FIG. 146.

382. Amount of Reflection—Reflecting Power.—The amount of radiation reflected from a surface depends (1) on the nature of the reflecting body, (2) on the state of its surface, and (3) upon the angle of incidence. As an illustration of the effect of the reflecting material, it may be shown that of light incident normally, water reflects $\frac{1}{55}$, plate glass $\frac{1}{25}$, crown glass $\frac{1}{23}$, and flint glass about $\frac{1}{15}$. Opake bodies have a much higher reflecting power; thus mercury reflects $\frac{2}{3}$ and speculum metal $\frac{2}{3}$ of the in-

cident light. Evidently the more highly polished the surface the higher its reflecting power. But no artificial surface can equal the natural surface in reflecting power. The amount of reflection increases with the incident angle. Thus water, which reflects only 0·018 per cent of the incident light falling upon its surface perpendicularly and only 0·019 at an incident angle of 30°, reflects 0·034 per cent at 50°, 0·145 at 70°, 0·333 at 80°, 0·503 at 85°, and 0·639 per cent at 88°. Glass, which at 0° incidence reflects only 0·043 per cent, reflects at 88°, 0·819 per cent. Fresnel from theoretical considerations has given the following equation representing the ratio of the reflected radiation I' to the incident radiation I for glass:

$$\frac{I'}{I} = \frac{1}{2} \cdot \frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{1}{2} \cdot \frac{\tan^2(i-r)}{\tan^2(i+r)}. \qquad [58$$

In this equation i and r are the incident and reflection-angles.

The measurement of reflecting power is made by placing the reflecting surface in line with the radiation, and receiving the reflected beam upon a photometer disk, or upon a bolometer or thermopile, placed upon an arm moving about the support of the reflector as a center. In the absence of any reflecting body, this arm is in line with the radiation and measures it directly. The ratio of the reflected energy to the direct energy is the reflecting power.

383. Diffusion of Radiation.—Besides regular or specular reflection, radiation may be irregularly reflected or scattered. It is then said to be **diffused.** It is by diffused light that we see non-luminous objects, since a perfectly reflecting surface is itself invisible. Diffusion is therefore due to roughness of surface, the irregularly reflected rays not forming an image. Moreover diffusion may be selective. A red ribbon in the red of the spectrum is most brilliant, while it is black in the green region. By means of his thermo-galvanometer Melloni

showed that diffusion both general and selective exists also for non-luminous radiations.

384. Reflection from Curved Surfaces.—A curved reflecting surface is said to be **convex** if the radiations fall on the side opposite to its center of curvature; **concave**, if upon the same side. Curved mirrors are in general spherical, although parabolic and cylindrical mirrors are used for special purposes. The **center of curvature** of a mirror is the center of the sphere of which it is a segment; the **center of figure** is the center of the mirror itself. The **field** of the mirror is the angle included between two radii drawn to the extremities of one of its diameters. An **axis** is any line passing through the center of curvature and incident upon the mirror. The **principal axis** is an axis passing also through the center of figure of the mirror. The other axes are called **secondary axes**. A plane containing the principal axis is called a **principal section**. If rays Aa and Bb (Fig. 147) parallel to the principal axis fall upon a concave mirror, they will be reflected so as to intersect that axis at the point F, which is called the focus for parallel rays or the **principal focus**. Since a radius is normal to a spherical surface and since

Fig. 147.

O is the center of curvature, the path of the reflected ray is obtained simply by making the angle of reflection FaO equal to the angle of incidence AaO. In the triangle FOa, $FO : Oa :: \sin OaF : \sin OFa$. But $\sin OFa = \sin aFC = \sin 2i$; since the angle OaF is equal to the angle OaA by the law of reflection, and the angle aFC is equal to AaF, being alternate. Hence $FO = R (\sin i / \sin 2i)$.

As the aperture of the mirror—in this case the angle aOb—grows smaller, the angle of incidence, which is one half that angle, decreases also. In consequence the angle of reflection will decrease and the point F will move out toward O. The limiting position of F is reached when the incident angle is indefinitely small. In this case we may write the arc for the sine, and the above expression becomes $FO = R(i/2i) = \frac{1}{2}R = FC$. In other words, the principal focal distance of a concave mirror of small aperture is one half its radius of curvature.

385. Conjugate Foci.—In order that the rays incident upon the mirror shall be parallel they must come from an object at an indefinite distance. Suppose now (Fig. 148) the two rays Aa and Ab come from a point A on

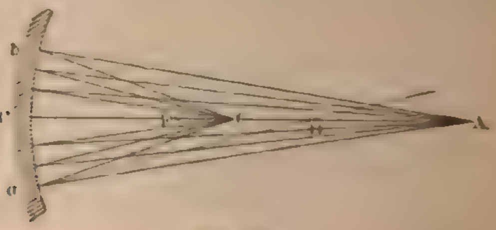


Fig. 148.

the principal axis at a finite distance from the mirror. Evidently the angle of incidence AaO being less than before, the angle of reflection faO will be less also, and the point of intersection of the reflected rays f will be nearer the center of curvature, O. Since the radius Oa bisects the angle Aaf, we have $Aa : fa :: AO : fO$. If the aperture of the mirror is small, we may write AC for Aa, and fC for fa. If we represent AC by f, and fC by f', r being the radius Oa, we have $AO : fO :: AC : fC$ or $f - r : r - f' :: f : f'$. Whence $f'r + fr = 2ff'$. Dividing by $ff'r$, we obtain

$$\frac{1}{f} + \frac{1}{f'} = \frac{2}{r}. \qquad (59)$$

the usual form of the expression. In words it states simply that the sum of the reciprocals of the conjugate focal distances is equal to the reciprocal of the principal focal distance; since $2/r = 1/F$, as has been shown above. The distances AC and fC or f and f' are called **conjugate focal distances** because of their mutual dependence. Thus if we put the above equation into the form $f = \frac{r}{2 - r/f'}$, we see that when f' is equal to $\frac{1}{2}r$, i.e., when the inner conjugate focus concides with the principal focus, the denominator of the fraction is 0 and f has an infinite value; the incident rays are parallel. As f' increases f diminishes, becoming equal to r when f' is also equal to r. Both the incident and the reflected rays now pass along the normal. As f' continues to increase, the two conjugate foci change places, f' being now greater and f smaller than r, until when f reaches the value $\frac{1}{2}r$, f' is indefinitely distant. These conjugate foci, like the principal focus, are called **real foci**, since they are formed by the intersection of the reflected rays themselves and can evidently be received on a screen.

386. Virtual Foci of Concave Mirrors.—Suppose the distance represented by f should continue to diminish after it has reached the value $\frac{1}{2}r$. Evidently the rays

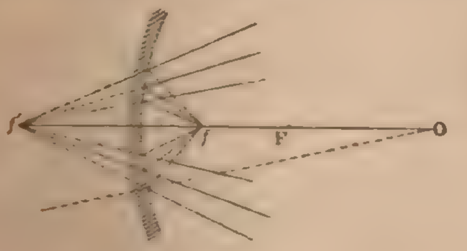


FIG. 149

from this focus would diverge after reflection, since the angle of incidence would be greater than is required for parallel rays. There can now of course be no focus con-

jugate to f in front of the mirror. But by prolonging the reflected rays backward (Fig. 149) these produced rays will be found to intersect on the principal axis at f'. Such a focus is called **a virtual focus** since it is wholly subjective, being formed by the projection of the rays by the eye to a point behind the mirror. The point f' is called the **virtual conjugate focus** of the mirror. Since, when both foci are on the same side of the mirror as the center of curvature, their distances from the mirror are all reckoned positive, we have only to change the sign of f' to adapt the general formula to the present case. Making f' negative we have $\frac{1}{f} - \frac{1}{f'} = \frac{2}{r}$ or $\frac{1}{F}$, as the expression for virtual conjugate foci in concave mirrors.

387. Convex Mirrors.—Since all rays reflected from a convex mirror are divergent, the foci of such mirrors are always virtual. If rays Aa and Bb (Fig. 150) be

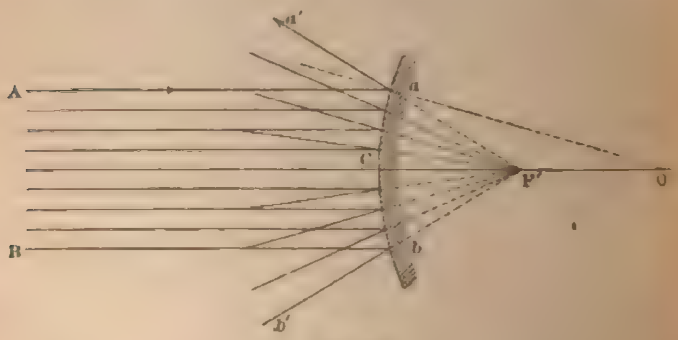


FIG. 150

incident on such a mirror, they are reflected to a' and b'; and these reflected rays if produced backward will appear to intersect the principal axis produced at a point F' behind the mirror. The point F' is called the **principal virtual focus** of the convex mirror. As the luminous points A and B approach or recede from each other, the incident angles increase or diminish, and the conjugate focus f' approaches or recedes from C (Fig. 151).

In the case of convex mirrors F and f' are both negative; and the general formula becomes $\frac{1}{f} - \frac{1}{f'} = -\frac{1}{F}$; or $\frac{1}{f'} - \frac{1}{f} = \frac{1}{F}$, which is preferable.

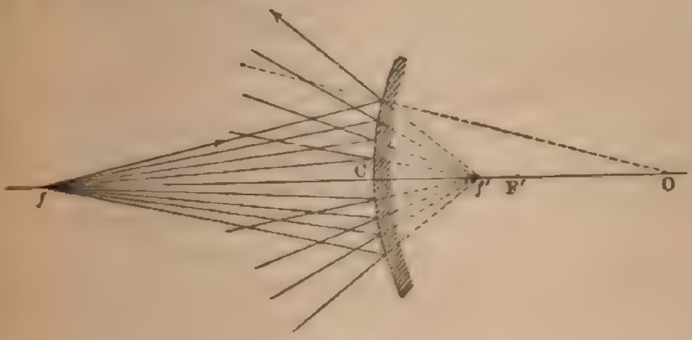


FIG. 151.

388. Experimental Determination of Foci. — (1) The radius of curvature of a mirror can be calculated from the value of its versed sine, which may be measured directly by the spherometer. If R is the desired radius, r the radius of the spherometer, and h the versed sine measured, the figure (Fig. 152) gives $R^2 = r^2 + (R - h)^2$; whence $R = (r^2 + h^2)/2h$.

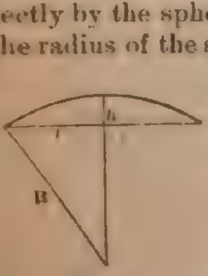


FIG. 152.

(2) A beam of sunlight consists of nearly parallel rays. By placing a concave mirror in sunlight, its principal focus may be obtained on a piece of card; and the measured distance of the card from the mirror is the principal focal distance. Or (3) a candle-flame may be placed in front of the mirror and moved along the axis until the flame and its image coincide. Both are then at the center of curvature. Again, (4) place a candle-flame at a known distance f from the mirror and on its principal axis, and measure the distance f' between the conjugate focus and the mirror. Then from the formula given above $F = ff'/(f + f')$. If the mirror be convex,

a parallel beam is allowed to fall on it through an opening in a screen. The screen is then moved away until the diameter of the reflected diverging beam is twice that of the original beam. Then we have the principal virtual focus of the mirror equal to the distance between the screen and the mirror. Since in all cases the principal focal distance is half the radius of curvature, either of these constants may be determined from the other.

389. Formation of Images in Mirrors.—Since an object may be regarded simply as a collection of luminous points, an image can be considered simply as the conjugate foci of these points. The image of each of the points of the object is on the same secondary axis as the point itself. Hence to construct an image of an object after reflection in a mirror it is necessary only to take a sufficient number of points of the object, to draw secondary axes through each of these points, and then to let fall upon the mirror rays parallel to the principal axis, continuing these after reflection, through the principal focus, till they intersect the secondary axes previously drawn. Thus let the object be an arrow AB (Fig. 153).

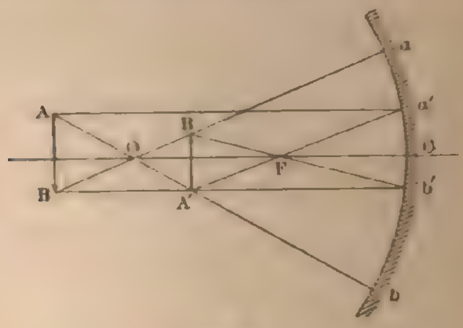


FIG. 153.

Draw the secondary axes Ab and Ba. The image of A will be on Ab, and the image of B on Ba. Let fall the parallel rays Aa' and Bb' on the mirror. After reflection they will intersect the axis at the principal focus F and finally cut the secondary axes at A' and B'. Since

image of A is on the line $Aa'FA'$, as well as on the ndary axis Ab, it must be at the intersection of these . The same being true of B, its image is at B'. image thus formed by reflection in a concave mir- is a real image, it is inverted in position, and is ller than the object; the size of the image being to of the object evidently, as the distance of the former the center of curvature is to the distance of the r.

Virtual images are obtained by similar constructions. a concave mirror let AB be the object and draw secondary axes OA and OB (Fig. 154), producing

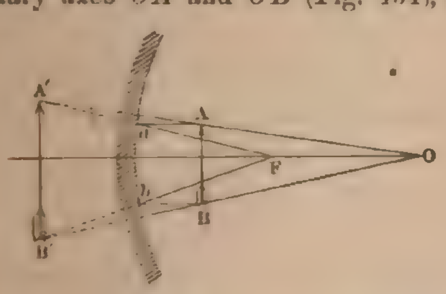


FIG. 154.

m indefinitely on the other side of the mirror. Draw the parallel rays Aa and Bb, which after reflection intersect at the principal focus F. Produce these backward to intersect the secondary axes at the nts A' and B'. The image will be $A'B'$. Here the ge is erect, larger than the object, and appears to be nd the mirror.

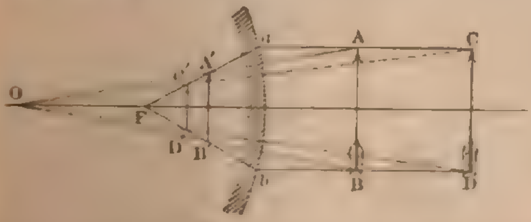


FIG. 155.

f the mirror be convex, draw the secondary axes and OB (Fig. 155) as before and let fall the parallel

rays Aa and Bb upon the mirror. Evidently the reflected rays will appear to intersect at the virtual principal focus F'. But before they intersect they cut the secondary axes at A' and B'. Hence the image $A'B'$ appears behind the mirror, is erect and is smaller than the object. If the object is farther from the mirror at CD, the image is smaller, $C'D'$. Since in both of these cases the image and object form the base of a larger or a smaller isosceles triangle whose sides are the secondary axes, the size of the one is to the size of the other as the height of the one triangle is to that of the other.

390. Spherical Aberration of Mirrors.—Caustics.—As we have seen, when a number of parallel rays fall on a concave surface, the reflected rays intersect the principal axis at points which are more and more distant from the mirror, in proportion as the incidence-points are

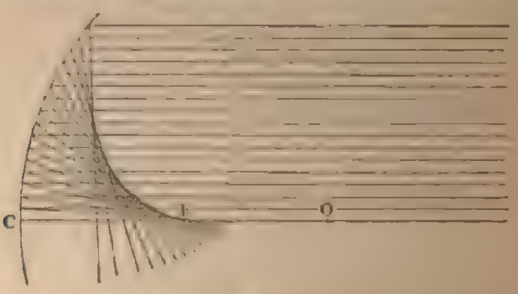


FIG. 156.

nearer this axis. Hence the intersections of these rays with each other form a peculiar curve known as a caustic. The figure (Fig. 156) shows the form of caustic produced by rays parallel to the principal axis.

To trace it, draw a semi-circumference within the hemispherical mirror (Fig. 157). Let a ray A fall on the mirror at a parallel to the principal axis, and join aO. Upon ad as diameter describe a circle. Its radius $O'd$ will be one half of Od. Since $eO'd = 2ead = 2AaO = 2aOF$ and since $Od = 2O'd$, the arc Fd must equal the arc ed. If, consequently, we suppose the circle aed to roll upon the circle Fd, the point e being originally at F, this point e will trace out the epicycloidal curve MeF on one side and FN on the other of the

axis, the cusp being at *F*. As the angle *aed* is a right angle, *de* is always perpendicular to the path of the moving point; and the reflected ray *ae*, which is perpendicular always to *de*, must always be tangent to the curve which is the locus of the moving point; i.e., to the epicycloidal caustic. All such parallel rays reflected from the mirror are tangent to this curve. If the radiant point be within the circle of which the reflecting mirror is a segment, the caustic curve is still an epicycloid, of the form known as the cardioid.

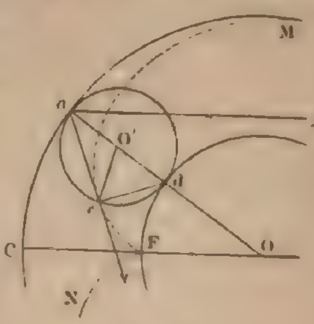


FIG. 167.

This caustic curve is well seen by placing a steel ring, polished interiorly, upon a white paper in sunlight. The appearance of this curve in milk, caused by the reflection from the walls of the vessel, is sometimes called the cow's foot in the milk.

In the formation of images in spherical mirrors, the non-coincidence of foci produced by rays reflected from different zones of the surface is a source of indistinctness. Not only are rays from the same point of the object reflected to different positions according to the part of the mirror on which they fall, but rays from different points of the object are reflected to the same focus. This indistinctness of the image is due to what is called the **spherical aberration** of the mirror, the distance measured along the principal axis, between the inner and the outer principal foci, being called the **longitudinal** spherical aberration. Since the two converging cones of reflected rays intersect along this distance, there is a point where the circle of intersection is a minimum. This is called the **circle of least confusion**, and its radius, the **lateral** spherical aberration.

By absorbing the luminous rays of the electric arc in a solution of iodine in carbon disulphide, Tyndall has shown not only that non-luminous rays may also be reflected to a focus, but also that they may form images.

At the outer focus of a concave mirror, these rays absorbed by blackened platinum heated this platinum to redness and produced an image of the carbon points placed at the inner focus. This production of luminous from non-luminous radiations of greater wave-length he has called **calorescence**.

It is a property of the parabola that a line drawn from the focus to any point of the curve, and a line drawn from this point parallel to its axis, make equal angles with a tangent to the curve at the point of incidence. Hence all the rays from a luminous point placed at the focus are reflected strictly parallel to the axis and to each other. The reflectors used in light-house illumination, and the specula of reflecting telescopes, are parabolic in form, securing in the latter case a sharp image, free from the defects due to spherical aberration.

C.—REFRACTION.

391. Refraction of Homogeneous Radiation.—Refraction is the change in direction which takes place when radiation passes from one homogeneous medium to another. It is usual to discuss refraction (1) under the condition that the radiation is itself homogeneous, i.e., contains but one wave-frequency; and (2) under the condition that it is more or less complex in this respect.

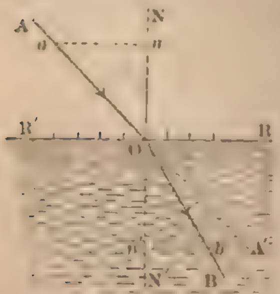


Fig 158

The laws of refraction are two in number: 1st, the refracted ray lies in the plane containing the incident ray and the normal; and 2d, the sine of the incident angle bears to the sine of the angle of refraction a ratio which is constant for the same two media. Thus (Fig. 158) let the ray AO in air be incident upon a surface of water RR'. A portion will be

flected, as we have seen. But a second portion will ter the water, suffering at the bounding surface a ange in its direction, so that it now passes on to B. he angle AON is the angle of incidence, BON' the gle of refraction; the latter angle under the circumances supposed being the smaller. The angle of eviation $A'OB$ is evidently the difference between the gles AON and BON'. Lay off on AO a radius Oa, d on BO an equal radius Ob. From a and b let fall e perpendiculars an and bn' on the normal NN'. These es represent the sines of the angles of incidence and fraction respectively, calling the radii unity; and nce by the second law their ratio is constant for ese two media, namely for water 1·336 or roughly $\frac{4}{3}$. bn represents a length of three units, an will be four its long on the same scale; and so on for all angles incidence. If μ represent this ratio for a vacuum d a given medium, we may write the second law thus: n i sin $r = \mu$; or in the equivalent form sin $i = \mu$ sin r. gain, if μ_1 be the ratio from a vacuum to a second edium, the ratio μ/μ_1 or n will be the ratio from the cond medium to the first. This ratio of the sines is lled the **index of refraction** for the two media. If one these media is a vacuum, the ratio is called an **absolute dex**, such as μ and μ_1 above. The ratio of two absolute dices is of course a **relative index**, e the n above given.

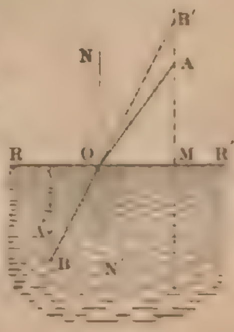


Fig. 159.

Since in the diagram annexed ig. 159) sin $i = OM/AO$ and sin $= OM/OB'$, (as $OAM = AON$ d $OBM = BON = BON'$,) we y write $OB' = n(OA)$; and, if e incident angle is small, $MB' =$ MA) approximately. Supposing e media to be air and water, MA uld be three fourths MB'. An e beneath the water would see at an object actually at A, one third rther above the surface. And conversely, an eye at A

would see an object which is actually at B in the water, at some point A' one quarter less deep.

This change of direction of a ray at the surface separating two media is a direct result of a change of speed at this surface. Let a wave whose front is OF (Fig. 160) be incident obliquely upon a bounding surface RR'.

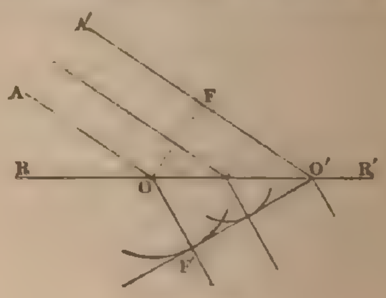


FIG. 160.

AO and $A'O'$ will be the normals to this wave-front, i.e., the rays. When the ray AO has reached O, the ray $A'O'$ is yet at F. So that while this ray is passing from F to O', the first ray has entered the second medium, in which by hypothesis it has a less speed, and has advanced to F'. Thus the new wave-front is $O'F'$; and normals to this represent the direction of the rays after refraction. Examining the triangles $OO'F$ and $O'OF'$, we observe that the angle FOO' is equal to the incident angle, and $OO'F'$ to the refraction-angle; and hence, since OO' is common, $\sin i : \sin r :: FO' : OF'$. But $\sin i / \sin r = n$, the index of refraction; and $FO'/OF' = s/s_1$ or the ratio of the speeds in the two media. The index of refraction, therefore, is simply the ratio of the speed of propagation of the radiation in one medium to that in the other.

The direct proof of the fact that the speed of propagation is less in proportion as the medium is more highly refractive, was made an *experimentum crucis* in the early days between the wave-theory, of which it was a necessary result, and the emission-theory, which required

exactly the contrary. This proof was made by Foucault in 1862 by measuring the relative speed of light in water and in air and finding it less in the water. Michelson in 1883 made this experiment quantitative and obtained 1·330 for the ratio of the speeds in air and water and 1·758 for the ratio in air and carbon disulphide; these values being very closely the indices of refraction of the media mentioned.

In the above cases the lower medium has been assumed as the optically denser medium; i.e., the more highly refractive. Under this condition the angle of refraction is less than the incident angle, the ray being refracted toward the normal. Obviously, if the direction of the ray be reversed, the incident angle will be in the denser medium and the refraction will be from the normal. It is usual to suppose the incidence to occur in the less highly refractive medium; so that the sine of the incident angle shall be the larger, and the ratio of the sines, i.e., the refractive index, shall be greater than unity. The index in the other direction is of course the reciprocal of this. Thus if the index from air to water is $\frac{4}{3}$, that from water to air is $\frac{3}{4}$.

392. Critical Angle.—Total Reflection.—Since the angle between the ray and the normal is always larger in the less dense medium, it is pertinent to inquire what will take place when this angle becomes 90°. Suppose a ray from the point B (Fig. 161) within the denser medium to be incident upon the surface at O. It will be refracted to A, apparently as if it came from A'. As the distance of the incident point from M increases the angle of emergence increases, until at some point such as O_1 this angle reaches 90°. Then the emergent ray will not issue from the medium, but will pass along the

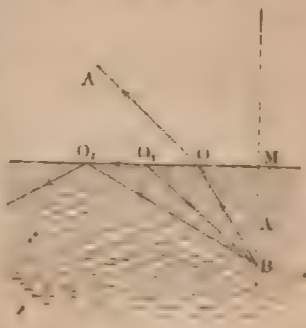


FIG. 161.

surface. If the distance of the incident point be increased to O_2, the ray from B can no longer emerge but is totally reflected back into the medium from which it came. Or, since $\sin i/\sin r = n$, if the angle in the less dense medium be 90° its sine will be unity, and the expression becomes $\sin r = 1/n$. The value of the angle in the optically denser medium, when the angle in the less dense medium is 90°, is called the **critical angle**, since it is the limiting angle of emergence. As the equation $r = \sin^{-1} 1/n$ shows, the critical angle is the angle whose sine is the reciprocal of the refractive index. In the case of water, for example, $\frac{3}{4} \sin i = \sin r$. And as the maximum value which $\sin i$ can have is unity, the maximum value of $\sin r$ is $\frac{3}{4}$. A ray cannot pass from water to air or from air to water so as to make with the normal to the surface an angle in the denser medium greater than the angle whose sine is $\frac{3}{4}$; i.e., the critical angle. This angle is 48° 35′; and therefore the eye when placed at B above can see through the surface only when the rays incident upon the surface are included within a cone having an angle of 97° 10′. Outside of this cone there will be a surface of total reflection, in which the images of objects at the bottom will be visible. Since under these circumstances the reflection is total, the image is always much brighter than that produced by simple reflection under the most favorable conditions. It should be kept in mind, however, that total reflection takes place only when the incidence is within the more highly refractive medium.

393. **Caustics by Refraction.**—Suppose a radiant point Q (Fig. 162) in an optically denser medium. The rays issuing from it will be refracted from the perpendicular at the surface of the rarer medium, the emergent angle being the greater as the point of incidence is farther from the normal QA; until at some point C or C' this angle becomes the critical angle and no radiation emerges. Prolonging the refracted rays backward, they will be seen to intersect the normal QA at points successively nearer the surface as the emergent angle

increases. The intersections of these prolonged rays form a virtual caustic curve which is the evolute of an ellipse and has a cusp at B; so that an eye placed at A

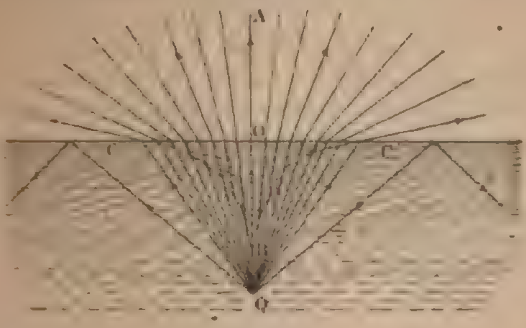


FIG. 162.

will see the point Q at B, such that $OQ = n(OB)$. This construction illustrates the apparent deformation of objects seen under water.

394. Refraction in Media bounded by Plane Parallel Surfaces.—Since the index of refraction from one medium A to another medium B is the reciprocal of that from B to A, it follows that the change of direction which a ray suffers on entering the second medium from the first is exactly reversed on emergence. Thus let RR' (Fig. 163) be a portion of any transparent medium with parallel sides. At the first surface we have $\sin AON = n \,.\, \sin nOO'$; and at the second, $\sin OO'n' = n^{-1} \,.\, \sin A'O'N'$. But the angles nOO' and $OO'n'$ are equal, being alternate. Hence $\sin A'O'N'$ is equal to $\sin AON$, the angle of emergence is equal to the angle of incidence, and the emergent ray is parallel to the incident ray. A ray passing through such a medium, as for example a plate of glass, suffers no change in its direction but is slightly

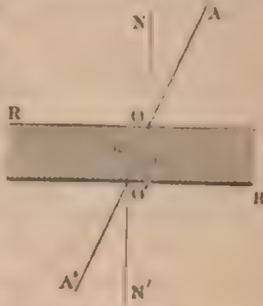


FIG. 163.

displaced parallel to itself, to an amount depending upon the thickness of the medium and the angle of incidence.

395. Refraction in Media bounded by Plane Inclined Surfaces.—Prisms.—If the plane surfaces bounding the transparent medium be inclined to each other, the change in direction of the ray, i.e., its deviation, instead of being destroyed at the second surface, is still further increased. We may define a **prism** as a portion of a transparent medium bounded by plane inclined surfaces. The line along which these surfaces meet is called the **refracting edge**, and the angle between them the **refracting angle** of the prism. The side opposite the refracting edge is called the **base** of the prism, and any section through it perpendicular to this edge is called a **principal section**. Let PQS (Fig. 164) be a principal section of an equilateral prism, P its refracting edge, and SQ its base. Let a ray AO be incident on the surface PQ at O. Draw normals at O and O'. At O the refraction will be toward the normal, $\sin r = n^{-1} \sin i$, and the ray will be refracted to O'. Here the second refraction takes place, $\sin A'O'N' = n \,.\, \sin rO'O$. But since the face PS is inclined to PQ, the second deviation is in the same direction as the first; i.e., toward the base of the prism. Evidently the deviation of the ray at the first surface is $AON - rOO'$, or $\delta = i - r$; and at the second is $A'O'N' - rO'O$, or $\delta' = i' - r'$. The total deviation is the sum of the partial deviations; or $\Delta = \delta + \delta' = (i - r) + (i' - r')$. That is, the total deviation in the figure, or the angle between the incident and emergent rays AIB or Δ, is equal to the sum of the partial deviations $\delta + \delta'$ or IOO' and $IO'O$; since the exterior angle of the triangle is equal to the sum of the two interior angles. Moreover, the deviation may be obtained in terms of the index and the prism-angle. Since $\Delta = (i + i') - (r + r')$ and α, the prism-

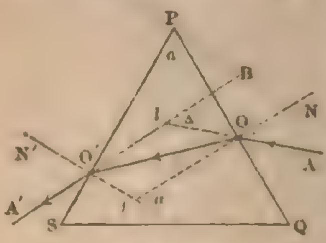


Fig. 164.

angle, is equal to $r + r'$, the deviation $\Delta = (i + i') - \alpha$. If the prism is thin, we may write $i = nr$ and $i' = nr'$; whence $i + i' = n(r + r')$ or $n\alpha$; and $\Delta = n\alpha - \alpha$ or $(n - 1)\alpha$; i.e., the deviation is equal to the prism-angle multiplied by the index less one.

396. Minimum Deviation.—In the last figure it is evident that the total deviation will be the same whether the ray passes through the prism from A to A' or from A' to A, the same angle being made between the incident and emergent rays in either case. Suppose now we let the incident angle i increase until it is equal to the emergent angle i'. This will be equivalent simply to reversing the direction of the ray, and the deviation will be the same as before. The deviation is the same, consequently, for two different angles of incidence. During the change it must therefore have passed through a maximum or a minimum value, corresponding to equal values of these angles; i.e., when $i = i'$. This result is easily verified by experiment; and the deviation is found to be a minimum when the ray traverses the prism parallel to its base; in other words, when the incident and emergent angles are equal.

397. Measurement of the Refractive Index.—When, therefore, the deviation produced by a prism is a minimum, the angles formed by the ray with the normals within the prism are equal, as are also the angles without it; i.e., $r = r'$ and $i = i'$. Consequently we may simplify the formula for deviation $\Delta = (i + i') - (r + r')$ by writing it $\Delta = 2i - 2r$. In the diagram (Fig. 165) we see that the angle at t formed by the two normals is for that reason equal to α, the angle of the prism. But, being exterior to the triangle $OO't$, it is equal to the sum of the two interior angles $r + r'$ or $2r$; i.e., $\alpha = 2r$ and $r = \frac{1}{2}\alpha$.

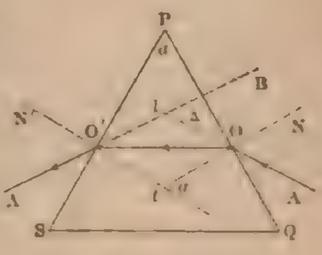


FIG. 165.

Hence $\Delta = 2i - \alpha$; and $i = \frac{1}{2}(\Delta + \alpha)$. By the formula for the index we have

$$n = \frac{\sin i}{\sin r} = \frac{\sin \frac{1}{2}(\Delta + \alpha)}{\sin \frac{1}{2}\alpha}. \qquad [60]$$

To determine the refractive index of a substance, therefore, it is necessary only to measure (1) the angle α of the prism and (2) the deviation-angle Δ when this angle has its minimum value.

These angles are both measured with the spectrometer. This instrument consists of a horizontal divided circle, about the center of which move two radial arms carrying telescopes. A parallel beam of light from one of these telescopes is incident upon one face of the prism placed vertically at the center of the circle and which is so adjusted as to reflect the beam into the other telescope. The position of the prism is then read on the circle. The prism is now turned on its axis until the same beam is reflected into the telescope from the second face, and a second reading is taken. The distance through which the prism has been turned is the supplement of α, the angle of the prism.

To measure the deviation the prism is adjusted so that the homogeneous beam from the one telescope is received by the other after refraction by the prism; this prism being previously carefully set so that the deviation of the beam is the least possible. If one reading be made when the telescopes are in line and the angle between them zero, and a second when the deviated beam passes through them, the difference of reading is Δ, the angular deviation. Whence by substitution in the above formula the index may be calculated.

The indices of liquids and gases are obtained by enclosing them in hollow prisms with parallel plates forming the sides, the same formula being used as with solids. In the case of gases, the index is so small that it may ordinarily be neglected.

he indices given in the following table were obtained sodium light of wave-length ·000589 mm. in air:

TABLE OF REFRACTIVE INDICES.

Substance.	Density	Refractive Index.	Temperature.
wn glass (soft).......	2·550	1·5146	
" (light)......	2·866	1·5410	
" (ordinary)...	3·658	1·6224	
" (very dense).	4·421	1·7102	
k salt		1·5442	17°
in (KCl)....		1·4903	20°
n................		1·4560	21°
nond..............		2·4700	
r spar.............		1·4339	
ber		1·5320	
er....	1.000	1·3324	15°
hol........	0·795	1·3638	15°
r..................	0·716	1.3536	15°
on disulphide.......	1·293	1·6442	0°
zene..........		1·4975	10·5°
roform.............	1·526	1·4490	10°
rogen.......... ...	0·0000896	1·0001387	0°
gen.......	0·0014298	1·0002706	0°
....	0·0012932	1·0002923	0°
on dioxide.........	0·0019774	1·0004544	0°
nogen.......	0·002440	1·0008216	0°

y employing waves of low vibration-frequency on and Perry found for ebonite the index 1·66. er, using a different method, obtained the value . And Kundt, by using thin layers, has deterd the index of silver for red light to be 0·27, of gold of copper 0·45, of platinum 1·76, of iron 1·81, of al 2·17, and of bismuth 2·61.

398. Condition of Emergence in Prisms.—Evidently cannot emerge from a prism if it is incident upon second face at an angle greater than the critical . We may inquire, therefore, what are the conns of emergence. In the figure (Fig. 166) the angles $r' + x = 180°$, and $x + a = 180°$, since the angle

external to x is equal to α. Hence $r + r' + x = x + \alpha$, and $r' = \alpha - r$; that is, the angle of incidence upon the second surface is the difference between the refracting angle of the prism and the angle of refraction at the first surface. When $r' = \gamma$, the critical angle, the emergent ray will pass along the face of the prism. This, therefore, is its maximum value, the equation becoming $r = \alpha - \gamma$. If the angle of the prism be 2γ, then $r = 2\gamma - \gamma$ or γ; i.e., when the angle of the prism is twice the critical angle, the entering ray will pass along one face, traverse the prism symmetrically, and emerge along the other face. If the angle be larger than this, no emergence will be possible, the ray being totally reflected within the prism. It is only when the angle of a prism is less than twice the critical angle, therefore, that a ray can pass through the prism. If the angle of the prism be equal to the critical angle, a ray incident perpendicularly on the first surface will meet the second at the critical angle. All rays incident on the prism between this perpendicular incidence and a parallel incidence will traverse the prism. Since for crown glass of index 1·5 the critical angle is 41° 48′, the maximum possible angle for a crown-glass prism is 83° 36′; and no ray can traverse a crown-glass prism whose refracting angle is 90°. Twice the critical angle for water, however, is 97° 10′; and hence a water prism of 90° will transmit a ray. Moreover, since $\sin i = n \sin r$ we may calculate the incidence under which transmission is possible, for a given prism-angle, say 60°, the prism being of crown glass of index 1·5. From the equation $r = \alpha - \gamma$ we have $r = 60^\circ - 41^\circ 48' = 18^\circ 12'$. Whence $\sin i$ is equal to 0·4685 and the angle of incidence is 27° 56′. That is, no ray can traverse a crown-glass prism of 60° except those making an angle of inci-

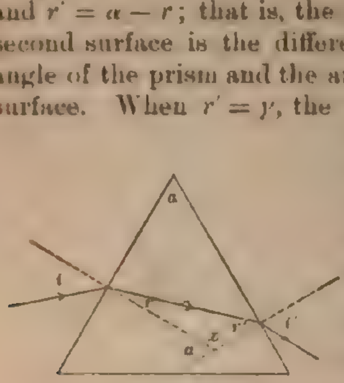


Fig. 186

dence greater than 27° 56′. The high refractive index of the diamond makes its critical angle small, only about 24°; consequently most of the light incident upon it is internally reflected. Hence its brilliancy.

399. Absolute Refractive Power.—On the hypothesis that in a vacuum the density of the æther is unity, the density in a medium of refractive index μ is μ^2; since δ varies inversely as (speed)^2 and therefore directly as μ^2. The difference $\mu^2 - 1$ is called the **refractive power**, and the quotient of this by the density or $(\mu^2 - 1)/\delta$, the **absolute refractive power**; since it was found by Biot and Arago to be constant for air and other gases. Gladstone and Dale have given the name **refractive energy** to the value $\mu - 1$; and **absolute refractive energy** to the quotient of this by the density, or $(\mu - 1)/\delta$. The product of this value by the molecular mass is called the **molecular refractive energy**. It is found to be constant for the same substance. Moreover, its value may be calculated from the chemical formula of an organic substance containing carbon, hydrogen, and oxygen by means of the empirical expression $5a + 1{\cdot}3b + 3c$, in which a, b, and c are the number of atoms of these three elements in the molecule. It follows from this that isomeric bodies have the same refractive energy.

400. Refraction through Curved Surfaces.—Lenses.—A lens is any portion of a transparent medium bounded by curved surfaces. These limiting curves may be spherical, elliptical, parabolic, or cylindrical in form, though they are generally spherical. Lenses are divided into two classes: **converging** lenses, A, B, C (Fig. 167), thicker at the center than at the edge; and **diverging** lenses, D, E, F, which are thinner at the center. If, as in D, E, and F, the centers of curvature of the two surfaces are on opposite sides of the lens and are distant from each other by a quantity greater than the sum of the radii, the lens is a **concave** one; if distant by a less quantity than this, as in A, B, and C, a **convex** lens; A being double convex and D double concave. If one of the centers is at an indefinitely great distance, the corresponding surface is

plane, and the lens is either **plano-convex** B or **plano-concave** E. If both centers are on the same side of the lens and the radii are of different values, the lens is said to be a

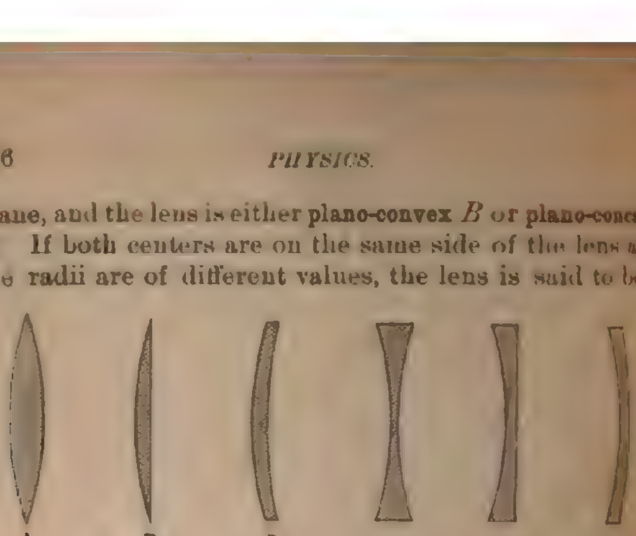


FIG. 167.

meniscus: a converging meniscus C if the surface of least curvature is farther from the center than that of greatest curvature; and a diverging meniscus F if the former is less distant.

An **axis** is any line drawn through the optical center of a lens. If it passes through the center of curvature also, it is called the **principal axis**; if not, a **secondary axis**. The **optical center** of a lens is a point on its principal axis such that a ray whose direction within the lens passes through this center suffers no angular deviation. It may readily be found by drawing a line connecting the ends of two parallel radii until it intersects the principal axis. Thus in the figure (Fig. 168) C_1a is the radius of the outer and Cb that of the inner surface, these two radii being parallel. Connect their ends with the line ab and produce it to intersect the principal axis at O. The point O is the optical center of the lens, and a ray Aa, whose direction ab within the lens passes through this center, is refracted along bB parallel to Aa, and suffers no change in its direction; evidently because the

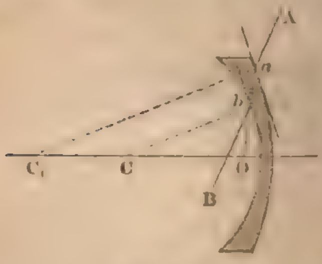


FIG. 168.

surfaces at the extremities of parallel radii are parallel tangents. Similar constructions will show that the optical center of a symmetrical convex or concave lens like A and D above (Fig. 167) coincides with its geometrical center; that for the plano-lenses B and E the optical center lies on the curved surface; and that it lies entirely without the lens in C and F, being on the convex side in the former case and on the concave side in the latter.

§ **401. Principal Focus of Converging Lenses.**—A lens may be considered as made up of an infinite number of prisms, the faces of each being tangent planes normal to the radius of curvature. Hence, as refraction always takes place toward the base of a prism, the deviation in a lens is toward that portion of it which is thickest; toward the center in converging and toward the edge in diverging lenses. Let a ray Aa (Fig. 169) be incident on the plane face of a plano-convex

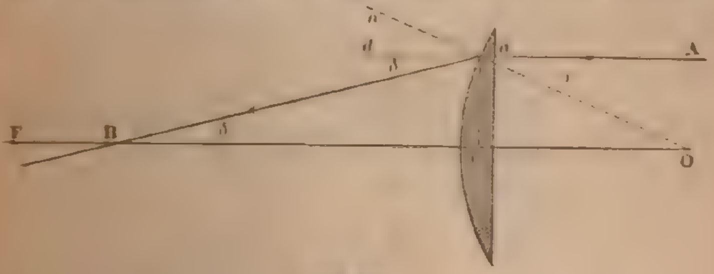


FIG. 169.

lens parallel to the principal axis OF. Since this incidence is a normal incidence, it will not be deviated but will meet the second surface at the angle OaA, which we may call i. At this surface it will suffer refraction toward the axis; the angle of emergence Bao or r, being in air, is of course larger than the angle of incidence i, in glass. We see that $\sin r = n \sin i$; and, since Bad or the deviation δ is equal to $Bao - oad$, i.e., to $r - i$, and since, further, $Bc = ac/\tan \delta$ and $ac = aO \sin i = r \sin i$, we have $BC = r \sin i/\tan \delta$ for the value of the principal focal distance. As the angle of incidence is made smaller this distance BC increases; so that the limiting

value of it when this angle is indefinitely small is FC. When the aperture of the lens is thus reduced, we may consider the arc as equal either to the sine or tangent and may write $FC = ri/\delta = ri/(r-i) = ri/(ni-i) = r/(n-1)$; that is, the principal focal distance of a plano-convex lens is equal to its radius of curvature divided by its index of refraction less one. Suppose, for example, the lens to be made of crown glass of index 1·5. The value of $r/(n-1)$ is then $r/(1·5-1)$ or $r/0·5 = 2r$; i.e., the principal focal length is twice the radius of curvature.

A similar discussion in the case of a double convex lens will show that the principal focal distance is $r/2(n-1)$, or one half that in the first case; clearly the result of the two curved refracting surfaces. If made of crown glass, the focal length will be $r/2(1·5-1) = r$; i.e., the principal focus coincides with the center of curvature. It is to be noted that in all cases the principal focal distance of a lens is a function of its radius of curvature and also of the index of refraction of its substance.

402. Conjugate Foci of Convex Lenses.—Suppose a ray fa (Fig. 170) to be incident upon a concave surface

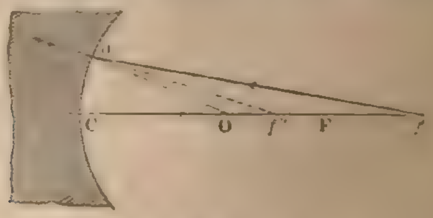


FIG. 170.

at the point a. Since the incident angle is faO, and the angle of refraction $f''aO$, we have by the law of refraction $n \sin r = \sin i$ or $n = \sin faO/\sin f''aO$. In the triangle faO we have $\sin faO : \sin fOa :: Of : af$; and in the triangle $f''aO$ we have $\sin f''aO : \sin f''Oa :: Of'' : af''$. Dividing the first proportion by the second we obtain $n : 1 :: Of/af : Of''/af''$; and hence $nOf''/af'' = Of/af$. As the incident point a approaches C, the position of f''

approaches a limiting value OF; so that at the limit we may write OF and CF for Of'' and af'', and also Cf for af; and then $n \,.\, OF/CF = Of/Cf$. If we represent the outer conjugate focal distance Cf by f, the inner CF by f'', and the radius CO by r, then Of will be $f-r$ and OF will be $f''-r$; whence $n \,.\, (f''-r)/f'' = (f-r)/f$. This readily reduces to the expression $(n-1)/r = n/f'' - 1/f$.

Now by the principle of reversibility we may consider the course of the ray emerging from the second surface of the lens precisely as if it entered the refracting medium through this surface; bearing in mind of course that the index is now the reciprocal of the former index. Representing $C'O'$ (Fig. 171), the radius of the second

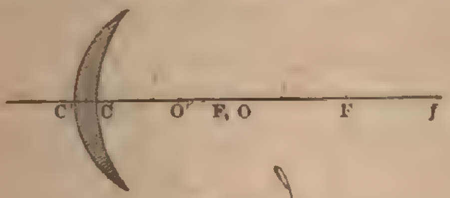


FIG. 171.

surface, by s, the distance $C'F$ of the outer focus by f'', the distance $C'F_1$ of the inner focus by f', we have from the above formula $(n-1)/s = n/f' - 1/f''$. Since the ray is emergent, the ray passes from a denser to a less dense medium and the index is $1/n$. Substituting we have $\dfrac{1/n - 1}{s} = \dfrac{1/n}{f'} - \dfrac{1}{f''}$, or, multiplying both members by n, $(1-n)/s = 1/f' - n/f''$, as the expression for the change of direction at the second surface. The total change is of course the sum of the changes at the two surfaces; and, adding the above equations, we have $(n-1)\left(\dfrac{1}{r} - \dfrac{1}{s}\right) = \dfrac{1}{f'} - \dfrac{1}{f}$ as the complete expression for the conjugate focal distances in terms of the radii of curvature and the refractive index. It will be seen that the thickness of the lens has been neglected, f'' having been used to represent $C'F$ as well as CF.

If in the above equation the distance f be supposed infinite, $1/f$ will be zero, the rays from it will be parallel, and f' will be the principal focal distance. Hence $1/f' = 1/F$; and the equation becomes $(n-1)\left(\frac{1}{r} - \frac{1}{s}\right) = \frac{1}{F}$. Replacing now in the general equation the expression $(n-1)\left(\frac{1}{r} - \frac{1}{s}\right)$ by its value $1/F$, we have $1/F = 1/f' - 1/f$. If, as is frequently the case, the two conjugate foci are on opposite sides of the lens, the relation is $1/f + 1/f' = 1/F$; i.e., the sum of the reciprocals of the conjugate focal distances is equal to the reciprocal of the principal focal distance.

403. Foci of Combined Lenses.—Since a positive lens brings parallel rays to the principal focus, it must evidently bring a convergent beam to a focus nearer the lens. In this case both foci f and f' will be on the same

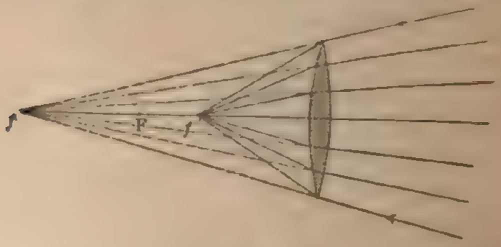


FIG. 172.

side of the lens (Fig. 172) and the formula connecting them is $1/f' = 1/F + 1/f$. If now two thin lenses A

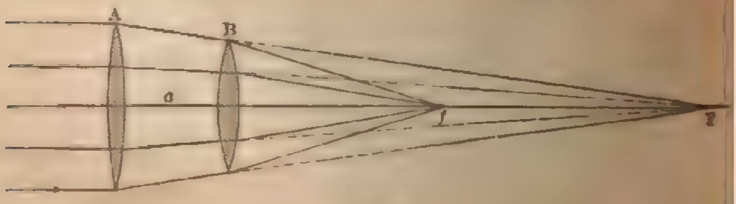


FIG. 173.

and B (Fig. 173) be placed at a distance a from each

other, the parallel rays falling on A would be brought to a focus at F if it were not for the lens B. The rays falling on B therefore form a converging beam whose virtual focus is at F, at a distance $F - a$ from B. Hence we have $1/f' = 1/F' + 1/(F - a)$, in which F' is the principal focus of lens B. Evidently if the lenses are in contact, $a = 0$ and $1/f' = 1/F' + 1/F$; and if they are of the same focal length $f' = \frac{1}{2}F$; or in other words, the focal length of the combination is one half that of either lens.

404. Virtual Foci of Lenses.—When the point from which the rays emanate is nearer the lens than the principal focal distance, these rays diverge after refraction and appear to come from a point on the same side of the lens as the actual point but farther from it. Such a focus is of course a virtual focus, since the rays do not actually pass through it.

In the case of a concave or diverging lens even parallel rays are caused to diverge by its action. And the point from which they appear to diverge is again the principal virtual focus of the lens. In all cases the general formula $1/F = 1/f' + 1/f$ applies, regard being had to the signs of the several quantities; those directions being considered positive which are measured from the lens in a direction opposite to that of the incident light.

405. Gauss's Method.—Cardinal Points.—The treatment of the more complex lens-problems may be much simplified by a method devised by Gauss in 1843. Every possible lens-system, if well centered, when taken in connection with the surrounding media, is found to possess six characteristic optical points called **cardinal points**, arranged in three pairs; two being called **focal points**, two **principal points**, and two **nodal points**. All rays moving toward the lens and traversing the first focal point become parallel to the axis; and conversely all parallel rays incident on the lens are so refracted by it as to pass through the second focal point. The second principal point is the image of the first; so that those

rays which in the first medium pass through the first principal point, pass through the second after refraction. So of the nodal points; a ray in the first medium which

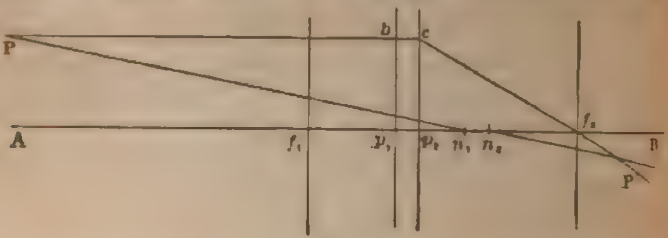


FIG. 174.

is directed toward the first nodal point, passes through the second nodal point after refraction, the direction of the rays before and after refraction being parallel. These points are shown in the diagram (Fig. 174), in which f_1 and f_2 are the focal points, p_1 and p_2 the principal points, and n_1 and n_2 the nodal points. Planes passing through the principal points are called **principal planes**, and those through the focal points **focal planes**. The distance f_1p_1 from the first focal point to the first principal point is called the first **principal focal distance**, and is positive, since the light comes from A. The distance f_1n_1 from the first focal point to the first nodal point is equal to the second principal focal distance f_2p_2, and the distance n_2f_2 from the second nodal point to the second focal point is equal to the first principal focal distance f_1p_1. Whence it follows: (1) that the difference between the two focal distances is equal to the distance between either principal point and its respective nodal point, or $f_2p_2 - f_1p_1 = n_1p_1 = n_2p_2$; and (2) that $p_1p_2 = n_1n_2$, or the distance between the two principal points is the same as that between the nodal points. Moreover, the ratio between the two principal focal distances is the same as that of the refractive indices of the first and second media, or $f_1p_1/\mu_1 = f_2p_2/\mu_2$. If therefore the two media are the same $\mu_1 = \mu_2$, the two principal focal distances are equal and consequently the principal points coincide with the corresponding

nodal points. To find, for example, the image of a point P by means of these cardinal points, draw the ray Pc parallel to the axis, and the line Pn_1 through the first nodal point. From c, the point where the ray intersects the second principal plane, draw cf_2 through the second focal point and prolong it until it intersects a parallel to Pn_1 drawn through the second nodal point. The image of P will be at this intersection P'.

406. Formation of Images by Lenses.—The image produced by a lens may be constructed by taking as many points on the object as may be necessary and constructing the images of these points separately, each on its own secondary axis. Thus for a double convex lens (Fig. 175) let the object be AB at a distance greater than the principal focal distance. Draw from

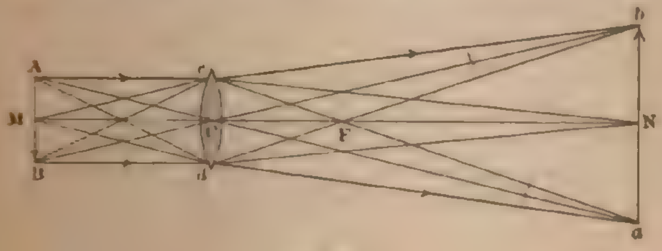


FIG. 175

A and B the two secondary axes Aa and Bb, passing through the optical center C of the lens. Let fall on the lens the parallel rays Ac and Bd from the points A and B. They will intersect the principal axis at F obviously. Continue these refracted rays until each of them intersects the secondary axis drawn from its own point; say at the points a and b. The image ab will be found at this intersection. To complete the figure draw the lines Ada, Bcb, McN and MdN. The image is seen to be real and inverted; and since it is farther from the lens than the object, it is larger than this object. By the similar triangles CAB and Cab, ab is to AB as CN is to CM; i.e., as the distance of the image is to the distance of the object.

A similar construction may be made in the case of virtual images, the object being now within the principal focus (Fig. 176). Draw the secondary axes CA and CB

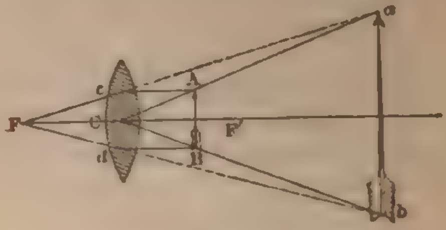


FIG. 176.

and continue them indefinitely. Draw the parallel rays Ac and Bd. They will apparently intersect at the principal focus F. The eye placed there will see the image of A on the upper line prolonged until it intersects the secondary axis at a. The image of B will be projected to b. Virtual images in converging lenses are therefore erect, are on the same side of the lens as, and are larger than, the object. Hence the use of convex lenses as magnifiers, the image being as much larger than the object as it is farther from the lens. A similar construction gives the image ab formed by a diverging

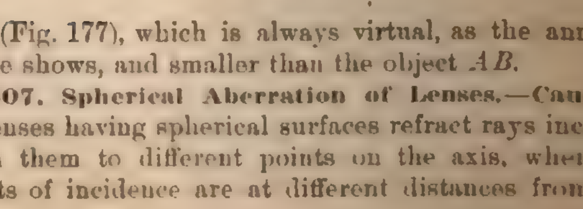


FIG. 177.

lens (Fig. 177), which is always virtual, as the annexed figure shows, and smaller than the object AB.

407. Spherical Aberration of Lenses.—Caustics. —Lenses having spherical surfaces refract rays incident upon them to different points on the axis, when the points of incidence are at different distances from this

axis. Thus the figure (Fig. 178) shows that rays incident near the circumference of the lens meet the axis at a

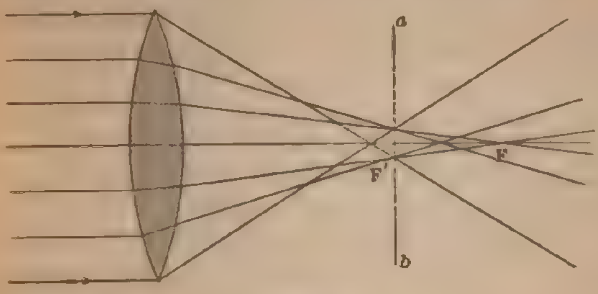


FIG. 178.

point F'; while when the incidence is nearer the axis, the focus is more distant, reaching its limiting maximum value at F when the aperture of the lens is indefinitely small. This fact is called the **spherical aberration** of a lens. The distance FF' is called the **longitudinal spherical aberration**; the minimum cross-section of the refracted beam, as at ab, the **circle of least confusion**; and the radius of this circle, the **lateral spherical aberration.** The rays thus refracted at different points of the lens are tangents to a curve which is called a caustic by refraction (393) and which is the evolute of a conic section. Lenses corrected for spherical aberration are called **aplanatic.** The form of the lens, the ratio of the radii of curvature, and the direction of the incident light upon it all affect the spherical aberration. The spherical aberration of a plano-convex lens along the axis is 4·5 times the thickness of the lens when parallel rays fall on its plane side, but only 1·7 this thickness when the rays fall on the convex side. That of a double convex lens of equal radii is 1·67 times its thickness. If its radii are as 2 to 5, the result is the same as above given for a plano-convex lens, according as the rays fall on the more or on the less convex side. With radii as 1 to 6, the spherical aberration is 1·07 times the thickness when the light is incident on the more convex and 3·45 times when on the less

convex surface. A meniscus lens refracts all rays incident on its convex side to a single focus, provided that the ratio of the focal distance from the first surface to the radius of that surface is the refractive index of the glass of which it is made. The result is even better if a combination of lenses be employed. Two plano-convex lenses of the same curvature placed with their convex sides toward each other, have an amount of spherical aberration equal to only 0·603 of the thickness of the lenses; and if their focal lengths are as 2·3 : 1, the lens of lesser curvature being turned toward parallel rays, the spherical aberration is only 0·248 times their united thickness. According to Sir John Herschel, if a double convex lens of radii 5·833 and − 35 be combined with a meniscus of radii 3·688 and 6·291, the focal length of the former being 10 and of the latter 17·829, the focal length of the compound lens will be 6·407 and the lens will be entirely free from spherical aberration; provided that the double convex lens be turned toward the object.

Inasmuch as rays from the same point are in this way brought to foci at different distances on the axis, and moreover as rays from different points of the object are brought to a focus at the same point on the axis, the effect of spherical aberration in a lens is to produce indistinctness in the image.

408. Experimental Determination of Focal Length. —As a beam of sunlight may be assumed to be composed of parallel rays, it is necessary, in order to determine the focal length of a converging lens, only to place the lens in such a beam and to measure the distance from the lens to the point where the image is a minimum. Or, as a second method, the radii of curvature may be measured directly by the spherometer and then, knowing the index of refraction of the material, the focal length of the lens may be calculated by the general formula above given. Again, if such a lens be used to form an image, and the distances (1) of the object and (2) of the image from the lens be measured,

the focal length may be calculated from the formula $F = ff'/(f + f')$; i.e., by multiplying these distances together and dividing by their sum. Or, if the image and the object be adjusted so that while they are equidistant from the lens, they are the minimum distance apart, the focal length is one fourth of this minimum distance.

If the lens be a diverging one, place it in a beam of sunlight and at such a distance from a screen that the circle of light on the screen is twice the diameter of the lens. Then the virtual focal length of the lens is equal

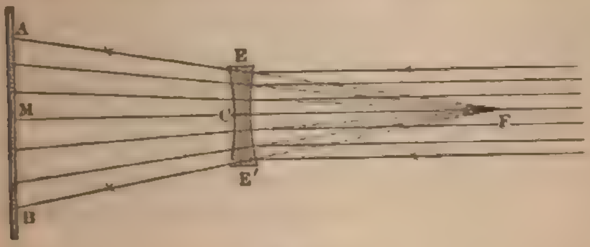


FIG. 179.

to the distance between the lens and the screen. For evidently from the figure (Fig. 179), the diameter of the illuminated circle AB is to the diameter of the lens EE as the distance FM is to the distance CF. Whence if $AB = 2EE'$, $CF = CM$.

D.—DISPERSION.

409. Refraction of Non-homogeneous Radiation.—Dispersion.—The speed of propagation in a vacuum appears to be the same for all wave-frequencies. But Fresnel suggested, and Cauchy showed mathematically, that this cannot be true in ordinary matter unless the sphere of action of the molecules is indefinitely small in comparison with a wave-length. The constitution of matter thus required, however, appears not to accord with fact. The most homogeneous medium, such as water, has, as we have seen, a grained or heterogeneous

structure whose dimensions are not incomparably smaller than the average length of a wave of light. Assuming this, Cauchy was led to suggest a relation between the wave-frequency and the speed of propagation, which expresses the latter value as a function of the former.

Since the index of refraction of any substance is the ratio of the speed of propagation of radiation in vacuo to its speed in that substance, it follows that if the speed in such a medium be a function of the wave-frequency, the index must be so also. And this is found experimentally to be the fact. Every wave-frequency has its own refractive index; and since in the case of light wave-frequency corresponds to color, every simple color has its special index. In discussing refraction thus far, we have assumed the radiation to be homogeneous; i.e., made up of vibrations of one rate only; using, when necessary to specify, the mean visible wave-frequency which is about 508 to 510 million million vibrations per second.

Moreover, since the deviation produced by refraction through a prism is a function of the refractive index, being, when the angle of the prism is small, the product of this index, less one, into the angle of the prism, it follows that the deviation produced by refraction must be different for every wave-frequency; e.g., for every color. From this we may conclude: 1st, that if homogeneous radiation of any one wave-frequency be incident on a refracting surface, it will be deviated by an amount special to itself; and 2d, that consequently, if complex radiation, containing many wave-frequencies, be so incident, these wave-frequencies will all be separated by the differential refractive action and will be arranged in the order of their refractive indices; i.e., in the order of their refrangibilities. Such a succession of wave-frequencies sorted out by a prism and arranged in the order of their refrangibilities is called a **prismatic spectrum**; and the production of such a spectrum by this differential refractive action is called **dispersion**.

ILLUSTRATION.—Thus if a complex radiant beam AO (Fig. 180) be incident on a refracting surface RR' at the point O, each of its component wave-frequencies will suffer different speed-changes, the less rapid undergoing a less diminution in speed and therefore being less deviated. For visible rays the red will appear at r, the violet at v, the intermediate colors completing the spectrum.

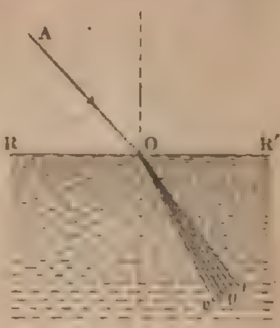


FIG. 180.

Newton was the first to discover the true nature of the visible spectrum. Placing a prism in a beam of sunlight AO (Fig. 181), he produced a solar spectrum consisting, as he assumed, of seven primary colors, red, orange, yellow, green, blue, indigo, and violet, in the order given, the red being the least and the violet the most refrangible. Submitting these colors separately

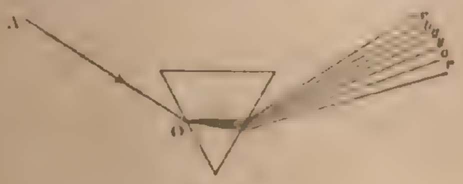


FIG. 181.

to the action of a second prism, he observed that they were homogeneous, and that each had a refrangibility special to itself, corresponding to its position in the spectrum. Hence his two theorems: "Lights which differ in color differ also in degrees of refrangibility;" and "The light of the sun consists of rays differently refrangible."

410. Dark Lines in the Solar Spectrum.—The radiation from a black body at the highest temperature obtainable artificially, as, for example, that from the positive carbon of the electric arc, contains a practically uninterrupted series of wave-frequencies, comprised be-

tween 10 and 1600 million million vibrations per second (Langley). Hence its spectrum consists of a similar interrupted series of wave frequencies, arranged side by side in the order of their refrangibilities; thus forming a continuous spectrum. In 1802, while Wollaston was observing a solar spectrum of especial purity, he noticed certain dark lines crossing it, perpendicular to its length, thus proving solar light to be deficient in certain wave-frequencies. These lines were more closely examined by Fraunhofer in 1814, who mapped 576 of them. Eight of the most prominent he designated by the first eight letters of the alphabet, the lines *A* and *B* lying in the extreme red, *C* in the orange-red, *D* in the yellow, *E* in the green, *F* and *G* in the blue, and *H* in the violet. Subsequently he added the line *a* in the red and the line *b* in the green.

The following are the wave-lengths in air (in centimeters), and also the wave-frequencies, of the Fraunhofer lines:

WAVE-LENGTHS AND WAVE-FREQUENCIES (BELL.)

Line.	Wave-length.	Wave-frequency.	Line.	Wave-length.	Wave-frequency.
A	$7{\cdot}621 \times 10^{-5}$	$3{\cdot}936 \times 10^{14}$	E_1	$5{\cdot}269 \times 10^{-5}$	$5{\cdot}694 \times 10^{14}$
B	6·884 "	4·358 "	b_1	5·183 "	5·788 "
C	6·563 "	4·571 "	F	4·861 "	6·172 "
D_1	5·896 "	5·088 "	G	4·307 "	6·965 "
D_2	5·890 "	5·093 "	H	3·968 "	7·560 "

Since these dark lines indicate the absence in solar light of definite wave-frequencies, they constitute fixed points of reference in the spectrum, which are of great use in measuring the changes which the refractive index undergoes for given variations in wave-frequency, and also the changes which this index suffers in different media for the same wave-frequency. Thus, for example, the spectra of sunlight given by prisms of the same angle, made of flint glass, crown glass, and water, respectively, as shown

in Figure 182, are quite different. For a given deviation of the Fraunhofer line *B*, the differential refraction between the lines *B* and *H*, which measures the dispersion, is more than twice as great in flint glass as in crown, and more than three times as great as in water; thus indicating the value of these lines as fixed points of reference.

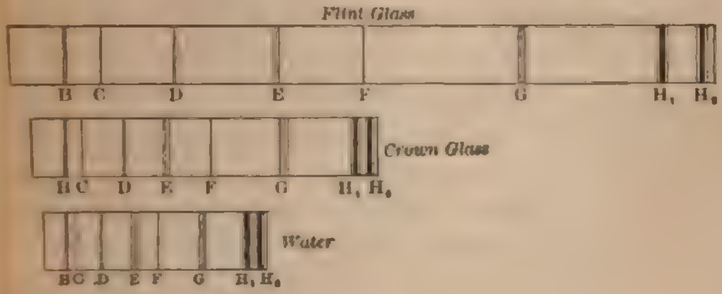


FIG. 182.

The indices of refraction given in the table on page 433 are the indices for the particular wave-frequency represented by the Fraunhofer line *D*.

REFRACTIVE INDICES FOR THE FRAUNHOFER LINES

Substance.	A	B	C	D	E	F	G	H
Crown glass	1·5089	1·5109	1·5119	1·5146	1·5180	1·5210	1.5266	1·5314
Flint glass (dense)	1·6965	1·7010	1·7034	1·7102	1·7121	1·7272	1·7482	1·7565
Rock salt	1·5366	1·5392	1·5405	1·5442	1·5490	1·5532	1·5613	1·5682
Alum	1·4500	1·4520	1·4530	1·4560	1·4589	1·4614	1·4656	1·4691
Water	1·3284	1·3300	1·3307	1·3324	1·3347	1·3366	1·3402	1·3431
Carbon disulphide... ...	1·6142	1·6207	1·6240	1·6333	1·6465	1·6584	1·6836	1·7090
Air ...	1·0002895	2899	2902	2911	2922	2931	2948	2963

411. Dispersive Power.—Inasmuch as dispersion is simply the differential deviation, by refraction, of rays of different wave-frequencies, it is evident that we may measure the total dispersion of any substance in terms of the extreme angular deviations produced; or if δ_H be the deviation for the *H* line and δ_A that for the *A* line, $\delta_H - \delta_A$ will represent the total dispersion. But we have seen above that $\delta = (n - 1)\alpha$; and consequently if $\delta_H = (n_H - 1)\alpha$ and $\delta_A = (n_A - 1)\alpha$, the dispersion $\delta_H - \delta_A$ in terms of the refractive indices will be $(n_H - n_A)\alpha$.

The **dispersive power** of a body is defined as the ratio of the total dispersion to the mean dispersion; or to $(\delta_H - \delta_A)/\delta_E$. Since $\delta_H - \delta_A = (n_H - n_A)\alpha$, and $\delta_E = (n_E - 1)\alpha$, we have for the value of the dispersive power $(n_H - n_A)/(n_E - 1)$; in other words, the dispersive power is equal to the difference of the extreme refractive indices divided by the mean index less one. It is independent of the prism-angle.

Thus for crown glass the dispersion is $0{\cdot}0225\alpha$ and the dispersive power 0·0434; while for carbon disulphide the dispersion is $0{\cdot}0948\alpha$ and the dispersive power 0·1466. Hence for the same mean deviation the spectrum produced by a prism of carbon disulphide is between three and four times as long as the spectrum produced by a prism of crown glass of the same angle.

412. Irrationality of Dispersion.—The differential deviation for the intermediate Fraunhofer lines is called **partial dispersion**: which for prisms of the same angle is proportional to the differences of the refractive indices for these lines. On comparing together the partial dispersions of different substances for the same portion of the spectrum, it is found that they do not agree even when the total dispersion is the same for both. Thus while the difference between the lines B and C in flint glass is 2·562 times this difference in water, the difference between the lines G and H is 3·726 times. Evidently, therefore, if two spectra produced by different refractive substances be superposed, they will not correspond in their wave-frequencies, even if they are made to have exactly the same total length. This want of proportionality in the spectra given by different refractive media is called **irrationality of dispersion**. In its extreme form it may even invert the order of the wave-frequencies; and it is then called **anomalous dispersion**.

413. Achromatism in Prisms and Lenses.—When non-homogeneous radiation is incident upon a refracting surface, it is dispersed; i.e., its constituent wave-frequencies, being differently refracted, are separated from one another. On emerging from a second surface par-

allel to the first, however, both refraction and dispersion disappear and the ray emerges as it entered. If two prisms of the same material be taken and one be reversed in position, their refracting surfaces will be parallel and there will be no deviation. The deviation, however, is a function, not only of the excess above unity of the refractive index, but also of the prism-angle. So that if these two quantities vary inversely as each other, their product, i.e., the deviation, will be constant. A prism of flint glass of mean index 1·66 and angle 20° will deviate a ray $(n - 1)a$ or 13·2 degrees; while one of water of the same angle, whose index is 1·33, will deviate it only 6·6°. Hence if a flint prism of 10° be combined with a reversed water prism of 20°, the radiation will pass through the system without suffering any deviation of the mean ray. Moreover, if the dispersion were proportional to the mean refraction, then the dispersion also would be corrected by this arrangement. But this is not the fact. The dispersion of the flint-glass prism of 10° above mentioned will be $(n_H - n_A)a$ or about 0·5°; while that of the water prism of 20° is about 0·3°. Consequently under these conditions there will be a residual dispersion due to the flint glass of about 0·2°. That is, the combination will give a spectrum whose angular breadth between the *A* and *H* lines will be 12 minutes of arc. Such a combination, therefore, gives dispersion without deviation. It is called a **direct-vision prism**.

If, on the other hand, the angle of the more highly refractive prism be adjusted so as to produce, not the same mean deviation, but the same difference of extreme deviation, i.e., the same dispersion, as that given by the less refractive one, then the dispersion will be corrected but not the deviation; and we shall have a compound prism which deviates a ray without dispersing it. In the case of light, such a combination is called an **achromatic prism**, since its image is devoid of color. The condition required, which is called the condition of achromatism, is in theory a simple one, being only that the sum of the dispersions shall be equal to zero. Suppose two prisms

of dispersions $(n_H - n_A)\alpha$ and $(n_H' - n_A')\alpha'$; from the above condition we have $(n_H - n_A)\alpha + (n_H' - n_A')\alpha' = 0$. Whence $\frac{\alpha}{\alpha'} = -\frac{n_H' - n_A'}{n_H - n_A}$; or the prism-angles must be inversely as the extreme index-differences of the two media employed for the prisms. The two members of the equation are of opposite sign. This means only that the two prisms must face opposite ways. Owing to the irrationality of dispersion, however, correction for extreme rays does not in general correct for the mean ones; and there remains, therefore, a residuum of color.

In the case of a lens there may evidently be as many foci as there are wave-frequencies in the incident radiation. When white light falls on a converging lens, for example (Fig. 183), the violet component being most

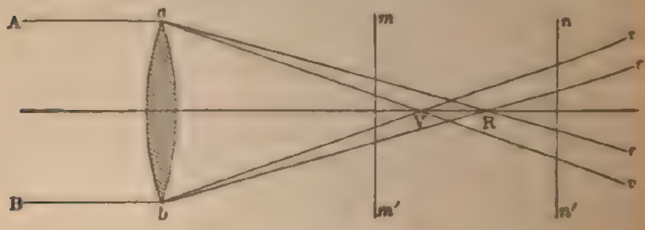


FIG. 183.

refracted will form its principal focus at V; while the focus of the red will be at R. The intermediate colors of the spectrum will come to intermediate foci. Hence a cross-section of the beam at mm' will show a violet center, while at nn' the center will be red. This phenomenon is called the **chromatic aberration** of a lens. In place of a single and uncolored image, a number of overlapping images appear, of various colors, producing a confused and indistinct effect.

By combining with this converging lens a diverging lens of such focal length that the dispersion shall be destroyed without destroying the deviation, an **achromatic lens** (Fig. 184) may be produced. This condition is at-

ined when the focal lengths of the two lenses are irectly proportional to their dispersions. In eneral, however, the dispersion in this case is ot the differences of deviation of the extreme ays, but those of less refrangibility; say between he greenish blue and the orange-yellow. For xample, suppose we have a converging lens of rown glass AA' of 20 centimeters focal length. To correct the chromatic aberration of this lens o that the lines C and F shall be brought to he same focus, a flint-glass diverging lens BB' f 33 centimeters focal length will be required; ssuming the dispersions of the flint and crown glass for hese two lines to be in the ratio of 33 to 20. The focal ength of such an achromatic lens would be 51 centimeters.

FIG. 184.

414. Experimental Measurement of Dispersion.—The dispersive power of a substance may be measured directly in terms of that of some standard substance. Water is generally taken as the standard of reference. To make the comparison, a hollow prism is provided, whose glass sides are hinged near the refracting edge, so that they may be placed parallel to each other or at any convenient angle. The prism being filled with water and the sides being made parallel, a ray sent through it suffers no deviation. The substance to be examined is made into a thin prism of known angle and placed in the water, its refracting edge being in an inverted position with reference to that of the water prism. The ray is now deviated, of course toward the base of this prism. The angle of the water prism is then increased until the ray emerges colorless and the angle of the water prism is read. The ratio of the prism-angles is then the inverse ratio of the dispersive powers; since

$$\frac{(n_H - n_A)(n_F - 1)}{(n_H' - n_A')(n_E - 1)} = \frac{n_H - n_A}{n_H' - n_A'}, \text{ and } \frac{\alpha'}{\alpha} = -\frac{n_H - n_A}{n_H' - n_A'},$$

from the principles of achromatism already discussed.

415. Spectrum Analysis.—When non-homogeneous radiation falls upon a prism its different constituent

rays are differently refracted, and in this way are separated from one another, so as to form a spectrum made up of these wave-frequencies arranged in the order of their refrangibilities. Since the length of the spectrum is the distance between the extreme rays as they diverge from the prism, the spectrum is longer the farther the screen on which it is received is from the prism. It is evident that this spreading out of the constituents of a given radiation into a spectrum by a prism may be utilized to ascertain the wave-frequencies which it contains. Thus if the extremely complex radiation from a carbon rod at high incandescence be subjected to analysis by the prism, its spectrum will be found to consist of an indefinite number of wave-frequencies extending uninterruptedly from about 10 to 1600 million million vibrations per second. While on the other hand if the radiation from incandescent sodium vapor, produced when a little salt is placed in a non-luminous gas-flame, be examined, its spectrum is found to consist of but two fine lines in the yellow, having wave-frequencies of about 508·8 and 509·3 million million respectively. This analysis of a composite radiation by means of its spectrum is called **spectrum analysis**.

416. The Spectroscope.—The instrument used in spectrum analysis is called a **spectroscope**. In its simplest form (Fig. 185) it consists of a glass prism and two telescopes, so adjusted that their axes make equal angles with the two sides of the prism. The inner ends of these telescopes carry achromatic lenses. At the outer end of one of them is a slit, adjustable by means of a screw to any desired width, and placed at the principal focus of its lens. Such a telescope is called a **collimator**. The radiation enters the slit, is rendered parallel by the lens, traverses the prism at the angle of mean minimum deviation (and so is not distorted), is received after dispersion on the lens of the second telescope and is by it brought to a focus. The spectrum thus produced is viewed by means of the eyepiece. This spectrum is received in this case directly upon the retina of the eye

and may thus be distinctly seen even when the radiation is quite feeble.

Where greater dispersion is required than can be given by a single prism, a series of prisms is used, up to

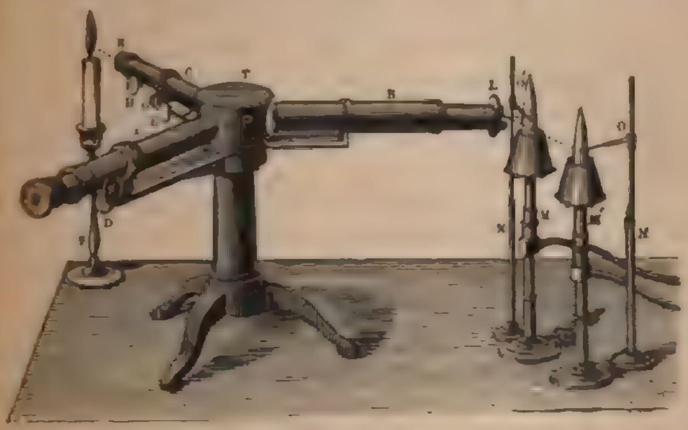


FIG. 185.

ten or more; these being adjusted, automatically in some cases, to minimum deviation. In such a case the total deviation of the ray may amount to more than an entire circumference.

Chemists had long known that certain substances when heated gave colored flames. The yellow of sodium, the crimson of lithium, the lilac of potassium, the red of strontium, and the green of barium are familiar to every one. In 1859, Bunsen was led to examine through a prism the light emitted by these flames and to map their spectra. He found that these spectra were characteristic and hence could be used for the identification of these elements either free or in combination. Moreover, the quantity of material required for the purpose was found to be exceedingly minute. One two-thousandth of a milligram of barium, one three-thousandth of a milligram of potassium, one thirty-thousandth of strontium, one fifty-thousandth of

calcium, one six-hundred-thousandth of a milligram of lithium, and one fourteen-millionth of a milligram of sodium suffices to produce in the spectroscope the characteristic lines when the substance is placed in the non-luminous gas-flame. But if the higher temperature of the electric spark be used to render these vapors incandescent, this extraordinary delicacy may be increased; so that now one ten-millionth of a milligram of calcium, one forty-millionth of a milligram of lithium, and one hundred-millionth of a milligram of strontium is sufficient to give the spectrum characteristic of these elements. This discovery placed in the hands of the chemist a method of qualitative analysis of far greater power than any hitherto in use. And one of the earliest of the results obtained was the fact of the extreme generality of diffusion of the elements in nature. Moreover, before the new method had left the hands of its discoverer he added two new substances to the list of the chemical elements, called cæsium and rubidium respectively, from the characteristic blue and red lines in their spectra. Subsequently the elements thallium, indium, gallium, and germanium were discovered by Crookes, Reich and Richter, Lecoq de Boisbaudran, and Winkler, in consequence of peculiarities in their emission-spectra.

The condition necessary for the production of the emission-spectrum is simply a high temperature. The substance to be examined must not only be converted into vapor, but this vapor must be dissociated and rendered luminous. And since the radiating power of gases is very feeble, this requires them to be strongly heated. In some cases where the substance is readily volatile, as sodium chloride, for example, a non-luminous gas-flame suffices for the purpose; the spectrum obtained being that of the sodium alone, the chlorine becoming luminous only at a much higher temperature. Strontium and calcium salts, however, are not completely dissociated in a gas-flame; and their spectra consist of bands due probably to their oxides volatilizing as such.

When, however, they are subjected to the higher temperature of the electric spark, complete dissociation takes place and the line spectra characteristic of the metals are obtained. In the case of gases they are enclosed in glass tubes of a form devised by Plücker (Fig. 186) consisting of enlarged cylindrical ends connected by a narrow capillary middle portion, in which the electric discharge is condensed. In general the emission-spectrum of an element becomes more complex as the temperature rises.

FIG. 186

417. Absorption-spectra.—The spectrum of an incandescent solid of high radiating power consists, as we have seen, of an uninterrupted band of color from the extreme red on the one hand to the extreme violet on the other. The spectrum of an incandescent gas, on the other hand, consists of isolated bright lines, greater or less in number according to the special gas employed. We have now to consider a third class of spectra, consisting of dark lines upon a bright and colored background; a continuous spectrum, in fact, from which certain wave-frequencies have been removed, thus causing gaps or dark spaces. Such spectra are due to absorption in a medium which the radiation has been caused to traverse. They are called **absorption-spectra.**

In a previous section attention has been called to the fact that absorption may be **general**; that is, exercised equally upon all wave-frequencies existing in the radiation; or it may be **special** or selective; i.e., confined to particular wave-frequencies. Evidently if complex radiation be transmitted through a medium possessing selective absorption, it will emerge with the loss of the absorbed wave-frequencies; and in its spectrum consequently there will be dark spaces corresponding to the absorbed radiations. Thus the absorption-spectrum produced from white light which has traversed a solution of didymium is as distinct and definite as any

emission-spectrum, its dark lines being of constant wave-frequencies. So, too, the absorption-spectrum of blood is perfectly characteristic, and the indication is so delicate that a single blood-corpuscle is sufficient to yield the spectrum in the micro-spectroscope. Moreover, apparently colorless liquids exert in some cases selective absorption and give absorption-spectra. Russell and Lapraik have observed dark bands in the light transmitted through columns about 2 meters long of water, alcohol, ether, and chloroform. And in the case of a saturated solution of ammonia in water, they observed as many as five absorption-bands. The absorption-bands due to the aqueous vapor, the oxygen, and the carbon dioxide of our atmosphere have been already mentioned (370).

418. The Solar Spectrum.—The spectrum of sunlight, as has been stated (410), contains a large number of dark absorption-lines, which were first observed by Wollaston (1802) and many of which were mapped by Fraunhofer (1814), who distinguished eight of them by the letters of the alphabet. Although a few of these spectrum lines are due to absorption in the earth's atmosphere, by far the greater number originate in the selective absorption of the solar atmosphere itself. And since this absorption relates to the same wave-frequencies as those which the absorbing substance would itself emit under suitable conditions, it is clear that identity of absent wave-frequencies indicates identity in the absorbing media. Hence the law of Stokes, developed in 1859 by Kirchhoff, enabled the process of spectrum analysis to be applied to the sun. The coincidence in position between the double solar line *D* and the double yellow line of sodium, as observed by Fraunhofer in 1817, is a proof that the wave-frequencies concerned are the same for both. And therefore, since no wave-frequencies are known to be common to two different elements, this coincidence in position is a proof that the double line *D* is produced by sodium existing in the sun. Kirchhoff therefore undertook an elaborate

investigation of the solar spectrum, mapping with great care nearly 3000 of its lines by means of a scale attached to his spectroscope. At the same time he undertook to compare the position of the bright lines of the spark spectra of several of the metals with that of the dark lines of the sun spectrum; using for the purpose a small reflection-prism covering the upper half of the slit. As a result of these comparisons he concluded that sodium, iron, calcium, magnesium, nickel, barium, copper, zinc, and probably potassium are existent in the sun; and that gold, silver, mercury, aluminum, cadmium, tin, lead, antimony, arsenic, strontium, lithium, and silicon are not present there. In the case of iron, for example, he mapped no less than 460 lines in its emission-spectrum; and to every one of these bright lines, there corresponded exactly in position and width a dark line in the solar spec-

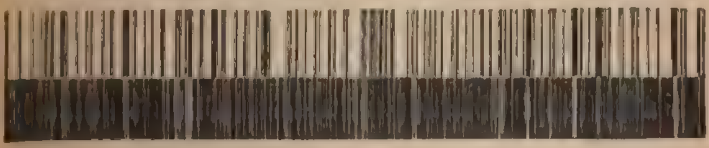


FIG. 187.

trum. This coincidence of the lines of iron with solar lines is well shown in Figure 187, which is taken from a photograph by Trowbridge. That this coincidence of 460 lines is a mere chance is of course possible; but the theory of probabilities teaches us that the chance-hypothesis is to the hypothesis that these lines are caused by iron in the sun's atmosphere, in the ratio of 1 to 2^{460}; i.e., equivalent to a positive demonstration.

Foucault, in 1849, had reversed the yellow double line in the spectrum of the electric arc, simply by reflecting the image of one of the carbon points through it; the spectrum then showing a double dark line in place of the yellow one. Kirchhoff interposed an alcohol-flame containing salt in the path of the rays from a calcium light and observed that the bright double line of sodium was reversed; i.e., was converted into a dark

one. On transmitting sunlight through the alcohol-flame, its spectrum showed the dark double line *D* greatly increased in intensity. Kirchhoff thereupon became convinced that the phenomenon of dark-line production was the same upon the sun as upon the earth; and was thus led to his celebrated theory of the solar constitution. This theory supposes a central luminous nucleus or **photosphere**, emitting all wave-frequencies; and, surrounding this, a gaseous envelope, called the **chromosphere**, also intensely hot, emitting only the wave-frequencies belonging to its elemental constituents. Evidently the radiation from the photosphere, in order to reach us, must traverse the chromosphere; and must there lose by absorption the wave-frequencies characteristic of the chromospheric elements. Could we see the photosphere alone, its spectrum would be continuous. Could we see the chromosphere alone, its spectrum would consist of bright lines. In fact, however, we see the former, only after its light has traversed the latter, in which an absorption has taken place so as to reverse the lines. During the solar eclipses of 1868 and 1869, when the bright disk of the sun was covered by the moon, the solar edge was seen to consist of a crimson layer, massed at certain points into protuberances, the spectrum of which did actually consist of bright lines. In 1872, Young observed 273 bright lines in the chromospheric spectrum, in the clear air of Sherman, W. T., at an altitude of about 2500 meters, and identified these lines with those of 27 elements.

The reflected radiation from the moon and planets is identical with that from the sun, except in so far as it is modified by selective absorption upon these bodies. In the case of Uranus this selective absorption is well marked. The radiation emitted by the fixed stars, however, while giving a spectrum full of dark absorption-lines, appears to be more or less special for each star, showing that while in general the fixed stars are suns constituted like our own, yet that they differ in the details of their composition. Thus Huggins has shown

that Aldebaran contains bismuth and tellurium, elements not found in the sun, while barium, an element contained in the sun, is absent from the star. Betelgeux and β Pegasi do not contain hydrogen, and the former star does not contain silver, mercury, cadmium, lithium, or tin. Moreover, the color of certain stars appears to be due to the massing of the absorption-lines in particular regions of the spectrum. Secchi and more recently Pickering have observed spectrum similarities in the stars which have enabled these astronomers to divide all of them into four groups: (1) White stars, such as Sirius; (2) Yellow stars, like the sun and Capella; (3) Red and orange stars, as Betelgeux; and (4) Small red stars, giving banded spectra. The nebulæ proper give bright line spectra, showing that they consist of rarefied incandescent gas. The spectrum of comets is also a bright line or band spectrum, the bands showing a remarkable identity with those given by certain hydrocarbons.

419. Effect of Motion in the Radiating Body.—If the body emitting the radiation be itself in motion, either to or from the observer, evidently the effect of this motion will be to shorten the wave-length in the former case and to lengthen it in the latter. Since, other things being equal, refrangibility is dependent upon the wave-length, the result will be a displacement of the characteristic spectrum lines toward either the more or the less refrangible end. In a continuous spectrum this change of position could not be detected; but in a bright-line or a dark-line spectrum it has not only been detected, but measured. Thus, for example, Lockyer has observed changes in refrangibility of the *C* and *F* lines of the sun spectrum corresponding to a speed of 163 kilometers per second in the line of sight; and Young gives 400 kilometers a second as the maximum speed observed by him upon the sun by means of the displacement of the hydrogen lines. Zöllner contrived a reversion-spectroscope for the purpose of measuring the displacements in opposite directions, of

given lines in the spectra from the east and west limbs of the sun, and in this way of determining the time of the solar rotation. And Crew has observed a solar displacement corresponding to a speed of rotation of 3·9 kilometers per second. Huggins applied this method in an investigation upon the motion of the stars in the line of sight. He observed a displacement in the F line of the spectrum of Sirius, for example, corresponding to an increase in the wave-length of this line of 0·11 millionth of a millimeter; thus proving that Sirius is receding from the earth at the rate of about 35 kilometers a second. Pickering has observed that the k line in the spectrum of β Aurigæ is alternately single and double at intervals of about 17 hours; thus showing this star to be double, the two components revolving about each other in less than four days with a speed of 240 kilometers a second.

420. Peculiarities of the Prismatic Spectrum.—The phenomena of the irrationality of dispersion and of anomalous dispersion have been mentioned (412). In both cases, even when the spectrum is of the same length, the distribution of its parts as to wave-frequency is different for the different substances of which the prisms are made, the order of refrangibilities being sometimes actually inverted. There is however another peculiarity of the prismatic spectrum, at least in the visible portion, which has been the cause of wide-spread error. This is the fact that the refrangibility here is not a simple linear function of the wave-length. If, for example, the position of a series of well-known lines, either bright or dark, be observed in a spectroscope provided with a scale, and if the scale-numbers of the lines thus obtained be used as abscissas, the corresponding wave-lengths, taken from the tables, being made ordinates, a curve will be obtained showing the distribution of wave-lengths in the spectrum of the particular prism employed. This curve will be convex toward the axis of abscissas; thus showing a [illegible]sing of wave-lengths in the less refrangible region. In [illegible]quence of this, the amount of energy in this region,

as measured by the bolometer, is greater for the same spectrum-length than in any other portion; a result which has given rise to the erroneous conclusion that the energy-maximum is in the ultra-red region. If, on the other hand, a spectrum be constructed by distributing its lines through it according to their wave-lengths, then a given difference of distance in every part of the spectrum will correspond to a given difference of wave-length and the curve representing this distribution of wave-lengths will become a straight line. Such a spectrum is called **a normal spectrum.**

The relation between wave-length and refrangibility has been investigated by several physicists. Among these Cauchy was one of the earliest; and he gave the formula $n = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$; in which n is the refractive index, λ the wave-length, and a, b, and c constants to be determined by experiment. The agreement between the results of observation and those calculated from this equation has been found very close within the limits of the visible spectrum; but Langley has shown that the formula gives impossible results when applied to the infra-red region of the spectrum. In fact, since b and c are both positive, the least possible value of n is a, obtained when b and c are both zero. But in the case of one of his prisms, in which $a = 1{\cdot}5593$, this least value of n corresponds to a deviation of 45° 35′; whereas with this prism he has actually made bolometric measurements in the sun spectrum as low as 44°. The best results he finds to be given by the formula of Briot, $\frac{1}{n^2} = a + b\frac{n^2}{\lambda^2} + c\frac{n^4}{\lambda^4} + k\frac{\lambda^2}{n^2}$; although at the limit of ·002356 millimeter he finds that the index of refraction becomes very nearly a linear function of the wave-length.

E.—APPLICATIONS OF OPTICAL PRINCIPLES.

421. Optical Instruments.—Instruments which depend upon optical principles for their mode of operation are called in the general sense optical instruments. Their essential element in most cases is a lens or mirror, by means of which the rays from an object are so deviated as to secure a visual advantage.

422. The Microscope.—As its name implies, the microscope is an instrument for viewing minute objects under an increased visual angle. The simple microscope consists of a single converging lens, or its equivalent, so placed that the object is between the principal focus and the lens. Under these circumstances, as we have already seen (406), the image formed by the lens is virtual and erect, and is larger than the object. Hence the lens is said to magnify the object. The amount of magnification is of course the ratio of the size of the image to that of the object, both being at the same distance from the eye. But since both object and image are contained between secondary axes, drawn from the center of the lens, the size of the image is to that of the object evidently as the distance of the image is to the distance of the object. The distance of the image, however, is fixed by the eye itself. For normal eyes, an object to be seen most distinctly must be placed at a distance of 25 to 30 centimeters; this distance being called ordinarily the distance of most distinct vision. The object, therefore, with reference to the lens must be placed so far removed from it that its virtual image is formed at a distance of 25 or 30 centimeters from this lens. This operation is called focusing. The magnifying power is then represented by the ratio $25/d$ or $30/d$, where d is the distance of the object from the lens. If the lens be of short focus, as is usually the case, so that its principal focal distance is practically the distance of the object, the magnifying power is simply the ratio of the distance of most distinct vision to the focal length.

Thus a lens of 10 centimeters focus would have a magnifying power of two and one half or three linear; the magnifying power being greater the less the focal length of the lens, and the greater the distance of most distinct vision of the observer. The reciprocal of the focal length is sometimes called the **power** of the lens.

Again, in the figure (Fig. 188) the image ab subtends at the eye E twice an angle whose tangent is ap/Ep.

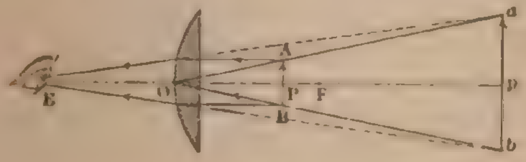


FIG. 188.

The object AB placed at the same distance would subtend twice an angle whose tangent is AP/Ep. Supposing these angles small, the magnifying power would be the ratio of ap/Ep to AP/Ep, or ap/AP; and $ap : AP :: Op : OP = f : f'$. But from the law of the lens $1/f - 1/f' = -1/F$; whence the magnifying power, which is represented by f/f', is equal to $1 + f/F$. According to this the magnifying power of a lens of 2 centimeters focus would be $\frac{30}{2} + 1$ or 16 diameters. By using two plano-convex lenses slightly separated, the convex sides facing the object, the spherical aberration, for the same magnifying power, is much diminished. This combination is known as **Wollaston's doublet.**

In the compound microscope, as well as in the telescope, it is the image of the object which is magnified by the eye-lens and not the object itself. In both cases this image is formed by means of a converging lens called the object-glass, the object being placed without the principal focus; hence it is a real image.

Under these circumstances f is negative and the expression for the magnifying power becomes $1 - f/F$. In the case of the microscope, however, the distance of the object is less than twice the principal focal distance,

while in the telescope it is greater. The magnifying power therefore will be greater than unity numerically in the former case, where $f/F > 2$; i.e., where $f' < 2F$. The object is consequently magnified in the case of the microscope; while in the telescope the real image is smaller than the object.

A compound microscope (Fig. 189) consists (1) of an object-glass or objective A, whose function is to form the real image of an object placed at s just outside its principal focus; and (2) of an eye-lens or eye-piece B, which produces a magnified virtual image of the real image, the latter being formed approximately at the principal focus of the eye-lens. In practice both the objective and the eye-piece are themselves compound. The objective may consist of two or three achromatic lenses, and the eye-piece of two or more simple lenses. The focal length of such an objective is that of the simple lens to which it is equivalent. Two forms of eye-piece are in use, one known as the negative or Huyghens eye-piece, the other as the positive or Ramsden eye-piece. The negative eye-piece has two converging lenses called the field-lens and the eye-lens, respectively, the focal length of the former being three times that of the latter, the distance between them being the difference of their focal lengths. The field-lens is placed next to the objective and is generally a meniscus; the eye-lens is plano-convex, the convexities of both lenses being turned toward the object. The positive eye-piece consists also of a field-lens and an eye-lens, both plano-convex and of the same focal length, but placed with their convexities facing each other, and at a distance apart equal to two thirds the focal length of either. The Huyghenian eye-piece is called negative because it can-

FIG. 189.

not be used directly to view an object. The real image to be viewed by it must be formed at the principal focus of the eye-lens, and therefore within the eye-piece itself; where also cross-wires, micrometers, and the like must be placed, which are to be seen simultaneously with the image. After its construction it was found to be achromatic when both lenses are made of the same glass. The focal plane of the Ramsden eye-piece on the contrary is outside of the field-lens and at a distance from it equal to one quarter of its focal length. This eye-piece, therefore, views directly the image formed by the objective, together with the cross-wires or micrometric graduations which are in the same plane with it. And hence this form of eye-piece is preferred for measurements.

The magnifying power of a compound microscope is the ratio of the angle subtended at the eye by the image to that subtended by the object, both at the distance of most distinct vision. Since both the objective and the eye-lens magnify, the conjoint magnifying power, being the product of that of the objective by that of the eye-lens, is greater than either. Experimentally, the magnifying power may be ascertained by placing a micrometer ruled to tenths of a millimeter upon the stage, and noting how many divisions of a millimeter scale at the same distance, one division of the micrometer covers. If the apparent size of the image, thus measured, be, for example, ten millimeters, the magnifying power is evidently one hundred diameters.

423. The Telescope.—The telescope is an instrument for increasing the angle under which a distant object is seen. The object-glass, which is achromatic, forms a real and reduced image, which is magnified by the eye-piece. The eye-pieces used with the telescope are the same as those used with the microscope. For astronomical purposes, the object-glass is large in order to secure as much light as possible, the light entering the eye being in the ratio of the square of the diameter of the glass to that of the pupil. The great refractor at the Washington Observatory has an aperture of 66 centime-

ters, that of Pulkova 76 centimeters, and that of the Lick Observatory, California, 91 centimeters; all these object-glasses having been made by the Clark Brothers in Cambridge. Recently, some notable object-glasses of Jena glass have been made by Brashear of Allegheny, from curves calculated by Hastings.

Mirrors have also been used in telescopes in place of lenses. The images which they give are free from chromatic aberration and they can be made of any desired size. The great reflector of Lord Rosse has a mirror of speculum metal 16 meters in focal length and 183 centimeters across. Foucault introduced silvered mirrors of glass and Draper constructed an admirable silvered glass reflector of seventy-one centimeters aperture. The glass reflector of the Paris Observatory is 122 centimeters and a new English reflector is 213 centimeters. The earliest form was Gregorian, in which the light, converged from the great mirror, fell on a secondary concave mirror placed in the axis of the tube and beyond its focus; and was by this reflected to the eye-piece through an opening in the center of the large mirror. Subsequently Newton placed a plane secondary mirror in the convergent beam near its focus, this mirror being inclined 45° to the axis; and thus threw the image into the eye-piece placed on the side of the tube and at right angles with the axis. Cassegrain about the same time modified the Gregorian form by using a convex secondary mirror placed within the focus of the reflector, thus shortening the telescope greatly. Finally Herschel tilted the mirror slightly so that the rays converged to a focus close to one side of the tube; the observer facing the mirror.

In the case of the object-glass of the telescope, $f' > 2F$ and hence $f/F < 2$; hence the magnifying power $1 - f/F$ is less than unity and the image is smaller than the object. In the case of the eye-piece, the magnifying power is $1 + f'/F'$ as the focus is virtual. Thus (Fig. 190) let ab be the image of the object AB formed by the object-glass O. The angle AOB subtended by the object

center of the object-glass is the same as aOb sub- by its image; and since the object is at a great e, this angle AOB will not materially differ from ubtended at the eye placed at E. Since the angle ded by the image is aEb, we have for the magni- power the ratio aEb/aOb; or since the angles,

FIG. 190.

mall, may be replaced by their tangents, the ratio $\frac{Eb}{Ob} = \frac{ap/Ep}{ap/Op} = Op/Ep$. In other words, the ying power is as Op to Ep, or in the ratio of the ngths of the two lenses. If, on the other hand, age and the object are assumed to be at the same e from the eye, i.e., at the distance EP, then mes A', the ratio is $\frac{A'P/EP}{AP/EP}$, and the magnifying is as $A'P$ to AP; or as the size of the image is to e of the object, both at the same distance. In the f the telescope the magnifying power is the ratio focal lengths of object-glass and eye-piece F/F'. e magnifying power m of the object-glass is y and that of the eye-piece m' is inversely as the ength. Hence the magnifying power of the com- n F/F' must equal mm'; or is equal to the mag- power of the eye-piece multiplied by that of the glass.

the Galileo telescope, now commonly known as ra-glass, the eye-piece consists of a single diverg- e, placed within the focus of the object-glass, the e between the two being the difference of their ngths. The convergent rays from the object-glass s made parallel to the secondary axis upon which

the point lies and the eye is enabled to form a virtual image of that point. The image in this telescope is erect and the instrument is much shorter in length.

In terrestrial telescopes a pair of convex lenses is introduced between the object-glass and the eye-piece for the purpose of inverting the image. They have the same focal length, are placed at a distance apart equal to the sum of their focal lengths, and the first is at a distance from the real image equal to its focal length. They are usually contained in the draw-tube which carries the eye-piece.

Other optical instruments might be described. But the practical application of optical principles to the construction of instruments has been sufficiently discussed for our purpose.

424. The Eye and Vision.—The optical portions of the eye are (1) the crystalline lens; (2) the vitreous humor; and (3) the aqueous humor. The crystalline lens l (Fig. 191) is situated near the anterior surface of

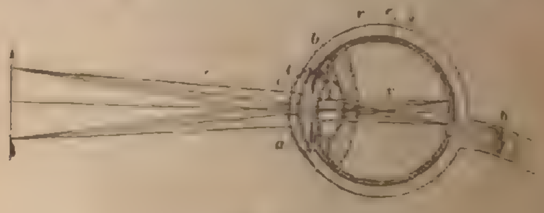


FIG. 191

the eye just behind the iris i. It is double convex in form and is 5 to 6 mm. in diameter and about 4 mm thick. The radius of curvature of the anterior surface, on the average, is about 10 mm., and that of the posterior surface about 6 mm. It consists chiefly of a globulin and its mean refractive index is 1·45. The vitreous humor v occupies four fifths of the volume of the eye, being bounded by the lens in front and by the retina r behind. It consists of a thin transparent albuminous jelly enclosed in a delicate membrane, and having a refractive index of 1·34. The aqueous humor a is con-

tained in the space between the anterior surface of the lens and the cornea *c*, this space being about 4 mm. wide. It is a dilute solution of sodium chloride containing some animal matter, has a density of about 1·006 and a refractive index of 1·34. The surfaces at which refraction takes place, therefore, are three in number: First, the anterior convex surface of the cornea, whose radius of curvature is about 8 mm.; second, the anterior surface of the crystalline lens; and third, the posterior surface of this lens.

The image formed by this optical apparatus is received upon the retina *r*, a membrane enveloping the outer surface of the vitreous humor *v* and through which ramify the final elements of the optic nerve *h*. Normally this image is brought to a focus exactly upon the retina, when the object is not nearer than 25 or 30 centimeters. But in some cases the antero-posterior diameter of the eye is so great that the image is formed before the rays reach the retina; in other cases this diameter is so short that the image is not formed at all, the rays not yet having come to a focus. In both cases the object is indistinctly seen. The former condition is called **myopia**, or short-sightedness; the latter **hypermetropia**, or long-sightedness. Evidently the use of a concave or diverging lens will remedy this defect in the myopic eye, and the use of a convex or converging lens, in the hypermetropic one. Common experience, however, teaches us that the eye is able to bring to a sharp focus upon the retina, the images of objects differing greatly in distance. In the case of an ordinary lens, this result is attained by focusing; i.e., moving the lens itself to or from the object. In the eye, however, this automatic adjustment for distance—which is called *accommodation*—is effected by a change in the form of the crystalline lens, produced by contraction of the ciliary muscle *b*, which, operating upon the suspensory ligament of the lens, relaxes it and thus allows the elasticity of the lens to act and increase its curvatures. This change may be readily observed by *looking obliquely* into an eye ad-

justed for a remote object, a candle-flame being suitably held upon the other side of it. Three images of the flame will be seen: (1) one, reflected from the cornea, bright, erect, and the first in order; (2) a second, reflected from the anterior and convex surface of the lens, also erect but less bright; and (3) a third image, reflected from the posterior concave surface of the lens, and therefore inverted in position. On changing the adjustment of the eye and looking at a near object no change will be noted in the first image; while the second and third images will become distinctly smaller, the change being much greater with the former. Hence it follows that during accommodation no change takes place in the cornea, but that the crystalline lens changes its form, chiefly on the anterior surface, becoming more convex as the object to be viewed is nearer. Sometimes defective vision results from the imperfect action of the ciliary muscle, so that the power of accommodation is diminished or lost. The practical effect of this is to increase the least distance of distinct vision. This condition of things is called **presbyopia**. It is common in old age. The range of accommodation in normal eyes is between 10 or 15 centimeters as a minimum—this value being called therefore the least distance of distinct vision—and an indefinitely great distance as a maximum.

Like other similar optical combinations with spherical surfaces the eye possesses spherical aberration. In front of the lens is a circular muscular curtain called the **iris** *i*, through which is an opening called the **pupil**. By contraction of the circular muscular fibers the pupil is contracted, by contraction of the radial fibers it is dilated. The iris thus acts the part of the diaphragm or stop in an optical instrument; limiting the incident rays to the central portion of the lens, and thus reducing the spherical aberration to the minimum consistent with the admission of sufficient light for distinct vision. The pupil always has the minimum opening necessary for this purpose; and hence automatically contracts when the incident light increases in brightness, and dilates

when it decreases. Moreover, the crystalline lens has its surface of greater radius of curvature toward the incident light; the position in which, according to Herschel, its spherical aberration is greatest. Again, though the lens decreases in density from the center to the surface, and therefore decreases in refractive power also, yet neither this nor the differences of curvature on its two surfaces destroy its spherical aberration.

Again, investigation teaches us that the eye possesses chromatic aberration also. If a distant light, such as that of a street-lamp, for example, be viewed through cobalt-blue glass, the image will appear red surrounded by a violet halo. If a normal eye be adjusted to bring red light to a focus on the retina from an indefinite distance, it can do the same for violet rays only when their source is at a distance of about sixty centimeters. Indeed the production of color in images formed by the eye may be observed directly by covering half the pupil with an opake screen and then viewing a white surface. The edge of the screen will appear colored.

In the next place, the curves of the eye are not truly symmetrical with reference to the optic axes. Both the cornea and the lens have different radii of curvature in different planes and hence their surfaces being more or less elliptical are not figures of revolution. Besides this they are not well centered. Hence arises the defect in vision known as **astigmatism**, present, to a marked degree, in many eyes otherwise normal; a defect which prevents horizontal and vertical lines, for example, from being in focus at the same time. If a series of concentric circles be looked at by an astigmatic eye, two sectors of indistinct and two of distinct lines will be perceived, the direction of which with the vertical will determine the position of the plane of maximum and minimum curvature. This defect may be corrected by the use of lenses with cylindrical surfaces.

The retinal surface upon which the light falls rests upon the choroid membrane *c*, which, as well as the external layer of the retina itself, contains dark brown or

black-pigment cells, as an absorbing layer. The point at which the optic nerve enters the eye is itself insensitive to light and is called the **blind spot.** The gap in the field of vision thus produced is about 6° in its angular horizontal dimension and 8° in its vertical dimension. It is therefore large enough to prevent our seeing eleven full moons if placed side by side, or a man's face at a distance of six or seven feet (von Helmholtz). Just outside of this blind spot, and two or three millimeters distant from it, placed exactly in the axis of the eye, is the seat of most distinct vision, the **yellow spot.** In the middle of this spot is a depression, the **fovea centralis,** where the retina is reduced to those elements alone which are necessary for exact vision. In angular diameter, it corresponds in the field of vision to that subtended by the finger-nail at arm's length. The condition of seeing an object distinctly is simply that its image shall be brought to a focus upon this precise spot; so that when we look at an object, we simply so direct the axis of the eye with reference to it, that this result shall be secured. Consequently the field of direct vision is exceedingly limited, the surrounding parts of the retina being able to afford only indirect vision. This defect is compensated for to a considerable extent by the rapidity and precision with which the eye can be directed to various parts of the field of vision in succession. Indeed so perfectly is this accomplished that it is only after much practice that we can avoid looking directly at an object which we wish to see indirectly, or can keep the eye fixed upon a given point for any length of time.

The elements of the retina which, being the end-organs of the optic nerve, are directly stimulated by the luminous disturbances falling upon them, are the rods and cones described by Schultze. Both consist of two portions; an inner one, fusiform or spindle-shaped, and an outer one, cylindrical in the case of the rods, slightly conical in the cones. In both the outer portion is double-refracting, the inner single-refracting. Both portions are longer in the rods than in the cones, so that the

former project beyond the latter. The average diameter of the cones is from ·004 to ·006 millimeter and that of the rods from ·0015 to ·0018 millimeter. While in the retinal surface generally, the rods are more numerous than the cones, they are entirely absent in the yellow spot; the cones in the **fovea centralis** being packed very closely together. If the retina has been kept in the dark, it will be found to be of a purplish-red color; and under the microscope this color will be found limited to the outer portions of the rods. Consequently the depth of retinal color will be in direct proportion to the number of rods in any given portion; and hence the yellow color of the central spot where only cones exist. This purple coloring matter, called by Kühne **visual purple** or rhodopsin, is readily bleached by light. So that Kühne obtained optograms of external objects by exposing the eye of a recently killed rabbit to a source of light such as the opening in a shutter; and then, after removing the exposed eye, hardening the retina and fixing the image with a solution of alum. On examining the surface, a clear white image of the opening in the shutter was visible, upon a beautiful rose-red ground.

425. Time required for Vision.—The time required for the production of a visual impression is dependent of course upon the intensity of the incident light: but is in any case very short. If the light be faint, as much as half a second may be required; while a flash of lightning lasting less than one millionth of a second renders visible an entire landscape. Langley has recently determined the total energy required to produce the sensation of vision in different parts of the spectrum. He finds that while the sensation of green can be excited by an amount of energy represented by one hundred-millionth (·00000001) of an erg, the sensation of crimson requires one thousandth (·001) of an erg; or one hundred thousand times as much. In other words, the same amount of energy may produce at least a hundred thousand times the visual effect in one color of the spectrum that it does in another. Moreover it appears from these

experiments that the eye can perceive lights whose intensities vary in the ratio of one to one thousand million million. The duration of an impression once produced is much greater than that of the time of action. Thus for strong lights, two successive flashes cannot be distinguished as separate unless they follow each other at an interval greater than from one thirtieth to one fiftieth of a second; while for ordinary lights, two sensations fuse into one if they are as far apart as even one tenth of a second. This phenomenon is known as the **persistence of vision** and is well seen in the simple experiment of moving a luminous point rapidly in a circle. Upon this principle are based the toys known as the thaumatrope, zoetrope, and phenakisticope, as well as the stroboscopic method of physical investigation.

426. Delicacy of Vision.—The limit upon the retina where two sensations cease to be distinct and appear as one, determines of course the distinctness of vision. Ordinarily two stars whose angular distance is less than sixty seconds appear as one. Two parallel lines whose angular distance is seventy seconds are indistinguishable as separate objects by even the best eyes. Now since an angle of seventy seconds corresponds to a retinal distance of ·005 millimeter, it would seem that the minimum visual area just given agrees very closely with the distance between the centers of two adjacent cones. Again, it has been shown that the smallest difference in length which the eye can detect between two lines, for example, is not an absolute quantity but a ratio. The smallest difference in illumination between two surfaces is about one hundredth of the whole. The minimum difference between two objects, one a centimeter long and the other a trifle less, bears the same ratio to the centimeter that the smallest detectible difference between two objects a meter long bears to the meter. In general, the law of Weber states that the smallest increase of sensation which the eye can detect, as the stimulus is steadily increased, remains always the same, so long as the ratio of increase to the total stimulus remains constant.

427. Color-sensation.—The sensation of color depends, as we have seen (353), entirely upon the wave-frequency of the incident radiation, the slowest vibrations producing the sensation of red, the most rapid that of violet. So that when complex radiation, containing all wave-frequencies within the range of vision, falls upon the retina, the sensation is that of white light. But the eye has no power of analyzing the light which it receives. It cannot distinguish, for example, between the white which is produced by the union of scarlet light with bluish green, that which is composed of yellowish green and violet, that which results from a mixture of yellow and ultramarine blue, or of red, green, and violet, or that which is obtained by mixing together all the colors of the spectrum (von Helmholtz). And yet a surface illuminated simultaneously with scarlet and bluish green, while appearing equally white with another upon which yellowish green and violet fall, would appear black in a photograph; the second surface appearing bright. In the same way a mixture of the wave-frequencies known as red and yellow produces the sensation of orange, not distinguishable by the eye from that of the spectrum. But these colors objectively are entirely different. The spectrum orange is a separate and distinct wave-frequency, incapable of production from any other wave-frequencies. "Each particular kind of yellow may be any one of an infinite number of different combinations of homogeneous rays" (Tait). Since, therefore, like sensations are excited by quite unlike stimuli, it would seem probable that all sensations, even those produced by a single wave-frequency, are of a mixed character. The methods adopted for studying the color-sensation produced by a mixture of colored lights are three in number: The first and simplest is that of von Helmholtz, which consists in placing a flat sheet of glass vertically, one of the colors being on one side of it and the other upon the other; the glass being so placed with reference to the eye that the light from the further color shall be transmitted through it, while that from the nearer color

shall be reflected from it, both along the line of vision. The second is the well-known color-top of Maxwell, in which pairs of disks of the various colors, slit radially so that the angular ratio of the two sectors may be indefinitely varied, are placed on the axis of a heavy top and so given a rapid rotation. The third method, which is also due to von Helmholtz, was used by him for mixing the pure colors of the spectrum. It consists in the use of a slit consisting of two portions at right angles to each other, each being inclined 45° to the vertical. The prismatic spectrum produced by each half of such a slit is of course oblique, the axis of the prism being vertical. These two oblique spectra are superposed, and in such a way that each color of the one, say the green for example, rests in some part of its length upon each of the colors of the other in succession. And in this way the compound color resulting from the superposition may be studied at the various intersections.

Since each color of the spectrum has its own wave-frequency, it is, as has been already stated, a simple color, incapable of any further decomposition. Hence any attempt to distinguish spectrum colors as primary and secondary is unscientific. But it is otherwise with color-sensations. It has been proved by the spectrum experiment of von Helmholtz above mentioned, that the sensation of yellow may be excited by the simultaneous action of red and green; and that of blue by the similar action of green and violet. Hence by selecting red, green, and violet as primary, all color-sensations may be produced from them. This theory of color-sensation originated with Young in the early part of the present century, but its present prominence in science is due to the researches of von Helmholtz. The theory supposes that there are in the retina three kinds of end-organs, one of which when excited produces the sensation of red, the second the sensation of green, and the third that of violet. Moreover, the first set are excited most strongly by low wave-frequencies, the second by waves of intermediate frequency, and the third by high wave-frequency. When

the first and second are simultaneously excited, the color-sensation of yellow results; and when the second and third are stimulated, blue is perceived.

428. Color-blindness. — Subjective Color. — Two striking phenomena are well explained by this theory. One of these is color-blindness; the other is subjective color. Color-blindness is a defect of vision which prevents the subject of it from distinguishing colors from one another. The most common form of it is red-blindness: a complete inability to recognize the color of red. To such persons the scarlet flowers of the geranium, or the rich red fruit of the cherry or strawberry, are distinguishable from the leaves only by their form. Scarlet, flesh-color, white, and bluish green appear to them identical, differing if at all only in shade. They cannot discriminate between the danger-signal of red and the safety-signal of green. All the colors visible to them are varieties of blue and green; i.e., those which result from mixtures of these colors. Evidently on the Young-Helmholtz theory, color-blind persons have eyes deficient in one or another of the three end-organs above mentioned. The end-organs excited by waves of long period are absent from the retina of a red-blind eye; or if present are inactive. Hence only those producing the sensations of green and violet remain effective; and the person sees all objects either uncolored or colored violet, green, or some mixture of the two. Green-blindness and violet-blindness have been observed, but have not been as completely studied.

Subjective color is due to retinal fatigue, upon the theory under discussion. If half of a white sheet of paper be covered with a black sheet, and the eye be fixed on a point of the former near the center for 30 to 60 seconds; then, on suddenly withdrawing the black paper, still keeping the eye on the point, the newly exposed half of the sheet appears of a brilliant white. Here evidently the exposed portion of the retina has become fatigued and the impression upon it enfeebled; as the superior brightness of the second half of the sheet proves,

when it is exposed to a fresh portion of the retinal surface. So fatigue may be partial. If the eye be fixed for thirty or more seconds on a strongly illuminated red wafer, and then views a sheet of white paper, a bluish-green image of the wafer will appear. Here the red end-organs, having been fatigued, are unable to supply their share of the effect produced by white; and the unfatigued green and violet end-organs produce a predominating effect. These after-images are cases of **successive** color-contrast. Again, if a screen be illuminated from two sources of light one of which is colored, it will be found that if an opake object be interposed between the uncolored source and the screen, its shadow upon the screen will have a color complementary to that of the other source. This color is purely subjective, since it has no external existence but is produced solely by the preponderance of the action of the unfatigued over the fatigued end-organs of the retina. This result, which is well seen on laying a piece of tissue-paper upon black letters printed on a colored ground, is called **simultaneous** color-contrast. From the same cause come the laws of the harmony and contrast of color. Complementary colors are most suitable to be associated together, because each of them strengthens the other. A red fabric appears much brighter after a green one has been looked at, but much weaker after an orange one.

Ingenious as this theory is, it must be regarded as purely provisional. Its basis is physical, not physiological or anatomical. No such distinction of end-elements as it requires can be found in the retina. "We are entirely in the dark," says Foster, "concerning the anatomical basis not only of color-sensations but also of vision as a whole." In the eyes of birds and reptiles, Schultze found retinal cones having a drop of ruby-red oil at the junction of the two portions; others having drops of orange-yellow and greenish-yellow oil at the same point. It has also been observed that the bleaching effect of light upon the visual purple is greatest for the greenish-yellow rays; i.e., those which are

most readily absorbed by the color itself. But the visual purple is absent from the cones; those end-organs which alone are found where vision is most distinct.

429. Visual Perception.—Behind the sensation there is a perception. The mind combines the sensations into an idea. The idea of size, for example, is derived from the visual angle, or the angle subtended in the field of vision by the object. The field of vision for a single eye is very large, being 160° laterally and 120° vertically. For both eyes it is somewhat more than two right angles, its extent being increased by movement of the eyes themselves up to 260° in the horizontal and 200° in the vertical direction. Obviously, at the nodal point or the optical center of the eye, the image on the retina subtends the same angle as the external object. From the actual size of the retinal image, therefore, determined by sensation, we may infer the angle which the object subtends; i.e., the apparent size. But this angle is a function not only of the real size but also of the distance. All objects subtend the same angle if the ratio of size to distance be constant. In order to form a judgment of the real size of an object, therefore, its distance must be known or assumed. The perception of distance by the eye is very imperfect, as is seen if we attempt to thread a needle with one eye closed, or to pass the end of a rod bent at right angles through a ring placed at arm's length. Hence our judgments being at the mercy of two variables are frequently unreliable. The moon, for example, appears larger upon the horizon than in the zenith, evidently because we assume unconsciously a greater distance for it in the former case. So, in the opinion of different individuals, the size of the moon high in the heavens varies from the size of a ten-cent piece to that of a cart-wheel, according to the idea previously formed of its distance, compared with that at which these common objects are generally viewed. And a fly moving close to the eye may by an error of judgment as to distance be mistaken for an eagle flying amid the clouds. Of two equal squares, one striped vertically

and the other horizontally, the latter will appear higher and the former broader than the other. Hence the effect of horizontal stripes in dress is to increase the apparent height of the individual, and stout persons avoid longitudinal stripes for a similar reason (Foster).

The perception of distance is greatly facilitated by the use of two eyes. The only means of estimating distance possessed by a single eye is that due to the muscular sense in producing the necessary accommodation. But with two eyes their axes are made to converge upon the object, this convergence being greater as the object is nearer. By this action the images in the two eyes fall on corresponding points in the retina, and so we perceive one object and not two. The muscular effort required to produce the necessary convergence by motion of the eyes themselves, is the basis upon which our estimate of distance rests. And the judgment founded on the muscular sense is of course a matter of education. The delicacy of this appreciation of distance appears in our estimation of solidity. A vertical circle, for example, is distinctly seen all at once because all portions of it are equally distant. But a sphere appears solid because we are conscious of the different optic convergence required to see its different parts distinctly. The perception of solidity is much assisted by the reflection of light from the object; so that raised surfaces may be made to appear depressed, and *vice versa*, by a suitable management of the light. This fact is fully recognized in drawing, since upon it the principles of shading depend.

430. Perception of Relief.—Another important fact contributing to the perception of solidity is that the two retinal images formed by a solid body are different, in consequence of the different positions of the two eyes with reference to it: these two different images being mentally combined into one. If a truncated cone be looked at with the two eyes, evidently the right eye will see the top displaced slightly to the left and the left eye will see it displaced to the right as in the diagrams (Fig. 192) given below. Conversely, if two circles be drawn

excentrically as above, and so screened by the finger, for example, that the right-hand figure *C* is seen only by the right eye and the left-hand figure *A* only by the left eye, then on opening both eyes the two figures will combine

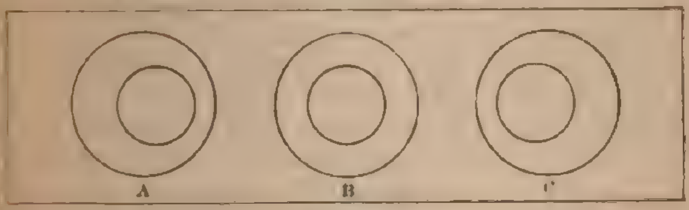


FIG. 192.

to produce the central symmetrical figure *B* but in which the central circle will appear raised above the plane of the outer one; i.e., the appearance of the truncated cone will be reproduced. Conversely, if the outer figures be exchanged so that the right eye sees the central circle displaced to the right and the left eye sees it displaced to the left, the combined image instead of being one of relief will be one of depression.

431. Reflecting and Refracting Stereoscopes.—This is the principle of the stereoscope, which is an instrument by means of which two slightly different pic-

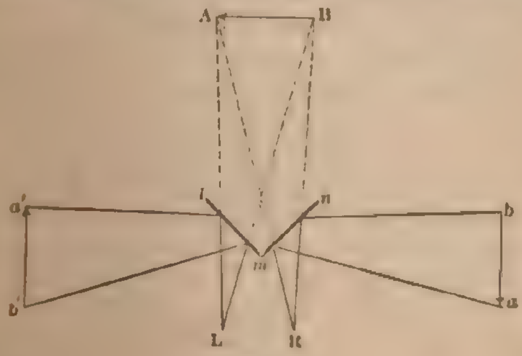


FIG. 193.

tures of an object may be combined so as to produce the effect of solidity or relief. In the reflecting stereoscope, the original form devised by Wheatstone (Fig. 193), two

plane mirrors lm and mn inclined 90° to each other are used to effect the combination, as shown in the diagram, in which ab and $a'b'$ are the two objects, combined by reflection into a single image AB. In the refracting stereoscope, subsequently devised by Brewster (Fig. 194), the images are combined by means of two prisms with curved surfaces m and n placed as shown in the diagram; a partition cd being so placed as to prevent either eye from seeing the object intended for the other. The pictures used in the stereoscope may be drawings, the two differing simply in the point of sight; or photographs, taken generally with a double camera, the distance between the lenses being equal to that between the eyes. Of course since the relief is greater as this distance is made greater, it may be exaggerated indefinitely. In Wheatstone's pseudoscope, the ordinary visual relations between the near and the distant parts of an object are reversed, that part of the object which is nearest the eye being made to produce a diminished convergence of the optic axis. In consequence the form of the object is inverted, so that a convex figure becomes a concave one, an impression of a seal resembles the seal itself, a bust seen in front appears as a hollow mask. Two printed pages, if exactly alike, appear flat in the stereoscope. But if in one there is a displacement of a type or a mark of any sort, not found in the other, that defect is made to stand out in bold relief. Dove applied this fact to the detection of alterations in bank-notes.

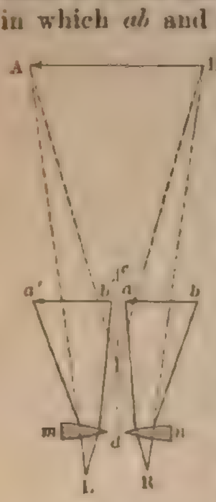


Fig. 194.

The subject of vision, however, belongs rather to the departments of Physiological and Psychological Optics, i.e., to Psycho-physics, than to pure Physics. We have attempted to give, therefore, only a rapid sketch of the subject so far as concerns the department of Physics proper.

F.—INTERFERENCE AND DIFFRACTION.

432. Interference of Radiation.—The general phenomena of wave-interference have already been discussed. Whenever two waves of the same period are compounded, the amplitude of the resultant wave is always the algebraic sum of the component amplitudes (66). Interference phenomena in the case of radiation were first observed by Young in 1801. Allowing sunlight to enter a dark room through a small opening in the window-shutter, he caused it to fall on two small holes pierced in a screen placed just behind this shutter. The light from these openings was received upon a second screen at some distance, and it was observed that where the two luminous cones overlapped there appeared a series of bands, light and dark alternately, which disappeared when one of the two holes was closed. This result Young attributed to **interference.**

Grimaldi, however, had observed as early as 1665 that the actual shadow of a small opake object placed in the cone of sunlight which came through a small hole in the shutter was much larger than that given by geometrical construction, based on the law of the rectilinear propagation of radiation. This result he attributed to a deviation of the rays whenever they passed near the edge of an opake body; and to this action he gave the name of **diffraction.** He also observed that this shadow was surrounded with three fringes of color. These phenomena were further studied by Newton, who explained them by assuming that this inflection of the rays of light in passing by the edges of opake bodies was in consequence of the attractive and repulsive forces exerted by the molecules of matter upon those of light before the two came into actual contact. The experiment of Young was therefore objected to by the Newtonians as inconclusive, since, inasmuch as the rays passed by the edges of the apertures, the result might be due, not to interference but to diffraction

Fresnel, in consequence, devised two methods of performing Young's experiment, the results of which were entirely free from diffraction phenomena. In the first method, two plane mirrors of black glass were employed, hinged and fixed at a large angle, nearly 180°. In the second, a bi-prism was used, the refracting angle of which was very large, differing but little from 180°.

433. Fresnel's Interference Methods.—The condition to be attained is simple. Two beams of light from the same source must be made to intersect at a small angle. Since this cannot be done directly, it must be effected indirectly; i.e., by reflection or refraction. If a source of light be placed between two inclined mirrors, a virtual image of it will be formed in each, the distance between these two images being less as the angle between the mirrors is greater (381). Thus rays from a source of light at A (Fig. 195) will be reflected from the

Fig. 195.

plane mirrors mO, $m'O$ to the point B, with precisely the same effect as if these rays actually came from the two points a and a', the virtual images of the luminous point A. So, rays from A (Fig. 196) on one side of the

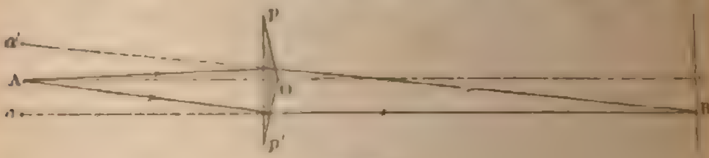


Fig. 196

bi-prism pOp' will be refracted to B upon the other side of the prism, the result being the same as if these rays came from the points a and a'. Since the rays in both cases come from a single source A, they start in

the same phase; and if the length of path pursued is the same for both, they will reach the middle point C of the screen in the same phase. Hence the two rays reinforce each other at this point, the amplitude of the disturbance here is doubled, and the intensity is quadrupled. If, however, the length of path is longer for one ray than for the other by half a wave-length, as is the case at the point B, they reach this point B in opposite phases, and therefore mutually destroy each other.

Analytically, if r represent the amplitude of one of the periodic disturbances and r' that of the other, the difference of phase being ϕ, the resulting amplitude is given by the expression $R^2 = r^2 + r'^2 + 2rr' \cos \phi$ (74). If both components are in the same phase, $\phi = 0$ and $R = (r^2 + 2rr' + r'^2)^{\frac{1}{2}} = r + r'$; if in opposite phases, $\phi = \pi$, and $R = (r^2 - 2rr' + r'^2)^{\frac{1}{2}} = r - r'$. If they have the same amplitude, $r = r'$; whence $R = 2r$ in the first case, and $R = 0$ in the second, as above.

The source of light in these experiments may be either a narrow slit illuminated with sunlight, or the linear focus of a cylindrical lens, placed symmetrically in the case of the bi-prism, but slightly to one side, in that of the mirrors. When the proper adjustments are secured, a series of symmetrical colored bands or fringes is seen on the screen, these fringes being parallel to the line joining the refracting or reflecting surfaces. If the radiation be homogeneous, these bands are alternately light and dark. If white light be used, each fringe consists of the colors of the spectrum in regular order. On examining the bands produced by different colors, it will be seen that those given by red light are the broadest, those given by violet light the narrowest; those of the intermediate wave-frequencies being of intermediate width. Moreover, the central band is bright; and hence the distance from the middle of this band to the next bright or dark band outside of it becomes smaller as the wave frequency increases. The fringes seen in the case of white light are therefore true spectra produced by interference, the colors being arranged from

the center outward in the inverse order of their wave-frequencies, violet, blue, green, yellow, orange, and red.

These results are readily explained. If a and a' (Fig. 197) be the sources of light, the central point B on

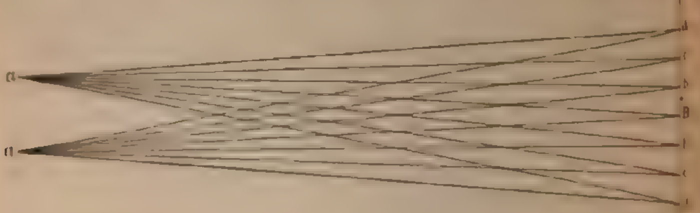


FIG. 197.

the screen will be illuminated by two beams in the same phase and therefore will appear bright. If another point b on the screen be taken, at such a distance from B that the path ab of the ray from a is half a wave-length shorter than $a'b$, the path from a', then it is evident that the two waves will reach b in opposite phases and their resultant will be zero; i.e., the point b will be dark. So if ac be longer by an entire wave-length than $a'c$, the point c will be bright; and if ad exceed $a'd$ by a wave-length and a half, d will be dark. Corresponding

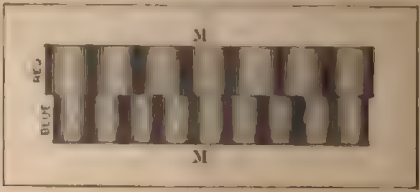


FIG. 198.

points are of course found on the opposite side of B, b' and d' being dark and c' bright. If a and a' be sections of the slit placed perpendicular to the plane of the paper, B, b, c, d, b', c', d' represent sections of the parallel fringes. It is clear that the distance Bb or Bb' from the middle of the central fringe to the middle of the first dark band is a function of the wave-length; since to

produce the required difference of path, it will not be necessary to go as far to the right or to the left of B for a short wave as for a long one. Hence therefore (Fig. 198) the bands are closer together the less the wave-length of the light.

434. Measurement of Wave-length.—From the data given by this experiment, the wave-length of a given radiation may be readily calculated. If b' (Fig. 199) be

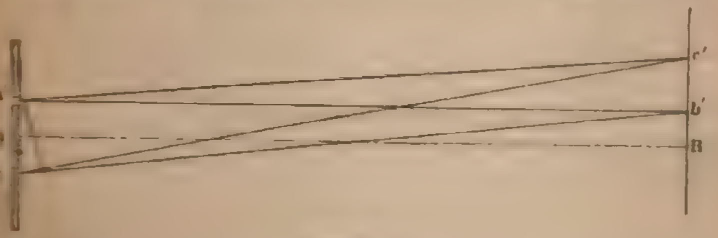


FIG. 199

the position of the first dark band in homogeneous light, we have $ab' - a'b' = \frac{1}{2}\lambda$. But since $a'b'$ is parallel to OB, $ab' = \sqrt{BO^2 + (Oa + Bb')^2}$ and $a'b' = \sqrt{BO^2 + (Oa' - Bb')^2}$; or calling d the distance BO of the screen from the line joining the luminous points, s the distance Bb' to the center of the first dark fringe, and $2l$ the distance aa' between the images, and remembering that s and $2l$ are both very small compared with d, we have $\frac{1}{2}\lambda = ab' - a'b' = (d^2 + (l + s)^2)^{\frac{1}{2}} - (d^2 + (l - s)^2)^{\frac{1}{2}}$; which becomes $\frac{1}{2}\lambda = \dfrac{(l + s)^2 - (l - s)^2}{2d}$ and reduces to $\lambda = 4ls/d$. Hence by measuring d, the distance to the screen, $2l$, the distance between the images, and s, that to the first dark fringe, we may obtain the wave-length. Since $l/d = \tan \frac{1}{2}aBa' = \tan \frac{1}{2}\alpha$, we may represent $\tan aBa'$ by $2l/d$ approximately. Hence $\lambda = 2s \tan \alpha$. The distance s may be measured accurately by means of a micrometer eye-piece in the focal plane of which the fringes are formed. Fresnel made a small hole in the screen at B and received the images after crossing, on a second screen, at a distance d'. Measuring the distance

$2l'$ between the images, the value $\tan\alpha = 2l'/d$. From the above equation $\lambda = 4l's/d$, we obtain $s = d\lambda/4l' = \lambda/2\tan\alpha$; in other words, the breadth of the bands increases with the wave-length and with the distance of the screen; and decreases as the luminous sources are separated; i.e., as the angle between the mirrors is increased. If the dark band measured be not the first one, but the nth, counting from the center, the above formulas become $n\lambda = 4l's/d = 2s\tan\alpha$; whence $\lambda = 4l's/nd$ or $2sn^{-1}\tan\alpha$.

435. Interference produced by Thin Plates.—The colors produced by thin layers of transparent substances are due to interference. The iridescence of Cyprus glass, of thin layers of oil upon water, and of the soap-bubble are familiar examples. If two clean pieces of plate-glass a few centimeters in area be strongly pressed together, bands of color will appear between them due to interference in the air-film separating their surfaces. The rays reflected from the lower surface of the upper glass meet those reflected from the upper surface of the lower glass; and since the path of the latter is the longer, they are in the condition of complete interference when as before this difference of path is half a wave-length for the incident light. Let ll' and uu' (Fig. 200) be these lower and upper surfaces, respectively. The ray ao incident on the surface ll' is partly reflected at o to a' and partly refracted to p. At p it is partly reflected from the surface uu', so as to be incident on the surface ll' at o', and is thence partly refracted to a''. Evidently this second portion has pursued a longer path than the first. The problem is to determine the distance between the plates required to produce complete interference for a given wave-length. The angle $oo'n'$ is equal to the angle of incidence, since their sides are

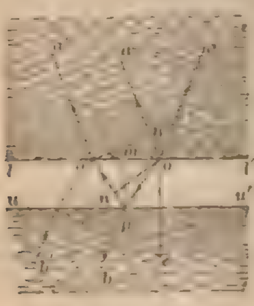


Fig. 200

perpendicular; and the angle $o'on$ is equal to the angle of refraction. Hence on' and no', being proportional to the sines of these angles, represent the ratio of the spaces traversed in the two media in the same time. The one ray is therefore behind the other by the distance $op + pn$. Produce the refracted ray $o'p$ to meet at e, a perpendicular let fall from o. Since $ep = op$, $op + pn = en$; and $en = oe \cos oen$. If we call δ the difference of path, t the thickness pm of the film, and ϕ the angle of incidence on the second surface (which is also the angle of refraction at the first), this expression becomes $\delta = 2t \cos \phi$. For complete interference this difference of phase must be a multiple of half a wave-length; i.e., $\delta = n.\frac{1}{2}\lambda$; where n is any whole number. From these two values of δ we get $t = \frac{1}{4}n\lambda \sec \phi$. Hence at normal incidence the thinnest interference-film is $\frac{1}{4}$ the wave-length. If n is even, however, the waves will meet in the same phase; if odd, in opposite phases, according to this formula. But in fact the act of reflection changes the phase by half a period, as will be shown later (437). Hence for thicknesses 1, 3, 5, 7, etc., there will be bright bands, and for thicknesses 2, 4, 6, 8, etc., dark ones. Moreover, the thickness evidently increases with the wave-length of the incident light; a soap-bubble giving a red color being thicker than one which gives a blue. Again, the thickness being proportional to λ, which in the substance of the film varies as the speed of light, is inversely as the relative index between the two media. Lastly, the thickness of the film required is directly as the secant of the angle of incidence upon the second surface, and therefore increases as the angle of incidence itself increases.

436. Newton's Rings.—In order to study this class of interference phenomena more precisely, Newton produced them by placing a convex lens of very long radius upon a plane glass surface. At the center where the air-layer is thinnest, a circular spot appears nearly uniform in color. This is surrounded by colored rings or fringes due to the uniformity of thickness of the air-layer at the

same distance in all directions; the different rings of course corresponding to different thicknesses. If the pressure be increased the circles all dilate, new ones appearing in the center, of colors denoting increased wave-frequency, until the central spot is black. With homogeneous radiation, the rings are alternately bright and dark and are very numerous; several thousand having been counted with sodium light. They are larger with red than with blue light, and hence when seen in white light they show the colors of the spectrum, in the inverse order of wave-frequency. They are known as Newton's rings. Calling r the radius of a given ring (Fig. 201), t the thickness of the air-layer at that point,

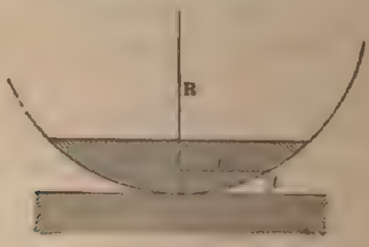


FIG. 201

and R the radius of the lens, we have $t : r :: r : 2R - t$, whence $r^2 = 2Rt - t^2$. As t is so small a quantity, its square may be neglected and we have $t = r^2/2R$; or the thickness of any ring varies as the square of its radius. Newton measured these diameters very accurately and observed that their squares and hence the thicknesses were in arithmetical progression; those of the bright rings being as the odd numbers 1, 3, 5, 7, etc., and those of the dark rings as the even numbers 2, 4, 6, 8, 10, etc. Moreover, he determined the absolute thickness of the fifth dark ring by measuring its diameter with great care, using the above formula; and he found it to be ·00143 millimeter. The first dark ring measured of course one fifth of this; or ·000286 millimeter. Consequently, since $\lambda = 2t \cos \phi$, we have for the wave-length of the light extinguished in this ring, the incident angle being about

4°, $2 \times \cdot 000286 \times \cdot 9976 = \cdot 00057$ millimeter; corresponding to yellowish-green light.

437. Change of Phase in Reflection.—It will be observed that the theory above given requires the rings to be bright when the thickness is an even multiple of the wave-length and dark when it is an odd multiple; whereas, as just stated, the fact is exactly the reverse. This is due to the special conditions of the experiment. In both the cases above given one of the reflections takes place at the surface of a rarer and the other at the surface of a denser medium. But Young showed that when the film between the glass surfaces has an index intermediate between the indices of the two glasses, as when oil of sassafras is placed between a lens of crown and one of flint glass, so that both reflections take place at the surface of a denser medium, the central spot is white and the rings whose thickness is an even multiple of the wave-length are bright, as the theory requires. The same is true if both reflections take place at the surface of a rarer medium. It would appear, consequently, that when reflection takes place from the surface of a rarer medium, the direction of motion of the æther-particles at the point of incidence is reversed; the consequence of which is that the wave leaves the surface reversed in position, producing the same result as if there had been a gain or loss of half a wave-length (65). There is obviously under these circumstances destruction where there should be reinforcement of light. Babinet produced interference by reflection from a silvered and an unsilvered surface which showed the same result.

438. Diffraction.—Diffraction is an interference phenomenon produced whenever radiation passes close to the edges of an opake screen. Newton had rejected the wave-theory of light on the ground that were it true, there should be a bending of the light-rays round the edges of obstacles, precisely as in the case of sound; so that a luminous point could be seen even if an opake object were placed between it and the eye, and shadows would not be possible. Were light a wave-motion like sound,

he argued, it would not travel in straight lines as we know it to do. The syllogism is legitimate, but the minor premise is untrue. The phenomena are identical in the two cases, provided the scale of the experiment is the same. An obstacle produces a light shadow ordinarily, simply because its size is enormous compared with the length of a light-wave. In proof of this, Rayleigh has produced sharply-defined sound-shadows, even by objects as small as the hand, by using sound-waves only 1·2 centimeters in length. And with an aperture of 14 centimeters or a disk of 15 centimeters in diameter the phenomena of sound-diffraction were satisfactorily obtained. Here the obstacle was about thirty wave-lengths in its diameter. If a sound-wave 1·3 meters long, like that of the middle *C*, had been employed, the obstacle must have been nearly forty meters in diameter, to produce the same effect; while if a light-wave ·00059 millimeter be used, the diameter of the obstacle need be only ·0177 millimeter. Moreover, ordinary experience teaches us that sound-shadows are well recognized phenomena in nature, where the obstacles are of large size compared with the sound-waves (229).

So, conversely, if homogeneous light be caused to impinge upon an obstacle of small size compared with its wave length, such as a hair, or be made to traverse a

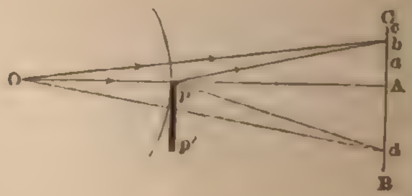


FIG. 202.

minute opening, such as a pin-hole, then the light is found not to travel in straight lines, apparently, but to bend round the edges into the geometrical shadow; i.e., the light is diffracted. The phenomena of diffraction in

ir simplest form are obtained when the light passes a single edge. The upper edge of the metal plate (Fig. 202) is so placed as to be grazed by the ray OA m the luminous point O to the screen BC. The light ould be homogeneous and the source very small; her a pin-hole, or better, the bright point which contutes the focus of a small convex lens of short radius. will be observed that at the point A which limits the ometrical shadow, there is not an abrupt transition m light to shade as rectilinear propagation would quire; but on the contrary there is a gradual diminun of the light from A toward B extending over a sible distance. On the other side of A, however, at points a, b, c, a series of alternate bright and dark nds appear, whose width depends upon the wavegth of the light employed, being greatest for red and st for violet. These are called **external** fringes; sim-r ones within the shadow as at d being called **internal** nges. If the light pass by two edges, as when a fine re or a hair is held in the path of the beam, then we

FIG. 203

ve the external fringes on both sides (Fig. 203) between and D and between B and C, and also a series of ernal fringes much less in width as at a between A d B.

If the two edges be those of an aperture, the appearces are still further modified (Fig. 204). Not only is width of the geometrical shadow AB increased, but e entire space is filled with bands alternately bright

and dark as at a, the width of which increases with the wave-length of the incident light. At the same time,

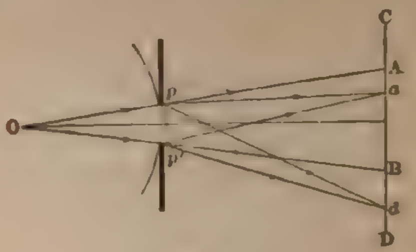


FIG. 204.

external fringes are formed as at d. The appearance of these fringes is shown in Figure 205. In the right-

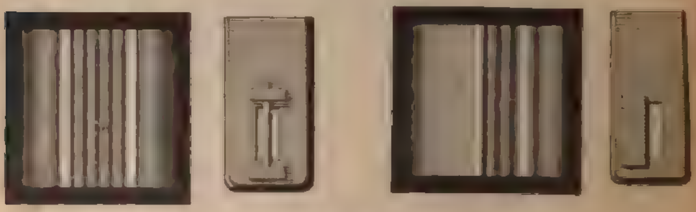


FIG. 205.

hand figure the diffraction is produced by a single edge and the fringes are external. In the left-hand figure a wire produces the diffraction and the fringes are internal. The apparatus used in observing these phenomena is called an **optical bench.** It consists of a pair of horizontal rails, upon which slide three stands, the first of which carries the short-focus lens, the second the diffraction-screens, such as are shown in the figure, and the third supports the micrometer eye-piece.

The explanation of these appearances we owe to Fresnel and to Schwerd, who proved them to be due to the interference of secondary waves produced at the edges of the obstacle, either with the primary wave or with

each other, as may be seen in the diagram (Fig. 206), where O is the luminous point, AB the obstacle, and ab the screen. The interior fringes are seen to be the result of the interference of the two sets of secondary waves

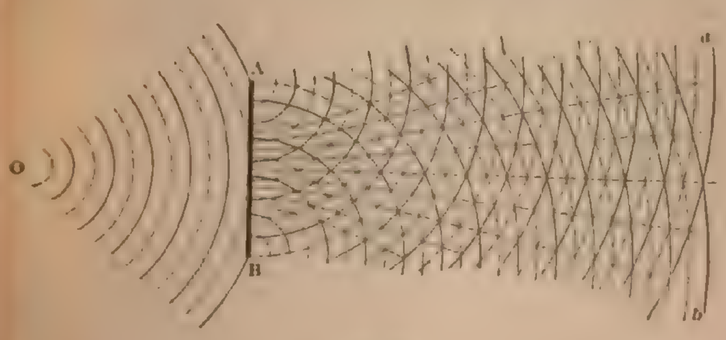


FIG. 206.

produced at the edges of AB. In the same way the exterior fringes result from interference between the primary waves from the point O and these secondary waves.

The character and position of these fringes may be

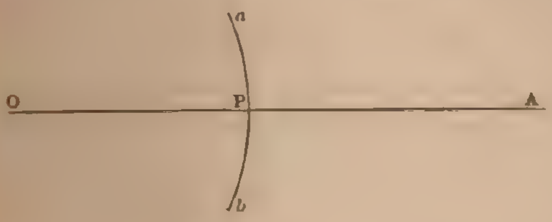


FIG. 207.

readily deduced upon the wave-theory. Every disturbance at a point in the æther is the resultant of all the disturbances which reach simultaneously the given point. If we suppose a spherical wave ab moving outward from a radiant point O, for example (Fig. 207), the effect at A will be not only the direct result of the original disturb-

ance at O, but also the indirect result of all the smaller disturbances which constitute the wave-front ab. Common experience teaches us apparently that an obstacle placed at P will entirely prevent the eye at A from seeing the luminous point O. But this is a question simply of the size of the obstacle, as compared with the radiation wave. If it be sufficiently minute, the radiation will bend round it in the same way that sound does round obstacles of ordinary size. To study the effect at A (Fig. 208) of the disturbances constituting the wave-front, let this front be divided into elements Pa, ab, bc, etc.,

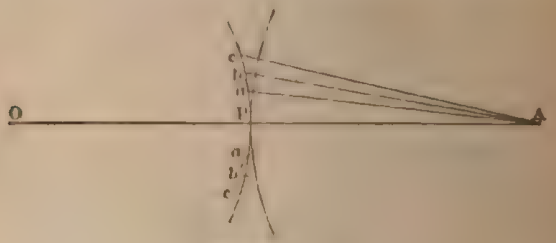


FIG. 208

such that the distances of each from A shall successively increase by half a wave-length; i.e., that $Aa - AP = \frac{1}{2}\lambda$, $Ab - Aa = \frac{1}{2}\lambda$, etc. Evidently the disturbance at A caused by the element Pa or Pa' will be opposite in its effect to that produced by the element ab or $a'b'$, since the disturbances will reach the point A in opposite phases (366). And hence, since the disturbance which any element produces at A is proportional jointly to the size of the element and to its angle of emission, it is clear that the effect at A of these elements as we go farther from P will ultimately be zero; leaving only the elements near P active in illuminating the point A. Since the waves are so minute, the distance about P, which alone contributes to the effect at A, is very small; so that practically no effect is produced there except along the bright line OA; i.e., the light is rectilinearly propagated (374).

439. Diffraction through a Slit.—Suppose now we allow sunlight to pass through a narrow slit *ab* (Fig. 209) and then to fall upon a screen *SS'* at some distance. As we have noted above, a series of bright and dark bands will be seen, parallel to the edges of the slit and having a bright central image. Evidently at some point *t* the distance traveled by the radiation from *a* will be half a wave-length longer than that from *b*; so that at this point these two marginal radiations will interfere in opposite phases and will destroy each other. If the perpendicular *bc* be let fall on *at*, the angles at *c* and *s* are right angles and the angle *abc* may be considered equal to the angle *srt*. Hence the triangle *abc* is approximately similar to *trs* and $ab : ac :: tr : ts$; whence $ac = (ab \times ts)/tr$; or, since *tr* differs only slightly

FIG 209.

from *rs*, *ac* or $\frac{1}{2}\lambda = \dfrac{\text{width of slit} \times \text{distance of 1st band.}}{\text{distance of screen}}$

Or if, with *t* as a center and radius *tb*, the arc *bc* be drawn, the angle at *c* will be a right angle and $ac = ab \sin abc$; or calling $ac = \frac{1}{2}\lambda$, $ab = a$, and *abc* or its equal $srt = \theta$, $\frac{1}{2}\lambda = a \sin \theta$. This result appears at first to resemble that of simple interference already discussed. It differs, however, in this regard, that while there are only the two outer rays to interfere in the former case, in the latter not only the marginal waves but all the waves entering through the slit *ab* contribute to the result. So that in fact experiment shows that the difference of path must be an entire wave-length in order that complete interference may take place at *t* and a dark band be produced

there. Consequently if t represents the first dark band, the distance ac represents λ.

440. Intensity of Diffracted Light.—The intensity of diffracted light at any point may be easily calculated (Muller). If the amplitude of two interfering waves be a and b, the amplitude of the resultant (36) will be $A = \sqrt{a^2 + b^2 + 2ab \cos 2\pi x/\lambda}$; which reduces to $A = a\sqrt{2 + 2 \cos 2\pi x/\lambda}$ when b and a are equal, and to $A = a\sqrt{2 + 2 \cos \beta}$ if β be taken to represent $2\pi x/\lambda$. Suppose the width of the slit to be divided into sixteen equal parts, each representing an element of the incident wave. If the difference of path for the extreme rays be assumed equal to half a wave-length, the difference for each element will be $\lambda/(2 \times 16)$. Hence $x/\lambda = \frac{1}{32}$ and $2\pi x/\lambda$, or β, $= 2\pi/32 = 180°/32 = 11° \ 15'$. Consequently we have $A = a\sqrt{2 + 2 \cos (11° \ 15')} = a\sqrt{3{\cdot}962}$ as the amplitude produced by two neighboring elements. The amplitude B, produced by two such pairs of elements, $= A\sqrt{2 + 2 \cos 22° \ 30'}$, since now $\beta = 2\pi/16$; the value of which is $A\sqrt{3{\cdot}848}$. That produced by the action of eight elements, $C = B\sqrt{2 + 2 \cos 45°} = B\sqrt{3{\cdot}414}$; and that by the action of the entire 16 or $D = C\sqrt{2 + 2 \cos 90°} = C\sqrt{2}$. Substituting in this the successive values of C, B, and A given above we have $D = a\sqrt{3{\cdot}962 . 3{\cdot}848 . 3{\cdot}414 . 2}$. But the light-intensity varies as the square of the amplitude; and hence we have $I_1 = D^2 = 104{\cdot}1a^2$ as the light-intensity at the point t (Fig. 209). At the point s, however, the amplitude produced by the sixteen wave-elements is $16a$ and the light-intensity $(16a)^2$ or $256a^2$. Calling this I we have $I_1 : I :: 104{\cdot}1a^2 : 256a^2$; whence $I_1 = 0{\cdot}406I$. In other words, at the point of the screen where the difference of path of the extreme lateral rays is half a wave-length, the light-intensity is 0·406 of that found in the central image. If however the radiant beam be so inclined to the plane of the slit that ac (Fig. 210) represents an entire wave-length, the figure shows that, dividing this beam into two equal portions, each of the rays of one portion differs from the corresponding

of the other by half a wave-length. The first element of the one portion, therefore, will destroy the effect the first element of the other, the second that of the cond, and so on; so that on the screen at the point where is beam is incident the resultant will be zero. In the ame way if the beam be still more inclined so that the ifference of path be one and a half wave-lengths, it may e divided into three portions in each of which each

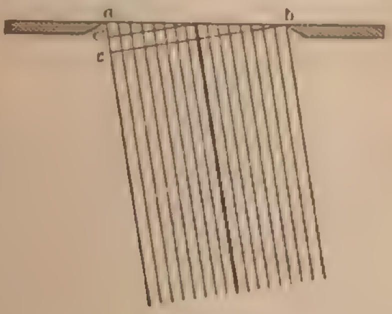


FIG. 210.

elemental ray is half a wave-length in advance of or behind the corresponding ray in the other portions. Evidently two of these portions will mutually destroy each other, only the third influencing the result. Since the amplitude is now only one third of what it was, the light-intensity is only one ninth; whence $I_s = \frac{1}{9}I$, or $0{\cdot}045I$. At the distance, then, on the screen where those rays are incident whose maximum difference of path is $\frac{3}{2}\lambda$, the light-intensity is 0·045 of that at the central image. So at the point where the outer rays differ by four half wave-lengths, the two pairs mutually destroy each other and the resultant is zero. If the values now obtained be plotted, a curve is obtained representing the light-intensity on the screen. Taking s, the central point (Fig. 211), as the origin 0, lay off distances 1, 2 to the right and to the left as the successive minima. At 0 erect an ordinate of one centimeter; and at the points $\frac{1}{2}$, $\frac{3}{2}$, erect ordinates corresponding to 4·06 and 0·45 millimeters. The curve drawn through these points as shown

represents the variations of light-intensity upon the screen on both sides of *s*. By an inspection of the formula above given, $ts = (ac \times rs)/ab$, it appears that the bands are wider, and hence farther apart, the greater the distance of the screen, and the greater the wave-

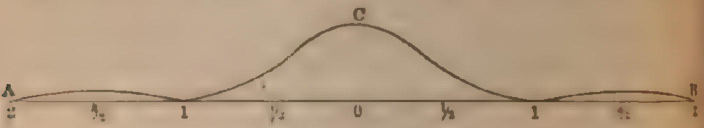


FIG. 211

length of the radiation employed. Consequently if white light be used in place of monochromatic light, the bands become colored fringes. Moreover, these bands widen and separate as the slit is made narrower.

441. Diffraction Gratings.—A series of parallel linear openings or slits separated by opake spaces is called a **grating**. The earliest grating was devised by Fraunhofer, who constructed it by winding fine wire round two screws with their axes parallel placed at some distance apart; the threads numbering 40 to the centimeter. Subsequently he used a plate of plane glass on which lines were ruled with a diamond. In 1843, Saxton ruled such gratings on glass and on steel for J. W. Draper. Nobert's gratings ruled on glass had as high as 500 lines to the millimeter, the ruled surfaces being two centimeters or more square. Rutherfurd's gratings had as many as 700 lines to the millimeter. They were ruled on glass and on speculum metal and were remarkable for the perfection of their construction. Some of the speculum-metal gratings had a ruled surface of 16 square centimeters or more. Recently Rowland has produced gratings far surpassing any previously made. The ruled surface on the largest of these is ten by fifteen centimeters and the lines in some cases exceed 1000 to the millimeter. Some of these gratings are concave, acting to produce images by reflection and without the aid of other optical apparatus.

The diffraction phenomena produced by a grating are capable of ready explanation upon the principles already given. Suppose a plane wave moving from right to left to be incident upon the grating AB (Fig. 212) made up

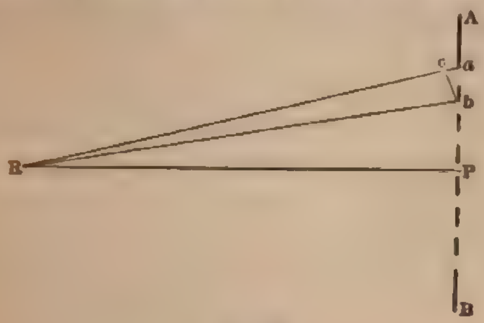


FIG. 212.

of alternate transparent and dark spaces. Suppose that from R lines be drawn Ra and Rb, so as to include a bright and a dark space ab, at such a distance from the point P that the difference in length of these lines shall be an entire wave-length. Then as we have seen above, if we divide the space ab into two equal portions, the two elements constituting these portions will entirely destroy each other's effect at R, provided that the entire space ab be transparent. Since, however, one half of this space is opake, this interfering element is stopped and the other half produces its total effect at R. Evidently at any point on the screen where the difference of path of the marginal rays is, for the particular radiation employed, an even number of half wave-lengths, there will be a bright band; and at any point where this difference is an odd number of half wave-lengths, the band will be dark. If the line bc be drawn perpendicular to Ra, evidently ac, which is a wave-length or λ, is equal to ab sin $abc = ab$ sin PRa; or calling α the angle made by the ray with the plane of the grating and a the space ab, $\lambda = a \sin \alpha$. At some point of the grating farther from P, the difference of path Ra' and Rb' will be 2λ; whence for this angle $2\lambda = a \sin \beta$; or in general

$n\lambda = a \sin \alpha_n$; whence $\sin \alpha_n = n\lambda/a$. In other words, for a given grating and given monochromatic light, the angles of deviation for the bright spaces vary as n; i.e., in arithmetical progression.

For a concave grating, theory shows that if the surface of the grating be a segment of a spherical surface of radius R, light from a source A situated on a circle of which R is the diameter, will produce a spectrum situated on the same circle; and further that in the position of minimum deviation, the source of light and its image are equidistant from the grating. Thus, for example, let GG' (Fig. 213) be a concave grating, O being its center of curvature and C its center of figure; the ruling being normal to the plane of the paper and symmetrical about C. If now a circle be described on OC as a diameter, and a source of radiation be placed at A, the image of the radiant source will be formed at B, and real diffraction spectra will be produced on the circumference of this circle. In Rowland's arrangement, the slit is placed at the intersection of two arms set at right angles, the grating and the observing lens respectively being placed at their extremities. The radius of curvature of the large Rowland grating is about 6·5 meters, the ruled surface being about 10 by 15 centimeters. The number of lines varies from 400 to 800 to the millimeter.

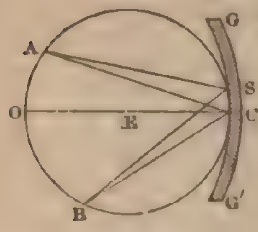


FIG. 213

442. Diffraction Spectra. — For different wave-lengths the expression $\sin \alpha = \lambda/a$ shows that for a given grating the sine of the angle of deviation varies directly as the wave-length. Hence the bright maxima will occur nearer the central image the shorter the wave-length of the incident light. So that if the incident light be white, its components will be distributed outward from the central image in the inverse order of their wave-lengths; thus forming what is known as a **diffraction spectrum.** More-

over, since in the above equation the angle α is a periodic function of the wave-length λ, the spectra thus produced repeat themselves indefinitely to the right and left of the central image; Fraunhofer having observed as many as thirteen on each side. Again, if the angle of deviation be small so that we may write $\theta \sin 1'$ for $\sin \theta$, we have $a\theta \sin 1' = \lambda$; so that for the deviations θ_1, θ_2, and θ_3 corresponding to different wave-frequencies, λ_1, λ_2, and λ_3, we have $\frac{\theta_2 - \theta_1}{\theta_3 - \theta_1} = \frac{\lambda_2 - \lambda_1}{\lambda_3 - \lambda_1}$; in other words, the intervals in a diffraction spectrum, under these conditions, are proportional to the wave-lengths, and the spectrum is a **normal spectrum** (420). A comparison of the diffraction spectrum with the prismatic spectrum will show the marked difference in the distribution of the wave-lengths. In

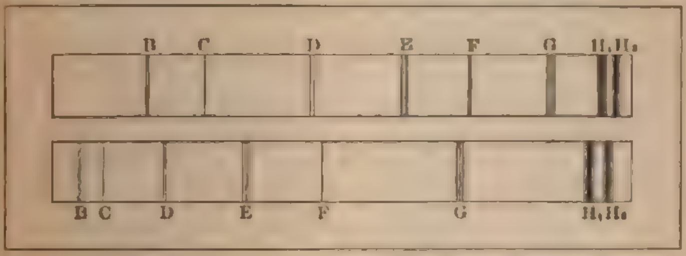


FIG. 214.

Figure 214, the upper spectrum represents the former and the lower spectrum the latter, the Fraunhofer lines being given in both to indicate corresponding wave-lengths. While the line *D* marks very nearly the middle point of the diffraction spectrum, the line *F* is nearest the center of the prismatic spectrum. In this latter spectrum, therefore, the radiation-waves appear to be condensed toward the less refrangible end and extended toward the more refrangible one. The successive spectra to the right and left are called spectra of the 1st, 2d, 3d, and 4th orders respectively, the length of each spectrum being greater and its intensity weaker than that of its predecessor. If sunlight be employed to form the spectra, the colors are

so pure that the Fraunhofer lines may be distinctly seen. It was by means of the superb concave gratings produced by Rowland, that he succeeded in photographing the solar spectrum with a perfection and an accuracy of detail before unattained. This magnificent spectrum is more than thirteen meters in length. In it the D lines are separated nearly 7·5 centimeters, and the B line is 6 centimeters in extent. A portion of this spectrum in the vicinity of the D lines is shown in the upper part of Figure 215. The lower spectrum shows the green band of carbon, taken with the same remarkable apparatus. Rowland has photographed also the spectra of many of the other elements.

443. Measurement of Wave-lengths. — It is by means of diffraction gratings that wave-lengths are usually measured. For this purpose the grating, supposed transparent, is supported upon the table of a spectrometer with its lines vertical, and its plane normal to the axis of the collimating telescope. The observing telescope being then so placed that its axis is parallel to that of the collimator, the image of the slit will bisect the cross-wires. The reading on the graduated circle gives its zero position. It is then moved to one side until the image of the slit, now illuminated with monochromatic light, say that of a sodium flame, again bisects the cross-wires. A second reading is now made; the difference between this and the first reading gives the deviation. Another similar determination is now made upon the other side of the zero; and the mean of the two is taken as the true deviation for the first spectrum. The value of the grating-spaces is determined by counting the lines in a given known space, under a microscope. So that if there are $n+1$ lines to the millimeter, the value of one grating-element, consisting of a line and a space, i.e., the distance from the center of one line to the center of the next one, will be $1/n$ millimeter. From the equation $\lambda = a \sin \alpha$, the wave-length is obtained by multiplying the value of one grating-element by the sine of the deviation angle. If a second mean be obtained from the two spectra of the second order, we have $2\lambda = a \sin \alpha_2$; and so on for higher

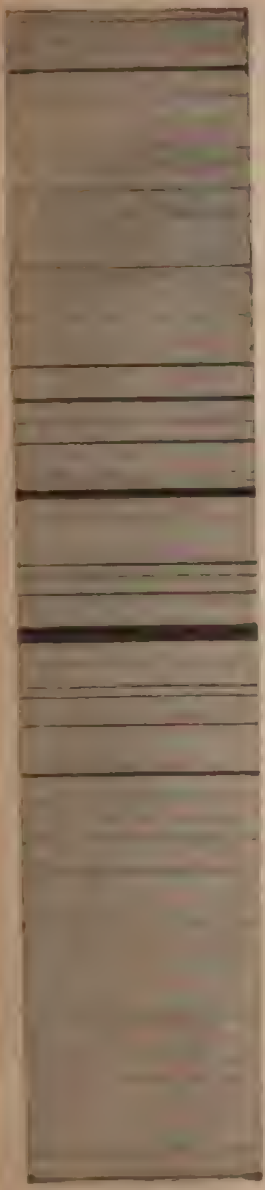


FIG. 215, A.—THE D LINES IN THE SOLAR SPECTRUM.

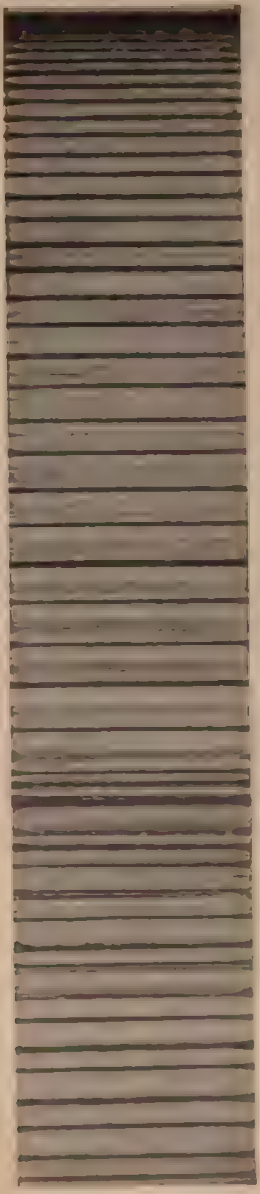


FIG. 215, B.—THE GREEN BAND IN THE SPECTRUM OF CARBON.

orders. Thus with a grating having 110·8 elements in each millimeter, the value of a is 0·009023 millimeter. If the deviation to the right produced by the first spectrum be 3° 44′ 30″ and to the left 3° 44′ 45″, the mean is 3° 44′ 37·5″, the sine of which is 0·0653. Consequently $\lambda = 0{\cdot}009023 \times 0{\cdot}0653$ or 0·0005892 millimeter, the wave-length of sodium radiation. Let the mean deviation of the spectrum of the second order be 7° 29′ 22·5″, the sine of which is 0·1304. Then $2\lambda = 0{\cdot}009023 \times 0{\cdot}1304$ or 0·0011761 and $\lambda = 0{\cdot}0005881$ millimeter. The mean of these two values is 0·0005887 millimeter.

G.—DOUBLE REFRACTION.

444. Phenomena of Double Refraction.—Since the speed of propagation of a disturbance in any medium is a function of the elasticity of the æther within that medium, it follows that if the medium be not isotropic, i.e., if it possesses different æther-elasticities in different directions, there must exist within it different speeds in different directions, and hence different refractions. The phenomenon of double refraction was first observed by Bartholin in 1669 in a transparent variety of calcite found in Iceland and hence known as Iceland spar. This substance crystallizes in the hexagonal system and cleaves readily into rhombohedrons, consisting of six equal rhombic faces having angles of 101° 55′ and 78° 5′; these faces forming with each other acute angles of 74° 55′ 35″ and obtuse angles of 105° 4′ 25″. They are united alternately three above and three below by the obtuse angles. The direction of the line joining the two solid angles thus formed is called the **optic axis**. It is also the crystallographic axis, the crystal being symmetrical about it. A plane containing this axis and perpendicular to one of the faces of the crystal is called a **principal section**, or principal plane.

If the upper surface of such a rhombohedron be covered with a card through which a small hole has been pierced (Fig. 216), and if sunlight be allowed to

fall on this card, two beams of light *CE* and *CO* will be seen to traverse the crystal and two images *E* and *O* to be formed on the farther side of it. Clearly, since

FIG. 216.

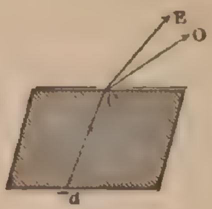


FIG. 217.

there are here two refractions, there must be two changes of speed due to two different elasticities. The crystal is said to be doubly refracting. Again, if the crystal be placed on a card on which is a black dot *d*, (Fig. 217), the same perforated card being upon its upper surface, it will be noticed that there are two positions of the eye in which the black dot can be seen; i.e., the positions *O* and *E*. Hence the radiations must travel along the line *dC* within the crystal with two different speeds, since on emerging into the air where the speed is the same for both, they are differently refracted. Moreover, of the two refractive indices thus resulting, the greater index corresponds to the slower speed within the crystal and the less index to the greater speed.

If the rhombohedron be placed over the black dot and, while viewed at a constant incident angle, be turned round on its lower surface, one of the two images of the dot will be seen to revolve about the other (Fig. 218), the line joining the two remaining always in the principal plane of the crystal; i.e., parallel to its shorter diagonal. The stationary image suffers equal refraction of course in all positions of the rhombohedron. Hence the refracted ray is always in the plane containing the normal and the incident ray, and the ratio of the

FIG. 218.

sines is constant. The ray which thus follows both the laws of ordinary refraction is called the **ordinary** ray, and its image the ordinary image. The other and revolving image is in the plane of incidence only when the principal plane coincides with this incident plane. When the principal plane is at right angles to the incident plane, the ray from this image, since it lies in the principal plane, does not lie in the plane of incidence. Moreover, in the two positions of the crystal in which it is in the incident plane, it is more refracted in one and less refracted in the other, than the ordinary ray. Since, therefore, this ray follows neither of the laws of ordinary refraction, it is called the **extraordinary** ray, and its image the extraordinary image.

445. Axis of no Double Refraction.—There is one direction in such a crystal along which a radiant beam may be transmitted without being divided into two. This is the direction of the optic axis, which is therefore called **the axis of no double refraction.** Starting from this axis, the difference between the ordinary and extraordinary rays increases until it reaches its maximum in a direction perpendicular to this axis. Hence in a plane perpendicular to the optic axis, the extraordinary index is constant. If a prism be cut from a crystal of Iceland spar, so that its refracting edge is perpendicular to the optical axis, then in the position of minimum deviation, the radiation will traverse it parallel to this axis and there will be but a single image produced, and that the ordinary one. The refractive index obtained under these circumstances will be the ordinary index, which for the line *D* and for this substance has the value 1·658. If, however, the prism be cut so that the refracting edge is parallel to the optic axis, then a ray traversing it in the position of minimum deviation passes through it in a direction perpendicular to the optic axis; and not only is divided, but the components have the maximum separation. If now the indices of refraction for the two rays be measured for the *D* line, one of them will be found to have the value 1·658 as before, while

the other or extraordinary index will have the value 1·486.

446. Uniaxial and Biaxial Crystals.—The phenomenon of double refraction is not peculiar to Iceland spar. All crystals possess this property except those belonging to the isometric or cubic system, though in a less marked degree. Quartz, for example, shows double refraction distinctly; but on measuring the two indices of refraction for the *D* line it is found that the extraordinary index in this case is the larger, being 1·553; while the refractive index for the ordinary ray is 1·544. Such crystals as quartz, in which the ordinary index is smaller than the extraordinary, are called **positive**; while crystals like calcite in which the converse is true are called **negative**. In the latter, the ordinary ray is the more refracted; in the former, the extraordinary ray.

Moreover, crystals belonging to the quadratic and the hexagonal systems, in which the vertical axis is symmetrical with reference to the lateral axes, have only a single direction along which radiation can pass without division. Hence such crystals are called **uniaxial**. Calcite, tourmalin, corundum, mellite belong to the uniaxial negative class; quartz, ice, and zirkon to the uniaxial positive class. In crystals belonging to the other three systems, there are two directions of no double refraction; i.e., two optic axes inclined more or less to each other. Such crystals are therefore called **biaxial** crystals. Niter, aragonite, borax, barite, topaz, and sugar are examples. Fresnel showed that neither of the two rays produced by a doubly refracting biaxial crystal follows the ordinary law. Both are therefore extraordinary rays.

447. Huyghens' Construction for Double Refraction.—In 1690, Huyghens investigated the phenomena of double refraction and gave a construction of remarkable acuteness for the path of the rays in a doubly refracting medium; this construction being founded upon his wave-theory. Evidently the wave-front of a disturbance radiating from a center in an isotropic medium is a sphere. The path of the ray after refrac-

tion at the surface of such a medium Huyghens determines as follows (Fig. 219): Let OF be a wave-front and

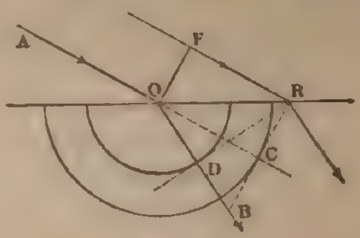


FIG. 219.

AO the normal to this wave-front incident on an isotropic surface at O. From the point O as a center draw two spheres whose radii OC and OD are equal, respectively, to the speeds of propagation S and S' of the disturbance in the first and second media, respectively; i.e., to the reciprocals of the refractive indices. Continue AO to C and draw a tangent plane at C to intercept the surface, say at the point R. Draw now from this point another tangent plane to the second sphere and let the point of tangency be D. The wave-normal OD through this point is the direction of refraction. For, evidently, the refracted ray is in the plane containing the normal and the incident ray; and in the triangles ROC and ROD, we have $OR = S/\sin\ CRO$ and $OR = S'/\sin\ DRO$; whence $S/\text{s in}\ CRO = S'/\sin\ DRO$, or $\sin\ CRO : \sin\ DRO :: S : S'$ or $\mu : 1$. The angle CRO is therefore the angle of incidence and DRO the angle of refraction.

In the case of an æolotropic medium, the speed of propagation will not be the same in all directions. In Iceland spar, for example, the speed of the extraordinary ray along the optic axis is different from the speed in a plane perpendicular to this axis, the latter being the greater. Moreover, the speed in this perpendicular plane is the same in all directions. Evidently, therefore, if a disturbance should radiate from the center in such a medium, the wave-front would be a flattened ellipsoid

of revolution. But in Iceland spar we have a compound medium, isotropic with reference to the ordinary ray, and æolotropic with reference to the extraordinary ray. Hence Huyghens' construction for both of these rays. Let SS' (Fig. 220) be the surface of a rhombohedron of

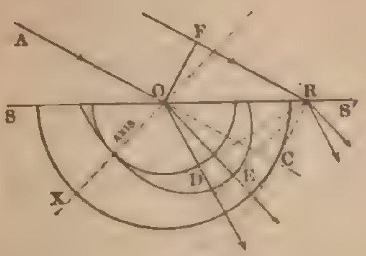


FIG. 220.

Iceland spar, AO being the incident ray, OF the wave-front, and OX the optic axis. The wave-front for the ordinary ray within the spar will be spherical as before and OD will be its direction after refraction. The wave-front of the extraordinary ray will be ellipsoidal; and drawing such an ellipsoid with the optic axis as its minor axis, its major axis representing the reciprocal of the extraordinary index, the direction of the extraordinary ray will be that of a line drawn from the point O through the point of tangency of a plane from R upon this ellipsoid; i.e., the direction OE.

In this case, however, the plane of incidence is a principal section; i.e., contains the optic axis, and is perpendicular to the surface of incidence. But other incident planes may be taken. Thus, let the plane of incidence be perpendicular to the axis; then the axis at O (Fig. 221) will be perpendicular to the plane of the paper. The wave-surfaces in the plane of incidence both have circular sections, the radii being proportional to the speeds of the ordinary and extraordinary waves; i.e., to the reciprocals of their indices. The points of tangency to the circles representing these wave-surfaces are evidently the points through which the normals to

the wave-fronts pass; in other words, OD is the ordinary and OE the extraordinary refracted ray. It will be

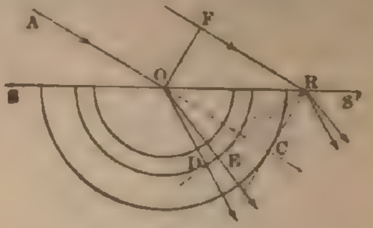


FIG. 221.

noticed that in this plane both of the indices are constant.

A third direction may be assumed for the incident plane; i.e., a direction parallel to the optic axis. Let the normal to the wave-front be incident at O (Fig. 222),

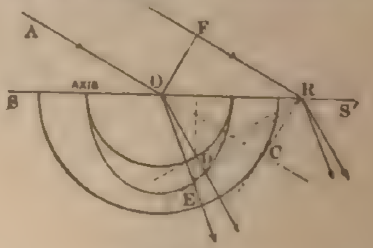
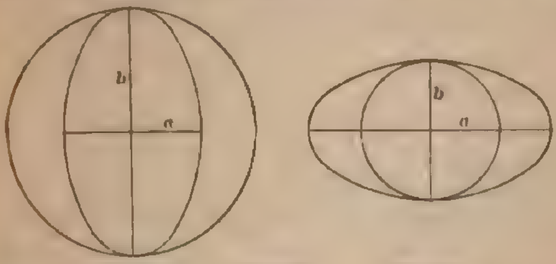


FIG. 222.

SOS' being the optic axis. Completing the diagram as before, it will be observed that now the extraordinary ray OE is more refracted than the ordinary OD; being repelled apparently by the axis. Hence the term repulsive, applied to negative uniaxial crystals.

In the Iceland-spar ellipsoid, the semi-axes a and b are equal to $1/m_e$ and $1/m_o$, the reciprocals respectively of the extraordinary and the ordinary refractive indices. But since b is not only the polar semi-axis of the ellipsoid, but also the radius of the sphere, it is evident that the ellipsoid is tangent to the sphere at the extremities of this axis; and further, that as m_e is smaller than m_o, a must be longer than b, and hence the ellipsoid must be exterior to the sphere. Obviously in the case of a posi-

tive crystal such as quartz, in which the ordinary index is smaller than the extraordinary index, a must be shorter than b. Moreover, b is now an axis common to the ellipsoid and the sphere. Consequently, while the two are



Positive. FIG. 223. Negative.

tangent at the extremities of the axis of revolution, the ellipsoid is prolate and the sphere envelops it (Fig. 223).

H.—POLARIZATION.

448. Change of Intensity in Double Refraction.—If two rhombs of Iceland spar be superposed with their edges parallel, the pair will act simply as a single plate of double thickness, and the separation of the images will be doubled. If, however, the upper rhomb be turned about a vertical axis, four images will appear, the two new ones becoming brighter and the two old ones fainter until the rotation reaches 45°, when all will have the same intensity. The change continues on rotation until 90° is reached, when but two images are seen, inclined 45° to both principal sections, which of course are now perpendicular. The ordinary wave from the lower crystal now becomes the extraordinary one in the second; and *vice versa*. As the rotation continues, four images appear, two decreasing in intensity and two increasing until 180° is reached, and the principal sections are again parallel, but the crystals are reversed in position. But one image is now visible, the upper crystal undoing the work of the lower. This change of intensity of the images was observed by Huyghens, who comments on the "wonderful phenomenon" that light has undergone such a change by refraction in the first rhomb that

its transmission through a second and similar rhomb depends on the position of the principal section of the second rhomb with reference to the first.

More than 100 years later, in 1808, Malus, engaged in verifying the Huyghenian law of double refraction, viewed through a double-image prism the light reflected from a distant window, and observed, on rotating this prism, that the two images varied in intensity. The ordinary image disappeared in two opposite positions of the prism, and the extraordinary image also in two opposite positions; but the second positions were 90° from the first. This variation of intensity was quite similar to that observed in the two rhombs of spar by Huyghens; and it became evident to him that light by simple reflection undergoes a change in its character similar to that which it suffers by double refraction. In short, that a beam of light thus treated is not alike upon all sides, but has certain relations to surrounding space other than direction. To this phenomenon Malus gave the name **polarization.**

449. Polarization by Reflection.—If the above experiment be suitably varied, it will be found that with a non-metallic mirror the effect produced reaches a maximum for a special angle of incidence. When the polarization is a maximum, Brewster showed that the tangent of the polarizing angle is the index of refraction between the media. Thus let the ray AO (Fig. 224) fall upon the surface SS' at the polarizing angle; then the angle AON, which is the angle of incidence i, will be this polarizing angle; and hence by Brewster's law, we shall have $\tan i = n$. From this we derive the expression $\sin i/\cos i = n$; and from the law of refraction, $\sin i/\sin r = n$; whence $\sin r = \cos i$, or $i + r = 90°$. Consequently, since $BOC + i' + r = 180°$, and $i' = i$, $BOC = 90°$. In other words,

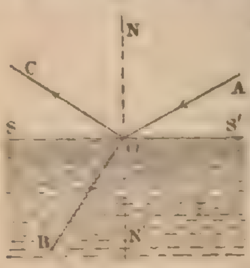


FIG. 224.

at the polarizing incidence the reflected and refracted rays are at right angles to each other. The **plane of polarization** for light polarized by reflection is defined to be the plane of incidence. This law of Brewster is independent of the direction of the ray with reference to the two media. The ray AO (Fig. 225) incident at the polarizing angle $\angle AON$ or i upon the upper surface of the glass plate RR' is reflected to B, the angle BOO' being a right angle. So the ray OO', incident upon the lower surface, and therefore incident in the denser medium at the polarizing angle $OO'N'$ or i', is reflected to O'', the reflected ray $O'O''$ making a right angle with the refracted ray $O'C$ as before. This must be so from the fact that the refractive index from the rarer to the denser medium is the reciprocal of that from the denser to the rarer, and hence if $\tan i = n$, $\tan i' = 1/n$; whence $\tan i = 1/\tan i'$, and the polarizing angle at the second surface is the complement of that at the first. This appears from the figure; for the incident angle at the second surface, $N'O'O$, is equal to $O'On$, its alternate; and since $O'On + O'OB + BON = 180°$ and $BOO' = 90°$, $O'On + NOB = 90°$; i.e., the two polarizing angles are complementary. It follows from this that if the angle of incidence at the first surface is the polarizing angle, the angle of incidence at the second surface will be so also. Consequently the total amount of light in the reflected beam from both surfaces will be completely polarized. At the polarizing angle, however, only a fraction of the total incident light, not more than a twelfth, is reflected. So that to obtain an intense beam, it is necessary to increase the number of reflecting surfaces. Hence the use of a number of thin plates of glass as a polarizing apparatus, ten or twelve being in general sufficient. Of course knowing the refractive index, the polarizing angle

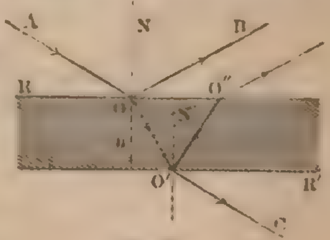


FIG. 225.

can be calculated by Brewster's law. Thus for water, whose mean index is 1·332, $\tan^{-1} 1·332 = 53° 6'$; for crown glass, $\tan^{-1} 1·515 = 55° 58'$; for flint glass, $\tan^{-1} 1·622 = 57° 45'$, and for diamond, $\tan^{-1} 2·470 = 67° 57'$. Evidently, however, since the index for a given medium differs with the wave-frequency of the incident radiation, the polarizing angle must be different for each kind of radiation.

If the radiation be incident at some other than the polarizing angle, the reflected beam still contains completely polarized radiation, but this constitutes only a fraction of it, the rest consisting of ordinary radiation unmodified by the reflection. If such a partially polarized beam suffer subsequent similar reflections, the proportion of polarized radiation continually increases until, as Brewster has shown, the beam becomes practically completely polarized; the number of reflections which are necessary increasing in proportion as the angle of incidence is more removed from the angle of polarization.

450. Polarization by Refraction.—But not only is the reflected portion of the incident beam in the above experiments polarized; the transmitted portion is polarized also, but with this difference, that the plane of polarization which, in the former case, is the plane of incidence, in the latter case is at right angles to this plane. Hence Arago's law: When radiation is partly reflected at, and partly transmitted through, a transparent surface, the reflected and transmitted portions contain equal portions of polarized radiation, the planes of polarization being at right angles with each other. As in the reflected portion, the fraction of polarization in the refracted beam increases with successive refractions, until the transmitted beam is completely polarized.

451. Transmission and Reflection of Polarized Radiation.—It follows from what has been said that if a beam of polarized light be incident upon a reflecting surface at the polarizing angle in such a way that its plane of polarization coincides with the plane of incidence,

it will be wholly reflected; while if its plane of polarization be at right angles to the plane of incidence, it will be wholly transmitted. So that if we may assume ordinary radiation to be made up of two equal portions each polarized in a plane perpendicular to the other, the process of polarization by reflection and refraction resolves itself simply into a sort of sifting process, the one portion or the other being transmitted or reflected according as its plane of polarization is at right angles to, or is parallel to, the plane of incidence.

Thus if two flat plates of glass ab and cd (Fig. 226)

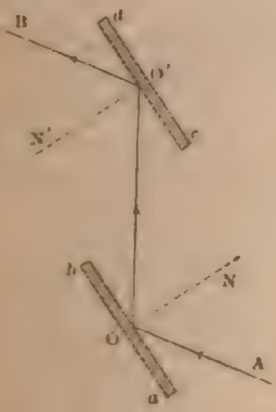


FIG. 226.

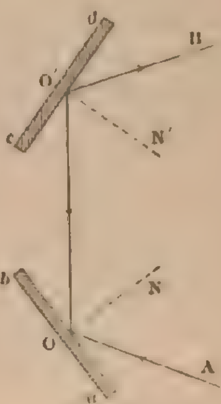


FIG. 227.

be so placed that the first receives the radiation AO at the polarizing angle AON and reflects it to the second, it is clear that the reflected beam OO' is polarized in the plane of incidence; i.e., the plane of the paper; and therefore being incident upon the second plate with the plane of polarization parallel to that of incidence, that it will be reflected along the path $O'B$. The same will be true if the second mirror be revolved about OO' as an axis through 180°, since the same condition remains true (Fig. 227). But if the rotation is only 90° in either direction, then it is evident that the plane of polarization and the plane of incidence on this

second mirror are perpendicular to each other and the beam will be transmitted instead of reflected. By attaching the mirrors to the outer ends of two tubes, one of which slides within the other, so that the normals to the mirrors form with the axis of the tubes the polarizing angle, and throwing a beam of light upon one so that it will be reflected centrally through the tube, it will be observed that the source of light can be seen by reflection from the second mirror only when the incident planes to the mirrors are parallel; and that it can be seen by transmission (when a bundle of plates is used) only when these incident planes are perpendicular to each other.

452. Variation of Intensity.—Law of Malus.—If the angle between the planes of incidence on the two mirrors be intermediate, the intensity of the radiation reflected from the second surface will be found to vary as the square of the cosine of the angle between these two planes; a law due to Malus. So that if α denote this angle and I the maximum intensity, we have for the actual intensity $I \cos^2 \alpha$; which becomes I when $\alpha = 0°$ or $180°$; and zero when α equals $90°$ or $270°$. Indeed if a beam containing two equal portions of polarized radiation, whose planes are perpendicular to each other, be incident upon a reflecting surface so that the polarization plane of one makes an angle α with the incident plane and that of the other makes an angle $90° - \alpha$ with it, the intensity of the reflected beam in the first case will be $I \cos^2 \alpha$ and in the second $I \cos^2 (90° - \alpha)$ or $I \sin^2 \alpha$. The combined intensity of the two will be $I (\cos^2 \alpha + \sin^2 \alpha)$ or I; i.e., the intensity of either beam alone when at its maximum. Hence the intensity of a beam thus composed will be constantly the same after reflection whatever the azimuth of the incident plane with reference to its axis. This fact, as common observation teaches, is true of common light.

453. Polarization by Double Refraction. — The change in the intensity of the images produced by double refraction when the upper of two Iceland-spar crystals

is revolved upon the lower, is of the same kind as that just described and follows the same law, the intensity varying as the square of the cosine of the angle made by the two principal sections of the crystals. Moreover, if we examine the two beams produced by double refraction, we find that the ordinary beam is capable of reflection only when the principal section of the crystal and the incident plane of the reflecting surface are parallel. The ordinary beam therefore must be polarized in a plane parallel to the principal section. So, since the extraordinary beam is transmitted under these conditions, or is reflected when the principal section is perpendicular to the incident plane, it must be polarized in a plane perpendicular to the principal section. Hence the two beams which issue from a doubly refracting crystal are polarized in planes at right angles to each other.

Certain doubly refracting crystals have the property of absorbing or destroying one of the two beams into which ordinary light is resolved by them. Tourmalin is such a crystal. If a plate be cut from a tourmalin crystal parallel to its optic axis, and a beam of common light be passed through it, the emergent beam will be found to be polarized in a plane perpendicular to this axis. But since two beams must have been produced by the double refraction, it follows that the one of these beams whose plane of polarization is parallel to the optic axis has been absorbed.

454. Double-image Prisms.—Nicol Prisms.—We have seen that the maximum difference between the ordinary and extraordinary indices is in a plane perpendicular to the optic axis; and hence that the maximum separation of these rays is obtained when the crystal is cut into a prism such that the refracting edge is parallel to the axis, so that the light traverses it in a plane perpendicular to this axis. But owing to the dispersion produced by such a prism, it is necessary in practice to achromatize it for either the ordinary or the extraordinary ray by means of a prism of glass reversed in po-

sition. Or better, as Wollaston suggested, by means of a second prism of the same material whose refracting edge is perpendicular to the optic axis. Such a device is called a **double-image prism.**

Owing to the complete polarization produced by double refraction, and to the fact that Iceland spar can be procured in large masses, this substance is commonly used for polarizing purposes. But it is often necessary to suppress one of the two beams. This was first done by Nicol by cutting a parallelopiped of Iceland spar whose length is twice its thickness, by a plane passing through its obtuse solid angles, polishing the surfaces, and cementing them together again as before by a layer of Canada balsam (Fig. 228). The index of refraction of the balsam lies between the ordinary and the extraordinary indices of Iceland spar. So that if a beam incident in a direction parallel to the lateral edges of the prism be divided into two within the prism, the ordinary beam will meet the surface of the balsam at an angle greater than the critical angle and will be totally reflected and thrown out; the extraordinary beam passing through the prism. This is insured by giving the end faces of the prism an angle of 68° with the blunt lateral edges of the parallelopiped in place of 71°, the natural angle; so that these end faces are perpendicular to the artificial plane. This device is called a **Nicol prism.**

FIG. 228.

455. Polariscopes.—A polariscope is an instrument for producing and testing polarized light. It is composed of two characteristic parts, the **polarizer** and the **analyzer**; which, since polarized light is examined by the same means that are used to produce it, may be any of the devices already mentioned for producing it. Thus the two glass plates already described, placed on a tube about the axis of which they can rotate, and inclined to

this axis at the complement of the polarizing angle, is known as Biot's polariscope. Either mirror at pleasure may be used to polarize the light, the other being then used to analyze it. So a bundle of thin plates, a double-image prism, a plate of tourmalin, or a Nicol prism may be used as the polarizer, and either of these in turn may serve also as the analyzer. The Nicol prism is generally preferred for both uses. Evidently when the incident planes or the principal sections of the polarizer and analyzer are crossed, no light is transmitted through the polariscope and the field is dark.

456. Nature of Plane-polarized Radiation.—The polarized radiation thus far studied is characterized by having a plane of polarization and hence is said to be **plane-polarized.** As we have seen, it is capable of reflection only when its plane coincides with the plane of incidence, and of transmission through a doubly refracting crystal only when its plane is parallel to the principal section of this crystal. The wave-theory, which in the hands of Young had so admirably explained the phenomena of interference, entirely failed for a long time to account for polarization. Even as late as 1821, the time of Fresnel, æther-waves were regarded as waves of compression and rarefaction, the particles vibrating in the line of propagation and differing from sound-waves only in length and in the speed of propagation. By the introduction of the idea of transverse vibrations, Fresnel was able to explain by means of the wave-theory the most complex phenomena of polarization as readily and completely as he had already accounted for those of interference. For it is obvious that now a ray, consisting of a single row of particles transmitting radiation, may be considered the axis of a wave-form, all the particles vibrating transversely to the direction of propagation, but successively. If, further, we suppose these transverse vibrations of successive particles to be executed all in one plane, evidently such a ray will be differently reflected or refracted according as the plane of vibration is parallel or perpendicular to

that of incidence. Such a ray, therefore, is plane-polarized. Opinions differ, however, on the question whether the plane of vibration coincides with the plane of polarization or is perpendicular to it. The weight of evidence at present is in favor of the latter view.

457. Circular and Elliptical Polarization.—In the wave-front of plane-polarized homogeneous radiation, all the vibrating particles move in straight lines, perpendicular to the plane of polarization. Suppose now two such wave-fronts to meet, each having the same amplitude but in one of which the vibration-plane is perpendicular to that in the other, and one of which is one quarter of a wave in advance of the other. The resulting motion of the æther-particle thus simultaneously acted on by the two waves will be circular; just as when two pendulums vibrating in perpendicular planes and differing a quarter of a period in phase are compounded (58). While, therefore, each particle in the wave-front describes a circular path whose plane is perpendicular to the wave-normal, the successive particles constituting the wave lag slightly each behind the other; so that the wave has the form of a helix, circular in cross-section; and the radiation is said to be **circularly polarized.**

The experimental production of circularly polarized light corresponds exactly to that above described. Let a beam of plane-polarized light fall on a plate cut from a doubly-refracting crystal, the faces of the plate being parallel to the principal section, and the plane of polarization inclined at 45° to the axis. Two equal beams will be thus produced, plane-polarized at right angles to each other. But since the extraordinary beam has travelled over a longer path through the plate, the two beams are not in the same phase; so that if the plate be cut of such a thickness that this difference of phase is one quarter of a period, we shall have the required conditions; two waves polarized in perpendicular planes and differing in phase by quarter of a period. Hence the beam emergent from the crystal plate (which is called a quarter-undulation plate) is circularly polar-

ized. Evidently, the thickness of such a plate, being a function of the wave-length, is different for each different color and must be used in monochromatic light. In some cases a pair of plates is used, superposed on each other; their action being differential.

Fresnel's rhomb is another device for effecting a retardation of one quarter of a wave. It is based on the fact that when a plane-polarized beam is incident on a reflecting surface so that the plane of polarization is not coincident with nor perpendicular to the incident plane, it is resolved into two beams polarized respectively in and perpendicularly to this plane of incidence; the intensity of these beams being equal when the plane of polarization of the original beam is inclined 45° to the incident plane. Moreover, we have already seen that there is a change in phase in reflection. Fresnel observed that in St. Gobain glass, whose index is 1·51, there is a difference of phase of one eighth of a wave-length when the polarized light is totally reflected from its surface at an angle of 54° 37′; so that by two such reflections, a difference of $\frac{1}{4}\lambda$ will be obtained. On making a rhomb of such glass (Fig. 229) the acute angles of which are 54° 37′, and which is rectangular in section, a beam of polarized light whose polarization-plane is 45° to the incident plane entering normally will suffer two reflections successively, and will emerge normally circularly polarized; having been resolved into two equal beams oppositely polarized and differing by a quarter of a period. Circularly polarized light is capable of reflection equally in any azimuth like common light. But on passing it through a second rhomb, the difference of phase is doubled and the emergent beam is again plane-polarized, its plane of polarization being now inclined 45° to the plane of the original incident beam; since two equal rectangular plane vibra-

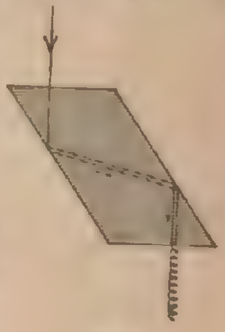


FIG. 229.

tions differing by half a period, compound into a single plane vibration intermediate between the two in direction.

Evidently in all the cases above considered, if the amplitudes of vibration be not equal for the two beams or if the difference of period be intermediate between the values given (i.e., between 0 and $\frac{1}{4}\lambda$ or between $\frac{1}{4}\lambda$ and $\frac{1}{2}\lambda$), the resultant wave will be elliptically polarized, the axes of the ellipse in the former case being parallel to the polarization planes of the constituent beams, and in the latter inclined to them. Light reflected from the surfaces of substances of high refractive power, such as the diamond, and from metallic surfaces, is found to be elliptically polarized; its intensity varying from a maximum to a minimum as the Nicol prism is rotated, but never becoming zero. Hence for such surfaces there is an incident angle of maximum, but not an angle of complete, polarization. By receiving a beam of elliptically polarized light upon a double-image prism and turning it until the two beams have the maximum difference of intensity, the principal section of the prism will have the direction of one of the axes of the ellipse; their lengths being proportional to the square roots of their intensities.

458. Interference of Polarized Radiation.—If two polarizing devices, such, for example, as two Nicol prisms, be placed with their principal sections crossed, no light will traverse them. But if a thin plate of some doubly refracting substance be inserted between the two prisms, the light will be transmitted. Closer observation will show that there are two positions of the thin plate at right angles to each other in which no effect will be produced; these being when the principal section of this plate is parallel or perpendicular to the plane of polarization of the incident light. At all intermediate positions, light pases through the analyzer, its intensity reaching the maximum when the principal section of the plate makes an angle of 45° with the plane of polarization. This effect is one of the most delicate tests of

double refraction in a substance. Moreover, it will be found that if the plate be very thin and if white light be employed, the transmitted beam is colored, the color depending on the thickness of the plate; and that if the plate be rotated in its own plane, the color vanishes four times in every complete revolution. If, however, the analyzing Nicol be rotated, the color given by the plate will be observed to pass gradually into its complementary color every 90° of rotation. The substances most commonly used for the thin plate are mica and selenite, since both these minerals readily cleave into thin laminæ.

These phenomena are due simply to interference. Two polarized beams can be made to interfere, as Fresnel has shown, only when their planes of polarization are coincident. Suppose now a plane-polarized beam to fall on a doubly refracting plate whose principal section is at 45° to the plane of polarization. Evidently it will be split into two beams polarized in perpendicular planes. Each of these beams falling on the analyzer at 45° will be split into two others also polarized perpendicularly, those components of both pairs alone being transmitted which are perpendicular to the principal section of the analyzing Nicol. These transmitted beams are in the same plane, therefore, and their effect upon each other will depend upon their difference of phase. Now the speeds within the thin plate are different for the two beams; and hence for a given wave-length there is a certain thickness of plate which will produce a difference of phase of half a wave-length between the two; a condition which will cause the two beams to destroy each other.

459. Ring-systems of Uniaxial and Biaxial Crystals. —If a plate cut from any uniaxial doubly-refracting crystal perpendicular to its axis be examined in a convergent or divergent pencil of plane-polarized light, produced by means of a lens, a series of circular colored fringes or rings will be seen, intersected at the center by a white or a black cross (Fig. 230, A). Since the incident

light is oblique, the thickness of the film traversed increases with the distance from the center and so causes interference in the inverse order of the wave-length; thus producing an inverted spectrum repeated at definite intervals. Again, since the beam of light is conical and is incident symmetrically about a center, these phenomena appear symmetrically as a series of circular fringes

FIG. 230.

or rings of color. Since no light traverses the analyzer in one fixed plane nor the polarizer in a perpendicular one, when the two are crossed the black intersecting cross represents the two polarization planes of the polariscope. If the section be cut parallel to the axis, the fringes are hyperbolas.

If a plate cut from a biaxial crystal be thus examined, a series of curved fringes will also be seen, the form of which will be different according to the relation of the section to the axes. If cut so as to be perpendicular to one of the axes, they are closed rings; if so as to be parallel to the plane of both the axes, they are hyperbolas; and they are lemniscates if they are cut perpendicularly to the bisector of the angle between the axes (Fig. 230, B). In the latter case, the fringes representing the ends of the optic axes approximate ovals in form, the angular distance separating them as the plate is turned about an axis perpendicular to the plane of the optic axes, being the angle between these axes. Dark bands cross these fringes when the polarizer and analyzer are crossed,

assuming the form of hyperbolas or intersecting crosses according to the position of the line joining the optic axes, with reference to the plane of polarization.

When a plate of quartz cut perpendicular to the axis is placed in such a conical beam, the colored rings or isochromatic lines belonging to uniaxial crystals appear but without the intersecting cross at the center. This results from the fact that the two rays into which quartz doubly refracts light are not plane-polarized but circularly-polarized. And the union of these two circularly-polarized rays by the analyzer produces a plane-polarized wave which has a different azimuth for different wave-lengths; and so these different colors appear at the center as the analyzer is rotated.

460. Rotatory Polarization.—In 1811, Arago observed that when plane-polarized light is transmitted through a plate of quartz in the direction of its optic axis, the plane of polarization is rotated through a certain angle depending upon the thickness of the plate and the color of the light. Biot subsequently showed that certain varieties of quartz rotated this plane to the right, others to the left; and he called the former dextrogyrate or right-handed, the latter levogyrate or left-handed. Other substances, both liquids and vapors as well as solids, were found to possess this property of rotation. Thus using sodium light and a plate one millimeter thick, the rotation produced by quartz is 21·67°, by cinnabarite 32·5°, and by sodium chlorate 3·67°. For liquids in tubes a decimeter long, calling right-handed rotations positive and left-handed negative, we have for turpentine − 29·6°, mint − 16·14°, aniseed − 0·7°, lavender + 2·02°, fennel + 13·16°, citron + 55·3°, Seville orange + 78·94°. For solutions, cane-sugar in water (a 50% solution) + 33·64°, quinine in alcohol (6%) −30°. The phenomenon of rotation depends on the production of two opposite circularly-polarized rays by the substance, which traverse it with different speeds and so when combined by the analyzer produce a plane-polarized ray inclined to the primitive one.

Practically the rotation of the plane of polarization by various substances may be turned to account in detecting or estimating these substances. Saccharimeters of various forms have been devised for estimating the commercial value of a sample of sugar by the amount of rotation produced by a solution of it of known strength; the various devices used to measure with precision the amount of rotation involving advanced scientific principles.

The **real specific rotatory power** α, of a substance, is the rotation, for a given wave-frequency, of a layer of the substance 1 mm. thick. The **apparent specific rotatory power** $[\alpha]$ is the rotation of the substance when in solution. If ϵ represent the amount of substance in 1 gram of solution, l the length of the column, usually given in decimeters, and ρ its density, $[\alpha] = \alpha/\epsilon l \rho$. Thus in the case of quartz and for the D line, $\alpha_D = 21{\cdot}67°$. For cane-sugar $[\alpha]_D = 67°$; for α-lactose $[\alpha]_D = 80°$; for β-lactose $[\alpha]_D = 54{\cdot}5°$. The **molecular rotatory power** is the product of the specific rotatory power by the molecular mass.

In the case of quartz, rotation depends on molecular aggregation, apparently; since dextrogyrate crystals exhibit hemihedral faces inclined to the right, and levogyrate crystals similar faces inclined to the left. Moreover in its amorphous form, as when fused, quartz has no rotatory action. In the case of tartaric acid, however, which is optically active when in solution, and of oil of turpentine and camphor, which are as active in the vaporous as in the liquid or solid state, rotation can be due only to a peculiar atomic structure within the molecule. Le Bel and Van't Hoff (1874) showed that all substances which, in the non-crystalline state, are capable of rotating the plane of polarization, contain an asymmetric carbon atom; i.e., a carbon atom whose four valences are saturated by four radicals of different kinds.

CHAPTER II.

ENERGY OF ÆTHER-STRESS.—ELECTROSTATICS.

SECTION I.—ELECTRIFICATION.

A.—HISTORICAL.

461. Electrics and Non-electrics.—Although, six hundred years before the Christian era, Thales appears to have been acquainted with the fact that amber, when rubbed, possesses the property of attracting light bodies, yet it was not until near the end of the sixteenth century that any real knowledge was acquired on this subject. Then Dr. Gilbert showed that many other substances, such as sulphur, resin, shellac, salt, alum, glass, rock-crystal, and the precious stones diamond, sapphire, ruby, amethyst and opal possess like properties with amber. And he gave the name **electrification**, from ἤλεκτρον, the Greek name of amber, to this property of these substances thus established. The substances in which electrification can be developed by rubbing he called **ideo-electrics**: in distinction from those in which no such property can be thus produced, which he called **anelectrics**. More recent investigation shows that all substances may be electrified by suitable means and are therefore electrics *per se*.

462. Conductors and Non-conductors.—In 1729, Gray observed that electrification can be transferred to an un-electrified body when brought into contact with an electrified one; i.e., that a body can be electrified by conduction. He succeeded in effecting this transference

through a wire more than 200 meters long, supported by silk threads; and he recognized in this way that bodies may be classified as good conductors and bad conductors, according to the readiness with which they permit this transference. Moreover, he observed that conductors as a class belong to the anelectrics of Gilbert; and hence that they show no sign of electrification when rubbed, because this electrification is conducted away. Since non-conductors do not permit the transference of electrification along them, they are called **insulators**. As a class the metals are the best examples of good conductors; while glass, resins, dry wood, silk, ebonite, turpentine, air, etc., are insulators.

EXAMPLES.—If a piece of amber or of jet, a stick of sealing-wax or of sulphur, a rod of glass or of hard rubber, be lightly rubbed with flannel, silk, or fur, it will be noticed that the body rubbed becomes electrified and will attract bits of paper, pith, or bran presented to it. A convenient form of light body for this experiment is a pith ball suspended by a linen thread; called often an electric pendulum. It acts as an electroscope to detect electrification; and by its means the greater electrification produced with some of the above substances over that with others may be shown. If such a pith ball be covered with gold-leaf and supported by a silk fiber, contact with an electrified body will readily electrify it by conduction. And then in its turn it will attract a second and unelectrified pith ball.

463. Electrification of Two Kinds.—Although in 1660 von Guericke had observed the repulsion of a light body after contact with an electrified one, it was not until 1733 that DuFay recognized the fact that electrification is of two kinds; and not until 1753 that Canton proved the rubbing body to be of quite as much importance as the body rubbed. To the electrification of glass Du Fay gave the name **vitreous** electrification; and to that of resin, **resinous** electrification. Wilke in 1757 arranged electrics in a series, each one being vitreous when rubbed with any one placed below it in the series, and resinous when rubbed with any one above it. Franklin in 1749 substituted the terms **positive** and **negative** for vitreous and resinous. Such a series, essentially that of Faraday, is the following:

ELECTRIC SERIES.

1. Cat's fur.	5. Glass.	9. Wood.	13. Resin.
2. Flannel.	6. Cotton.	10. Metals.	14. Sulphur.
3. Ivory	7. Silk.	11. Caoutchouc.	15. Gutta-percha.
4. Quartz.	8. The hand.	12. Sealing-wax.	16. Gun-cotton.

It will be observed that cat's fur is always positive and gun-cotton always negative, when rubbed with any of the other substances in the list. And further, that while glass for example is positive when rubbed with silk, it is negative when rubbed with flannel; sealing-wax on the other hand being negative when rubbed with silk, and positive when rubbed with gun-cotton. Moreover, in all cases the electrification of the body rubbed is equal in amount, and of opposite sign, to that of the rubbing one. These classifications must be accepted with some reserve, however, since the electrification developed is affected often by slight and apparently arbitrary causes, such as the physical condition of the substances, traces of foreign matters, temperature, etc. Thus a disk of ground glass is negative when rubbed against a similar disk of polished glass; a piece of white silk ribbon is positively and a piece of black silk ribbon is negatively electrified by being drawn between the dry fingers; if a piece of white ribbon be drawn transversely across a second piece, cut from the same roll, the two will be oppositely electrified. Metals may readily be electrified by providing them with insulating handles. Platinum, gold, and silver, for example, when rubbed with resins, silk, gun-cotton, or gutta-percha, are negatively electrified, while zinc and iron thus treated are positively electrified. Mercury, in which a glass tube is immersed, becomes negatively electrified on withdrawing the tube. And a jet of dry air electrifies positively a plate of glass against which it is directed. If two pieces of the same substance at different temperatures be rubbed together, that at the higher temperature will be negatively electrified.

EXPERIMENTS.—Bring near a pith-ball electroscope suspended by a silk fiber, a rod of glass excited by silk. The ball will be attracted.

will come in contact with the rod, will be positively electrified by this contact, and then will be actively repelled. A similar phenomenon will appear if a rod of sealing-wax excited by flannel be used to excite a second ball. Bring now the two pith balls near each other and they will be found to attract mutually. Moreover, the glass rod will attract the pith ball which the sealing-wax repels and *vice versa*. By taking the different substances given in the table, and rubbing them with silk for example, it will be observed that some of them by this treatment become positively and others negatively electrified; while the same substance may be positively or negatively electrified according to the character of the substance used as the rubber.

Hence the qualitative law of electrical attraction and repulsion announced by Du Fay: Bodies similarly electrified repel one another; bodies oppositely electrified attract one another.

464. Simultaneous Production of the Two Electrifications and in Equal Quantities.—Œpinus in 1759 took two disks of the same diameter, one of glass, the other of wood covered with cloth, both provided with glass handles, and having rubbed them together and separated them, brought each of them near an electric pendulum. The disks were found to be electrified, the glass disk being positive and the wooden disk negative. On repeating the experiment, but without separating the disks, it was observed that their joint action on a pith ball suspended by a linen thread, was zero; although on separation they were found electrified as before. Since the resultant action of two opposite electrifications is the algebraic sum of their separate actions, this resultant can be zero only when these electrifications are equal.

EXPERIMENTS.—Pour into a conical glass some melted sulphur and place a glass rod upright in the liquid to serve as a handle. When cold it will be found that while the system as a whole exhibits no electrification, the sulphur cone, on withdrawing it by means of the glass handle, is positively and the glass vessel negatively electrified. The effect is increased by covering the outside of the glass with tinfoil and connecting it to the ground while the sulphur is solidifying (Œpinus). Faraday's apparatus consisted of a stick of sealing-wax covered with a cap of silk at one end. After turning the stick round within the cap a few times, it may be presented to an electroscope; but no effect will be noticed. On removing the

silk cap, however, by means of an attached silk cord, the cap is found to be positively and the sealing-wax negatively electrified. A sheet of mica showing no electrification whatever yields two electrified laminæ when split, one of which is electrified positively, the other negatively.

B.—NATURE OF ELECTRIFICATION.

465. Mechanism of the Process.—In all the cases above considered two unlike substances have been electrified by being placed in contact with each other. If the substances are bad conductors it is obvious that to secure electrification over the entire surface, contact must take place at every point of it; or in other words, that the two must be rubbed together. The object of rubbing them together, therefore, is simply to secure an extended contact. The term electrification by friction is evidently a misnomer, since the effect is rather diminished than increased by friction. The principle may be laid down as an absolutely general one that when two dissimilar substances are brought in contact and then separated, they are equally and oppositely electrified. The phenomena are more striking in the case of bad conductors like glass and silk, because the electrification remains where it is produced and the effect accumulates; while when two conductors like zinc and copper are separated after contact, the electrification flows to the last point touched and so is dissipated by conduction from one to the other.

466. Theories of Franklin and of Symmer.—The phenomena of electrification were explained by Franklin (1749) by supposing the existence of "an electrical matter" resident in common matter. "Electrical matter differs from common matter in this," he says, "that the parts of the latter mutually attract, those of the former mutually repel, each other." "But though the particles of electrical matter do repel each other, they are strongly attracted by all other matter." "Common matter is a kind of sponge to the electrical fluid." "In common matter there is (generally) as much of the elec-

trical as it will contain within its substance. If more is added, it lies without upon the surface and forms what we call an electrical atmosphere; and then the body is said to be electrified." Symmer on the contrary (1759) considered the phenomena of electrification to be due to two electrical fluids, each self-repulsive and each attracting the particles of the other, and both attracting ordinary matter. Both of these hypotheses served provisionally to explain the phenomena for which they were created. But they were both deficient in that they assumed action at a distance and failed to recognize the importance of the intervening medium.

467. Modern Theory of Electrification.—Faraday first called attention to the important part which the intervening medium—called by him the **dielectric**—plays in the phenomena of electrification. He pictures two electrified bodies as immersed in an electrical field of force, the lines of force which connect the bodies tending to shorten and being apparently self-repellent. In such an electrical field there is always a stress parallel to the lines of force, of the nature of a tension; and a stress perpendicular to these lines, of the nature of a pressure. The name **electricity** is given to the agent upon which the phenomena depend. And although electrification is a form of potential energy, electricity itself is not a form of energy at all; it is rather a form of matter and this in the most refined sense. Like matter it can neither be created nor destroyed, though it can be moved and put under stress. It behaves like an incompressible fluid filling all space, and yet entangled in an æther having the rigidity necessary to propagate the enormously rapid and minute oscillatory disturbances which constitute radiation, while at the same time allowing the free motion through it of ordinary matter (Lodge). Touch together a disk of metal and one of silk; the contact transfers electricity from the metal to the silk. And now on separating them the medium between them is thrown into a state of strain, indicated by Faraday's lines of force. Electrification,

then, is to be viewed as a state of strain in the dielectric, produced by a transfer of electricity from one body to another and the subsequent separation of the two bodies thus electrified.

Section II.—Electrical Potential.

A.—Attraction and Repulsion.

468. Law of Electric Action.—We owe to Coulomb (1784) our earliest knowledge of the quantitative laws of electric attraction and repulsion. The instrument employed by him in his investigations has become classic and is known as the **torsion-balance** (Fig. 231). The needle *p* is made of shellac, carries a gilt ball *n* at one end, and is suspended by a fine silver wire from the torsion-head *e* at the top of the tube *d*. It is inclosed in a glass cylinder *o*, graduated around its middle portion *c*, and provided with a glass cover *A*. Within this cylinder is a dish containing drying material. Through the cover passes a wire *i* carrying a fixed ball at each end, the lower ball *m* being opposite the zero of the graduation. On electrifying the fixed ball, the movable ball, which has previously been brought by turning the torsion-head, just to touch this fixed ball, is repelled and after a few oscillations comes to rest, suppose at an angle of 36°. Since the couple of torsion is equal to the angle of torsion, the angle of deflection may be taken as approximately proportional to the force of torsion; i.e., to the force of repulsion. Coulomb found that to diminish the angular distance of the balls from 36° to

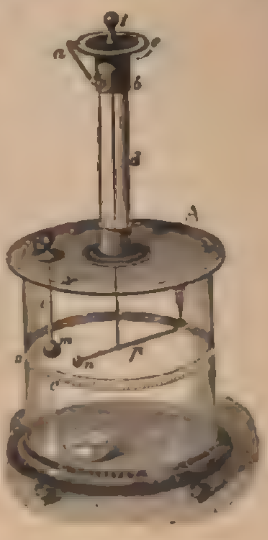


Fig. 231

18° required that the torsion-head be turned through 126°; and to reduce it to 8·5° the torsion-head must be turned through 567°. The total torsion in the first case is 36°, in the second 126 + 18 = 144°, and in the third 567 + 8·5 = 575·5°. Now these values are approximately 1 : 4 : 16; while the distances are 4 : 2 : 1; and hence the forces of repulsion are inversely as the squares of the distances. A rigorous investigation confirms this relation. By a similar method of experimenting, Coulomb proved the same proportionality in the case of attraction, the balls being oppositely electrified. If the fixed ball or the movable ball be touched by another ball of equal size, the electrification will be equally divided between the two; and now it will be found that only half the torsion will be required to bring the movable ball to the same distance from the fixed one. Hence by halving the electrification, the force of attraction or repulsion is halved; thus proving the force to be proportional to the amount of electrification. Similar results were obtained by Coulomb by oscillating a small shellac needle carrying a disk of gilt paper, at different distances from an electrified sphere. Since the force is inversely proportional to the square of the time of oscillation, the value of the force at any point is known when the number of oscillations per second is known. The law of electric action then is the same as that of any action from a center, such as gravitation, light, sound, etc., and may be enunciated as follows:

The force which is mutually exerted between two electrified masses is directly proportional to the product of their electrifications and inversely proportional to the square of the distance separating them.

Calling q the amount of electrification on the one body and q' that on the other, and representing by d the distance separating them, we have the force exerted between them represented by

$$F \propto qq'/d^2; \quad \text{or} \quad F = k(qq'/d^2).$$

in which k is a constant and represents the force exerted by unit electrification at unit distance. If, as is the case in an absolute system, this coefficient be made unity, then we may write

$$F = qq'/d^2. \qquad [61]$$

469. Unit of Quantity of Electrification.—From the above equation we have

$$q = Fd^2/q'; \quad \text{or if} \quad q = q', \quad F = q^2/d^2 \quad \text{and} \quad q = d\sqrt{F};$$

whence making F, d, and q' unity, we have q = unity also. In other words, a unit of electrification is that quantity of electrification which exerts unit of force, either attractive or repulsive, on a similar quantity of electrification placed at unit distance from it. In the C. G. S. system it is the quantity which acts with the force of a dyne on another similar quantity at one centimeter distance. This is called **the electrostatic unit of quantity.**

EXAMPLE.—Let two equal pith balls a and b (Fig. 232), each of mass 25 milligrams, be suspended by silk fibers 80 centimeters long and be similarly electrified. After contact they will repel each other; when in equilibrium let their centers be 10 cm apart. It is required to find the quantity of electrification on each ball. Calling the length ca or cb of each fiber l, and $2d$ the distance between the centers, m the mass of each ball, and q the quantity of electrification on it, the force of repulsion acting between the balls by the law just given is $q^2/4d^2$. By similar triangles, since the weight of each ball is mg, we have $q^2/4d^2 : mg :: d : (l^2 - d^2)^{\frac{1}{2}}$; whence

$$q^2 = \frac{4d^3mg}{l\left(1 - \frac{d^2}{l^2}\right)^{\frac{1}{2}}}. \qquad [62]$$

FIG. 232.

Substituting in this equation the numerical values given above, neglecting d^2/l^2, and taking g as 980, we have ± 12·37 as the value of q. There is then approximately 12·87 C. G. S. units of electrification, either positive or negative, upon each ball. (Gray.)

B.—ELECTRICAL DISTRIBUTION.

470. Electrical Charge.—The term **charge** is used to indicate the electrification of a body; as when a sphere is said to be charged with ten units of electrification. The term **surface-density** is employed to express the amount of electrification upon a unit of surface. It is generally represented by σ, and is obtained by dividing the total charge on a body by its surface. The expressions **linear** and **volume density** are also employed to indicate the amount of electrification per unit of length and per unit of volume.

471. Surface-distribution on Conductors.—Whenever a charge is communicated to a conductor, the electrification being free to move distributes itself over the conductor, reaching finally a condition of equilibrium. The first point to be observed is that the electrification, being self-repulsive, lies wholly upon the surface; i.e., there is no electrification whatever in the interior. Since this result, as we have already shown (124) in the case of gravitation, is a direct consequence of the law of inverse squares, very elaborate experiments have been made to ascertain its exactness. Franklin (1755) "electrified a silver pint cann on an electric stand and then lowered into it a cork ball of about an inch diameter hanging by a silk string, till the cork touched the bottom of the cann. The cork was not attracted to the inside of the cann as it would have been to the outside, and though it touched the bottom, yet when drawn out it was not found to be electrified by that touch, as it would have been by touching the outside." He adds: "The fact is singular. You require the reason; I do not know it. Perhaps you may discover it, and then you will be so good as to communicate it to me." Coulomb (1786) took a cylinder of wood supported on an insulating stand and bored holes in it a centimeter in diameter and a centimeter deep. After electrifying it, he applied to the surface and to the interior of the openings a little

disk of gilt paper supported on a shellac handle; testing the electrification of the disk by means of a torsion-balance so delicate that a force of 0·001 milligram would turn it through 90°. But not a trace of electrification could be detected beneath the surface of the cylinder. Faraday (1837) had a cubical chamber built, 2·66 meters on a side, covered with copper wire and tinfoil, to render its surface conducting, and well insulated. It was charged by means of the large electrical machine of the Royal Institution. "I went into the cube and lived in it," he says; "and using lighted candles, electrometers, and all other tests of electrical states, I could not find the least influence upon them, though all the time the outside of the tube was powerfully charged and large sparks and brushes were darting off from every part of its outer surface." Maxwell (1879) repeated Cavendish's well-known experiment (1773) of the two hemispheres and obtained a negative result, although the electrometers used by him were capable of indicating an electrification of one millionth of the quantity normally experimented with. Boys (1888) has shown that one soap-bubble entirely protects another bubble within it from electric action; thus proving the exceedingly minute depth to which the electrification on a conductor penetrates.

FIG. 233.

EXPERIMENTS.—A variety of experiments have been suggested to show the absence of electrification in the interior of charged conductors. (1) One of the simplest is made with a piece of brass tube 10 centimeters in diameter and about the same length mounted on an insulating stand with its axis horizontal, and having two pairs of small pith balls suspended, the one pair on the inside, the other on a small stem on the outside (Fig. 233). On electrifying the tube, the outer pith balls will diverge while the interior ones will not be affected. (2) Cavendish's apparatus, above mentioned, is a metallic sphere suspended by an insulating thread and provided with a pair of metal hemispheres enclosing it and furnished with insulating handles. If the sphere be electrified and then the hemispheres be placed over

it, so as to touch this sphere for an instant, it will be found on removing them that the sphere has entirely lost its electrification while the enclosing hemispheres are strongly electrified; the electrification having passed to the outside surface. The radius of the sphere should be somewhat smaller than that of the hemispheres. A modification of this apparatus is shown in the figure (Fig. 234). A hollow metal hemisphere B, resting on a hard rubber base, is electrified by striking it with cat's fur. Over this is then placed a similar but larger hemisphere A, having an insulated handle H. If now by pressing the knob S a spring is made to touch B for an instant, it will be found that all the electrification has passed to the hemisphere A. (3) A hollow ball with an opening at the top and supported on an insulating stand, may be used to repeat the experiments of Franklin and of Coulomb. A small disk of gilt paper on the end of a stick of shellac—called a **proof plane**—when inserted into the charged ball will receive no electrification whatever on touching the interior surface; as may be shown by testing it afterward with an electroscope. A cylinder of wire gauze on an insulating stand answers the same purpose. (4) Franklin placed a silver can on a wine-glass on the floor. Into the can be put about three meters of brass chain; fastening to one end of it a silk thread which passed through a pulley in the ceiling, and by means of which the chain could be raised. A lock of cotton, also suspended by a silk thread, was placed near the can; and was repelled when the can was electrified. On raising the chain "the (electric) atmosphere of the can diminished by flowing over the rising chain and the lock of cotton

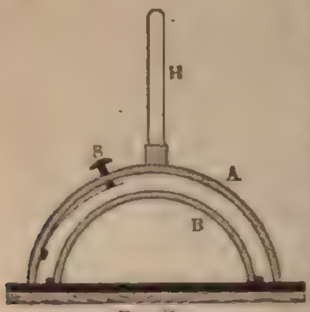


FIG. 234.


FIG. 235.

accordingly drew nearer and nearer to the cann." Upon lowering the chain the cotton was again repelled. Here as the surface increased, the charge remaining constant, the surface-density diminished and the repulsion decreased. (5) Faraday constructed a conical bag of linen gauze and attached its open end to an insulated ring, a silk string passing through the apex (Fig. 235). If this cone be electrified, no charge can be detected on the interior. But on turning the bag inside out by means of the silk cord, the electrification will pass through the gauze and again appear on the outside. (6) Sparks may be drawn from the exterior of an electrified bird-cage, while gold leaves placed within it are unaffected.

472. Influence of the Form of the Conductor.—Experiment shows that the distribution of electrification upon a conductor is entirely independent of its substance and is a function of the form of its surface only. Since a spherical surface is symmetrical about all its axes, a uniform distribution of electrification satisfies the law of electrical action. In the case of an ellipsoid, it can be shown that electrical equilibrium requires the distribution to be such that the surface-density at any point is proportional to a perpendicular let fall on the tangent-plane at the point. Thus (Fig. 236) the surface-densities at a, b, c, and d on the section of the ellipsoid shown in

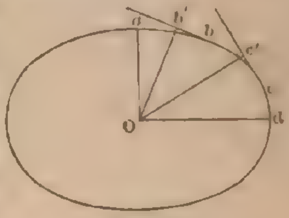


FIG. 236.

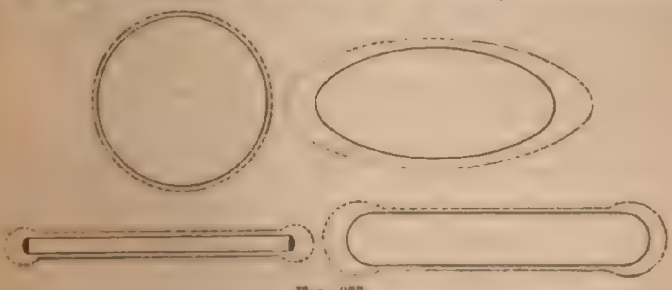


FIG. 237.

the figure are represented by Oa, Ob', Oc', and Od; these lines being perpendiculars on the tangents at a, b, c, and d. If the distribution on conductors of various forms be

examined (Fig. 237), it will be noticed that the surface-density is greatest where the curvature is smallest. In cases where the form of the conductor is not exceedingly simple, the calculation of the distribution is extremely difficult.

EXPERIMENT.—Fasten a disk of thin copper to a handle of shellac for use as a proof-plane. Bring the disk in contact with the surface of a charged sphere and then apply it to a suitable electrometer, the torsion-balance for example, and note the deflection. Upon repeating the experiment at various points of the surface, the same deflection will be obtained: showing that the charge of the proof-plate and therefore the surface-density on the sphere are the same at every point touched. If an ellipsoid be used instead of the sphere, the electrical surface-density will be greatest at its ends; and by making the experiments with care the results may be made quantitative. The surface-density upon the angles of an articulated parallelogram

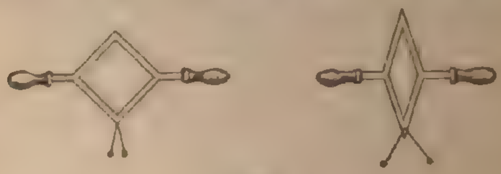


FIG. 238.

of silver paper (Fig. 238) increases as these angles become more acute (Mach).

Coulomb found in this way that using a cylinder 75 centimeters long and 5 centimeters in diameter, and calling the surface-density at the middle point unity, the density at 5 centimeters from the end was 1·25, at 2·5 centimeters 1·80, and at the end 2·30. For a disk 25 centimeters in diameter he found the surface-density at 10 centimeters from the edge 1·001 (that at the center being unity), at 7·5 centimeters 1.005, at 5 centimeters 1·17, at 2·5 centimeters 1·52, at 1·25 centimeters 2·07, and at the edge 2·90 centimeters.

473. Distribution due to Repulsion.—Since similar electrifications repel each other it is evident that there must be a repulsive effect due to an electrical charge on a conductor; in other words, that at all points of the

surface of such a conductor there must be a pressure acting outward against the surface of the surrounding dielectric. Since, in order that the electrical force in the interior of a conductor of any form shall be zero, it is necessary that the density shall increase as the surface decreases, it follows that where the radius of curvature is least and the density greatest, the outward pressure should be greatest. A soap-bubble blown on a metallic pipe and then electrified is found to expand from this repulsive force, and von Marum showed that a hydrogen balloon expands and becomes lighter when electrified. It can be shown (124) that the repulsive force thus exerted amounts to $2\pi\sigma^2$ dynes for each square centimeter of surface; i.e., that it varies as the square of the surface-density.

474. Effect of Pointed Conductors.—As the curvature of the surface increases, the density increases also; and the repulsion, which varies as the square of the surface-density, increases still more rapidly. When the density becomes about 100 electrostatic units per square centimeter the diminution of the air-pressure in consequence of the electrical repulsion is 68 grams per square decimeter, equivalent to about 66640 dynes per square centimeter. Then the electrification can no longer be retained upon the conductor and sparks fly from it into the surrounding air. Evidently this result takes place most readily where the density is the greatest; i.e., where the curvature is greatest, or at a point. The effect of points, therefore, as Franklin was the first to recognize, is to discharge into the air the electrification of any conductor of which they form a part. Electrical apparatus should therefore be free from points and from sharp edges. Since the air in contact with the point is similarly electrified and is repelled, there is continually passing away from a point an air-current, or an electrical aura.

EXPERIMENTS.—Place a needle upon a metallic sphere and then attempt to electrify the sphere. No charge will be received under these conditions, the density upon the needle-point being so great

that the electrification escapes there into the air. If a lighted taper be held near the point and in front of it, the flame will be deflected by the current of repelled air; and if this current be strong enough, the flame may be blown out. A windmill can be made to revolve by this current of air. Since this current is due to a mutual repulsion between the point and the air, the point may be made movable and will be driven in the opposite direction. A pair of pointed wires shaped like the letter S and supported on a central post (Fig. 239) revolves, when electrified, by this reactive effect.

FIG. 239.

475. Distribution on Two Conductors.—The same principles apply to the distribution of electrification upon two conductors in contact. Coulomb electrified a ball 15·5 cm. in circumference and found that it gave a torsion of 145°. After touching it with another ball 60 cm. in circumference, the torsion was reduced to 12°. Hence the charge on the second ball was to that on the first as 12 : 133 or as 1 : 11·1; the ratio of the torsions being as 1 : 12·1. The surfaces of the two balls were to each other as 1 : 14·8; and hence the density on the smaller ball being the quotient of charge by surface is 1·33 times that on the larger one. The experimental results agree with those obtained from theory by Poisson, both in this case and in that where the two conductors remain in contact. Thus with two equal spheres there is no electrification at the point of contact, nor for an angular distance of about 20° from this point. Then the density increases somewhat rapidly up to 90°, and afterward more slowly up to a maximum at a point opposite the point of contact. In the case of a cylinder and a sphere in contact, the ratio of the densities approaches a constant value as the length of the cylinder increases, the radii of the two bodies remaining the same. If the size of the sphere be increased, the mean density on the cylinder will be approximately proportional to the diameter of the sphere. Coulomb made use of this principle to calculate the electrical density at the end of the string

in Franklin's kite experiment. Calling the radius of the string 2 millimeters and supposing the cloud with which the kite was in contact to be the equivalent of a sphere 288 meters in radius, the ratio of the radii will be 1 : 144000; and the ratio of the mean densities, which is three sixteenths of this, will be 1 : 27000. Since the density at the end of an elongated cylinder is 2·3 times the mean density, it follows that at the end of the string the density is 62000 times as great as at the cloud; and hence the readiness with which sparks were drawn from the key, even assuming the cloud to have been but feebly electrified.

476. Electrical Convection.—If a pith ball be suspended between two conductors oppositely electrified, and be brought in contact with one of them, it will become similarly electrified, will be repelled by this one and attracted by the other, will come in contact with the other and will have its electrification reversed; and so will continue vibrating between the conductors, transferring the electrification of each to the other until they are both discharged. This transference of electrification by the actual motion of a charged body is called **electrical convection.** If a metallic lamp burning alcohol be placed on a charged conductor, it will rapidly destroy the electrification of the conductor; the products of combustion rising from the flame being charged and carrying away the electrification of the body from which they go. The readiest means of completely discharging electrified non-conductors, such as a plate of resin or of vulcanite, is to pass them rapidly through a flame like that of a Bunsen burner, connected with the earth. If an insulated conductor be placed in the rising column of heated and electrified air from the alcohol flame just mentioned, it will become charged. If the flame of a burner connected to earth be brought near a charged conductor, this conductor is discharged; but if the burner be insulated and be connected with a second conductor also insulated, this second conductor becomes similarly electrified with the first. The use of the water-dropping

collector of Thomson, and of a roll of burning touch-paper to obtain the electrical condition of the air at any point, depends upon the same principle.

EXPERIMENTS.—Various electrical toys are made use of to show electrical convection. "Suspend by a fine silk thread a counterfeit spider made of a small piece of burnt cork with legs of linen thread, and a grain or two of lead stuck in him to give him more weight. Upon the table over which he hangs we stick a wire upright as high as the phial [Leyden jar] and wire, two or three inches from the spider; then we animate him by setting the electrified phial at the same distance on the other side of him; he will immediately fly to the wire of the phial, bend his legs in touching it, then spring off and fly to the wire in the table; thence again to the wire of the phial, playing with his legs again both in a very entertaining manner, appearing perfectly alive to persons unacquainted." (Franklin.) A tumbler electrified and inverted over pith balls will set them dancing. Images may vibrate between two plates. And clappers may vibrate thus between bells hung near to them. In 1752, Franklin connected such a set of chimes to his lightning-rod so as to ring on the approach of a thunder-storm.

C.—ELECTRICAL WORK AND ENERGY.

477. Electrification a Form of Energy.—When two dissimilar bodies, such as mercury and glass, are brought into contact, they both become electrified; so that to separate them more force is required than is due to their gravitative action alone. To separate them to a given distance an amount of work must be done upon them which is represented by the product of the acting force by this distance; and this work thus done upon the system represents the increased potential energy of the system. So long as they may be still farther separated, so long may work be done upon them and so long may the potential energy of the system be made to increase. Evidently, then, the potential energy of a system reaches its maximum when the attracted and the attracting bodies are at an infinite distance from each other. But since the electrification of the system and its potential energy increase and decrease together, becoming zero at contact, increasing with distance and reaching a maximum at infinity, it is clear that what we have called the energy

of electrification of a system is nothing more than its potential energy.

If, on the other hand, the charged bodies be similarly electrified, the potential energy of the system has a positive value so long as they repel each other; and work will be done by the system, and its potential energy will diminish until they are separated to an infinite distance. In this case the potential energy is a maximum when the two repelling bodies are in contact, because the maximum work has been done upon them. It is a minimum when they are at an infinite distance.

478. Electrical Field.—The space surrounding an electrified body and through which the electrical force is exerted is called a **field of electrical force.** Faraday conceived this field of force to be filled with lines of force, indicating by their direction the direction of the resultant force and by their closeness the intensity of the field. A line of force then indicates the direction in which a unit of positive electrification would move if placed in the field. A field is said to be of unit intensity when a unit of electrification placed therein experiences unit force. This is represented in the C. G. S. system by one line of force to each square centimeter of surface, each such line of force representing a dyne. Hence a unit of electrification in a field of unit intensity would experience a force of one dyne, tending to move it along a line of force.

479. Electrical Potential at a Point.—Potential has already been defined (127) as a condition at a point, due to attracting or repelling masses in the vicinity, in virtue of which a unit mass placed at that point would possess potential energy. Hence if a positive charge be placed anywhere in an electrical field, its potential energy, due to the work done upon it to bring it there, will tend to diminish; and therefore it will experience a force tending to move it from the point where it is to another point where it would have less potential energy. In other words, it will move from a point of higher to a point of lower potential. The resultant force in such a

field therefore is in the direction in which the potential diminishes most rapidly, i.e., $F \propto -V$. But the potential energy possessed by the charge at any point, or, more strictly, possessed by the system in its present configuration, is measured by the work which has been done upon it to bring it to its present state from another state in which its potential energy was zero. That is to say, the electric potential at a point in a positive field is represented by the work which must be done upon a unit charge of positive electrification to bring it from an infinite distance to that point. Moreover, the difference of potential between two points in an electrical field represents the amount of work which would be done upon a unit positive charge in carrying it from the point of lower to the point of higher potential; or upon a unit negative charge in carrying it from the point of higher to the point of lower potential. A C. G. S. electrostatic unit of potential difference therefore exists between two points when an erg of work must be expended upon an electrostatic unit of quantity in order to carry it from one point to the other. If we represent potential by V, the potential at A by V_A, and the potential at B by V_B, then we have for the work done in carrying unit charge from A to B

$$W_A^B = V_A - V_B. \qquad [63]$$

In all cases the work done by or upon the electrical forces in transferring a unit charge from A to B is independent of the particular path followed in going from A to B.

Since in transferring unit charge through a difference of potential equal to unity, unit work is done, it is evident that by transferring two units of charge through unit difference of potential, or one unit charge through two units difference of potential, two units of work will be done; or four units of work if two units of charge are moved through two units difference of potential. In

general the work done in moving electrical charges is measured by the product of these charges into the difference of potential through which they are moved against the electrical action of the charges themselves. If Q units of positive electrification be moved from a point of lower to a point of higher potential, the difference between them being V, the work done $W = QV$.

480. Equipotential Surfaces. — If a unit positive charge be concentrated at a point and a unit negative charge at first in contact with it be moved away from it against attraction until unit of work shall have been expended, then it is evident that the difference of potential between them is unity. Moreover, the force being the same in every direction, the distance corresponding to unit difference of potential will be the same. So that about the unit positive charge as a center, we may draw a surface such that to carry a unit negative charge to it from that center will require the expenditure of an erg of work. So a second spherical surface may be drawn, the difference of potential between which and the first shall be unity and the value of the potential over the whole surface of which shall be the same. Such surfaces as these are called **equipotential surfaces** (130). In the case of spheres they are simply concentric surfaces drawn at such distance apart that one unit of work shall be done in transferring unit charge from one to the next. But in the case of more complicated distributions, these surfaces are more complex. Evidently since the potential is the same at all points of such a surface, no work is done in moving an electric charge over it. And since lines of force are the directions in which potential varies most rapidly, it is evident that the lines of force in any field must be perpendicular to the equipotential surfaces. The surface of a conductor in equilibrium is therefore an equipotential surface.

481. Energy Expended in Charging a Conductor.— When a conductor is positively electrified its potential is thereby raised to an extent dependent upon its shape, size, and form. The amount of charge required to raise

the potential of a conductor from zero to unity is called the **capacity** of the conductor. If Q units of electrification raise the potential of a conductor to V, its capacity C is

$$C = Q/V. \qquad [64]$$

In the case of a sphere, we have (127) for the potential at the center due to a charge Q

$$V = Q/R.$$

Combining these two equations, we have $C = R$; or in other words, the capacity of a sphere is numerically equal to its radius. We have seen above that the work done in raising a charge Q through a difference of potential V is QV units. But in charging the sphere the potential steadily rises as the electrification proceeds; so that only the last increment is raised through the potential V. Since the first increment is raised through the potential zero, the mean value of the potential through which the whole charge is raised is $(0 + V)/2$ or $\frac{1}{2}V$. Whence the total energy expended in charging the sphere is $\frac{1}{2}QV$. But since $Q = CV$ or $V = Q/C$, we have

$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \tfrac{1}{2}Q^2/C. \qquad [65]$$

From which it will be seen that the electrical energy of a conductor is directly proportional to the square of its potential or to the square of its charge.

482. Unit of Electric Potential.—The force exerted upon unit charge by a charge Q at distance R is, in the C. G. S. system, Q/R^2 dynes. The potential due to a charge Q at a distance R is Q/R ergs. If there be several acting masses q, q', q'', q''', etc., at distances r, r', r'', r''', etc., the total potential $V = \frac{q}{r} + \frac{q'}{r'} + \frac{q''}{r''} + \frac{q'''}{r'''}$, etc., $= \Sigma(q/r)$. Since numerically the work done between A and B in carrying a unit charge from one to the other is $V_A - V_B$ and is also $F(AB)$, we may equate these values: $V_A - V_B = -F(AB)$. Whence we have $F = -(V_A - B_B)/AB$; in which F represents the average or

mean force along AB. It will be observed that electric force is simply the rate at which the electric potential varies per unit of length.

Since potential represents work done, the unit of potential is the unit of work; i.e., the erg. Moreover, a unit difference of potential exists between two points when to bring unit charge from one to the other against the electric forces requires one erg of work to be expended. This is the case between two equipotential surfaces. Since relative potential is the analogue of level, any equipotential surface may be assumed as a surface of reference and called zero; potentials above this being called positive and below it negative. As the level of the sea is taken as the point from which heights are measured, so the surface of the earth is considered in practice to be at zero potential, electrically.

SECTION III.—ELECTROSTATIC INDUCTION.

A.—CHARGE BY INDUCTION.

483. Phenomenon of Induction. — Whenever an electrified conductor is brought into the vicinity of an unelectrified one, the latter becomes electrified; dissimilar electrification appearing on the side nearer the electrified conductor and similar electrification upon the farther side. Electrification produced in this way, by the presence of an electrified body and without contact, is called electrification by **induction**. Evidently in this case the two electrifications have been simultaneously developed, the similar electrification being repelled by the charge on the electrified conductor and the dissimilar electrification being attracted. On removing the electrified conductor, all signs of induced electrification disappear.

If, while the second conductor is still under the influence of the electrified one, its remote side be connected to earth, the electrification will disappear from that side; and on withdrawing the electrified conductor, the second conductor will be found to have a charge opposite in

kind to that with which the electrified conductor itself was originally charged. Hence electrification by induction produces an electrification opposite in kind to that of the exciting charge.

This inductive action plays an important part in all electrical attraction and repulsion. When a pith ball is attracted by an excited glass rod, the rod induces negative electrification on the nearer side, positive electrification on the more remote side. And as a consequence the attractive action being due to the opposite electrifications, which are nearer to each other than the similar ones, is stronger than the repulsion.

Further, if these two electrified bodies be allowed to come in contact, the two opposite electrifications will evidently neutralize each other, either wholly or partially; thus having both bodies similarly charged. It may therefore be quite impossible to distinguish the similar electrification of two bodies which is developed by induction, from the similar electrification produced by conduction. Faraday says: "Bodies cannot be charged absolutely but only relatively, and by a principle which is the same with that of induction. All charge is sustained by induction. Induction appears to be the essential function both in the first development and the consequent phenomena of electricity."

EXPERIMENTS.—An insulated cylindrical conductor with hemispherical ends whose length is about ten times its diameter (Fig. 240) is provided with two pairs of pith balls at its extremities, suspended by linen threads. (1) Bring near this conductor a metallic

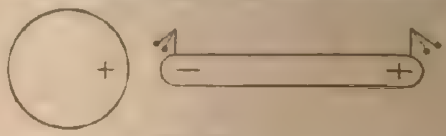


FIG. 240.

sphere charged positively. Both pairs of pith balls will diverge, the pair nearer the sphere being attracted toward it. (2) Examine now the condition of the cylinder by means of the proof-plane. It will be found that while a region near the middle part of the cylinder is entirely unelectrified, the end nearer the sphere is negatively, that farther from it is positively, electrified. (3) Remove the sphere from

the vicinity of the cylinder. All signs of electrification disappear, the positive and negative charges completely neutralizing each other; thus proving that they are equal. (4) Replace the electrified sphere and touch with the finger the remote end of the cylinder. This is equivalent to making the cylinder of indefinite length; moreover, the positive electrification being conducted to earth is so diffused that the charge on the cylinder becomes insensible. (5) Remove now the electrified sphere. The negative electrification, being no longer held by the attraction of the positive electrification of the sphere, diffuses itself over the cylinder and both pairs of pith balls again diverge, but now with the same electrification; i.e., negative. (6) Bring the electrified sphere into contact with one end of the cylinder and then remove it. The cylinder will be found to be electrified positively throughout, the negative electrification developed by induction upon the end nearer the sphere having been neutralized by the positive charge of the sphere itself, thus leaving the cylinder positively charged. (7) If two short insulated cylinders, placed in contact, be electrified by induction and then separated from each other, each will be found charged, but with opposite electrifications.

484. Gold-leaf Electroscope. — An electroscope is an instrument for detecting electrification. Franklin used a pair of linen threads as an electroscope, the two diverging when electrified. Canton placed a small ball of cork upon the end of each thread. Saussure used two fine silver wires each provided with a pith ball. Volta employed two pieces of straw; and Bennet (1787) replaced the pieces of straw by two strips of gold-leaf. Maxwell's form of gold-leaf electroscope is shown in Figure 241. The gold leaves l, l are suspended from a metal rod which passes through the top of an enclosing cylinder of glass G, G, and terminates in a metallic disk L. To screen the gold leaves from the action of outside electrification and to protect them from unequal distribution on the internal surface of the glass, a wire cage m, m is placed within the cylinder and is connected to a metal knob M upon the outside. The divergence of the gold leaves indicates a difference of potential between the leaves themselves

Fig. 241.

and that of the cage; so that if the cage be connected to earth its potential will be zero and the difference of potential will be the absolute potential of the gold leaves, due to the electrification upon them.

EXPERIMENTS.—(1) Bring a stick of sealing-wax which has been rubbed with flannel, near the plate of the gold-leaf electroscope. The gold leaves will diverge, being charged negatively by induction.

(2) Touch the plate for an instant with the finger and then remove the sealing-wax. The gold leaves will now be permanently charged positively.

(3) Bring a positively charged body, an excited glass rod for example, into the vicinity of the plate. The gold leaves will diverge still more. If a negatively charged body be brought near the plate the divergence of the gold leaves will be diminished.

(4) Insulate now the knob of the electroscope from the earth and bring a charged body near the plate. The phenomena will be all reversed, the divergence of the gold leaves being diminished if the body be positively charged, and increased if it be negatively charged.

(5) Repeat these experiments, using a glass rod rubbed with silk as the source of the electrical excitation. The indications of the electroscope will be the same as before with a change of sign in the electrification.

By means of the gold-leaf electroscope, therefore, not only may the electrification of a body be detected, but its positive or negative character relatively to that of the gold leaves may be determined; the gold leaves themselves being positively or negatively electrified according as they are of higher or lower potential than the surrounding cage.

485. Amount of Induced Electrification.—If an electrified body A be completely surrounded by a conductor B of any form whatever, there is produced upon the interior surface of the conductor B, by induction, an electrification of equal value with, and of contrary sign to, the electrification of the inducing body A, the distribution of which depends upon the form and position of A. There is also produced upon the exterior surface of the conductor B, by induction, an electrification of the same sign as that of A and of equal value, distributed regularly upon B as if no electrification existed in its interior. If the conductor B be put into communication

with the earth, the external electrification disappears, but the internal electrification is not affected; so that it exerts no action upon any exterior point. (Faraday.)

EXPERIMENTS.—(1) Place a suitable cylinder of metal *A* (Fig. 242)—Faraday used a pewter ice-pail—on an insulating stand, in connection with one electrode of a gold-leaf electroscope *B*, the other being put to earth, and lower into it a charged metallic sphere *C* suspended by a white silk thread. Observe (*a*) that the exterior of the cylinder is electrified, as is shown by the divergence of the gold leaves, (*b*) that this electrification is of the same kind as that of the sphere, and (*c*) that it ceases to increase as soon as the sphere is entirely within the cylinder. This is strictly true only when the cylinder is an entirely closed one such as is shown in Figure 243. Moving the sphere about within the cylinder has no effect upon the electrification.

FIG. 242.

(2) Touch the outside of the cylinder with the finger for a moment; the gold leaves will collapse. Observe that no electrification appears on moving the sphere about within the cylinder, but that on taking it out, the gold leaves diverge by the same amount as in the first experiment. The electrification, however, while equal, is now of the opposite sign to that of the sphere.


FIG. 243.

(3) Lower the charged metallic sphere until it comes in contact with the bottom of the cylinder. Observe (*a*) that no change takes place in the exterior electrification of the cylinder at the instant of contact; and (*b*) that on removing the sphere it is absolutely unelectrified. Thus proving that the unlike charge produced by induction on the interior of the cylinder and the like charge induced upon its exterior are exactly equal to the inducing charge upon the sphere.

(4) Compare the charges of two similarly electrified spheres (*a*) by letting them down into the cylinder alternately and noting the divergence of the gold leaves; and (*b*) by placing one of the spheres within the cylinder, touching the outside for an instant, then withdrawing the first sphere and introducing the second. If the charges are equal, there will be equal divergence in the first case; and in the second the electroscope will show no electrification.

(5) Place the two spheres oppositely electrified within the cylin-

der. If their charges are equal, the gold leaves will remain unaffected.

(6) Support a small electrified sphere within a suspended and closed cylinder, and discharge the external electrification. Then place the whole in a larger insulated cylinder connected with an electroscope. Observe (*a*) that the gold leaves will be unaffected even if the sphere be taken out of the smaller cylinder and moved about within the larger; and (*b*) that if either the electrified sphere or the smaller cylinder be removed from the larger, the electroscope will indicate either positive or negative electrification.

(7) Place this small charged sphere within the larger cylinder and note the divergence of the gold leaves. Then, without altering the charge, place the sphere within the smaller cylinder and both within the larger one, and again note the divergence. It will be the same in both cases; showing that the electrification induced on the outside of the smaller cylinder is the same in amount as that of the inducing charge on the sphere.

(8) Hang the smaller cylinder within the larger one, place the charged sphere in the inner cylinder, and connect the two cylinders. The larger one will have a charge equal to that of the sphere. Remove the sphere, take out the smaller cylinder and discharge it, replace it, put the sphere again within it, and again make contact. The outer cylinder now receives a second charge equal to the first, and by repeating the operation, any number of charges each equal to that of the sphere may be given to the cylinder. (Maxwell.)

486. Laws of Electrostatics.—The laws of electrostatic action are thus stated by Maxwell:

I. The total electrification of a body or system of bodies remains always the same except in so far as it receives electrification from, or gives electrification to, other bodies.

II. When one body electrifies another by conduction, the total electrification of the two bodies remains the same; the one losing as much positive or gaining as much negative electrification as the other gains of positive or loses of negative electrification.

III. When electrification is produced by any known method, equal quantities of positive and of negative electrification are produced.

IV. If an electrified body or system of bodies be placed within a closed conducting surface,

the interior electrification of this surface is equal and opposite to the electrification of the body or system of bodies.

V. If no electrified body be placed within a hollow conducting surface, the electrification of this surface as a whole and of every part of this surface is zero.

487. Electromotive Force.—We have already seen that a positive charge experiences in an electrical field a force tending to move it in the direction in which the potential diminishes most rapidly; i.e., in the direction of a line of force. Whenever a positive charge is placed on a conductor, its presence raises the potential of the conductor at that point above that of surrounding points; and hence a flow of electrification takes place until the potential is the same everywhere; i.e., until the surface of the conductor is an equipotential surface. So again, if two metal spheres at different potentials be connected by a wire, a transfer of positive electrification will take place from the one of higher to the one of lower potential, or a transfer of negative electrification from the one of lower to the one of higher potential, or both, until the difference of potential disappears. To any agency, whatever its nature, which tends to produce a transfer of electrification such as these the name **electromotive force** has been applied.

If a gilt pith ball suspended by a silk fiber and positively charged be placed in an electric field, it experiences a mechanical force tending to move it in the positive direction along a line of force; or in other words, in the direction of the electromotive force of the field. This mechanical force is found to be directly proportional, first, to the charge upon the ball; and second, to the electromotive force of the field at the point occupied by the center of the ball; and hence it is measured by the product of these quantities. If the charge upon the ball be one electrostatic unit, then the mechanical force which it experiences, measured in dynes, represents the electromotive force of the field. Hence "the electric or electro-

motive force at a point is the force which would be experienced by a small body charged with the unit of positive electrification and placed at that point, the electrification of the system being supposed to remain undisturbed by the presence of this unit of electrification." (Maxwell.)

B.—ELECTROSTATIC CAPACITY.

488. Effect of Induction upon Capacity.—The capacity of a simple conductor has already been defined (481) as the ratio of its charge to its potential; or in other words, as the amount of electrification required to raise its potential from zero to unity. In the case of a sphere at a distance from other bodies, the number of C. G. S. electrostatic units of electrification required to raise its potential from zero to unity is represented by the same number that expresses its radius in centimeters. Suppose now this sphere be surrounded by a spherical shell concentric with it, and connected to earth. It will be found that its capacity is very greatly increased, so that now a much larger quantity of electrification is required to raise its potential from zero to unity. Or what is the same thing, a comparatively small electromotive force is able to accumulate within such an apparatus a very large charge. The capacity of a conductor, therefore, is a function not of its own size and shape alone, but also of the form and position of neighboring conductors. A pair of conductors separated from each other by a small interval constitutes an apparatus termed **a condenser**, since in the language of the older theory so much electric fluid can be condensed into it.

489. Capacity of Condensers.—It is not difficult to calculate the capacity of some of the simpler forms of condenser. Take in the first place a sphere of radius a contained within a second and concentric sphere of radius b. If a charge q be given to the inner sphere, an equal and opposite charge $-q$ will be induced on the interior surface of the outer sphere. In the chapters on Potential

(127, 482) it was proved that the potential of a charge q at a distance r is q/r. Hence at a distance r from the center of the condenser the potential due to the inner sphere will be q/r and that due to the outer one will be $-q/r$. But these values are equal and opposite in sign, and hence at the point r, or at any other point outside the outer sphere, the potential is zero. At any point between the two surfaces, at a distance r from the center, the potential due to the inner sphere is of course q/r, since the point is outside of it. The potential due to the outer sphere, being constant at all points within it, and the same in value as on the surface, will be $-q/b$. Whence the total potential in this intermediate space, being the algebraic sum of the partial potentials, is $(qr^{-1} - qb^{-1}) = q(r^{-1} - b^{-1})$. At the surface of the inner sphere, the potential is $q(a^{-1} - b^{-1})$ since on this surface $r = a$. At all points within this inner sphere the potential is uniform and has the same value. The capacity of this inner sphere being the charge q when the potential is made equal to unity, we have from the equation $V = q(a^{-1} - b^{-1})$ by making $V = 1$, $q = 1/(a^{-1} - b^{-1})$; which reduces to $q = ab/(b - a)$; or $b - a : a :: b : q$. In other words, the capacity of a spherical condenser is a fourth proportional to the distance between the surfaces and the radii of these surfaces (Maxwell). If the distance $b - a$ between the spheres be called e, and ab be regarded as equivalent to a^2, then the capacity $= a^2/e = 4\pi a^2/4\pi e = S/4\pi e$; in which S is the surface of the inner sphere.

Evidently by making $b - a$ smaller, the capacity may be made greater; so that the capacity of a condenser is the greater the smaller the distance between its conducting surfaces. To illustrate this increase in capacity, take the case of a condenser having an inner sphere of 10 cm. radius and an outer sphere of 10·1 cm. The capacity of the inner sphere, away from all other bodies, would be ten electrostatic units. But in presence of the enclosing sphere connected to earth, it will take 1010 electrostatic units to raise its potential from zero to unity. In other words, its capacity has been increased 101 times; so that

it is now equal to that of a simple sphere more than twenty meters in diameter.

As a second case, let us take a condenser made of two flat plates separated by a layer of air of thickness e, much less than the dimensions of the plates, and let us restrict our investigation to the central portion. Calling A the potential of the upper plate and B that of the lower one, the mean electric force at any point between the plates will be $(A - B)/e$, acting from A toward B; since $Fe = A - B$, or the work done between the plates is equal to the difference of potential between them. According to the law of Coulomb, the force close to the surface of an electrified conductor is 4π times the surface-density; or $F = 4\pi\sigma$. Equating these two values of F we have $4\pi\sigma = (A - B)/e$; whence $\sigma = (A - B)/4\pi e$, the surface-density on the upper plate A. That on the lower is the same in amount but of opposite sign. If we assume an area S upon the upper plate as the area of the condenser-plate, the charge q upon such an area will be $S\sigma$, or $q = S((A - B)/4\pi e)$. If we make $A - B$, the difference of potential between the condenser-plates, equal to unity, the potential of plate A, since B is connected to earth and is zero, will be unity; and hence the capacity q will be $S/4\pi e$. Let us assume a disk of 10 cm. radius, as the upper plate of the condenser. Its capacity, supposing it to be separated one millimeter from the other plate, will be $100\pi/0{\cdot}4\pi = 250$. Or, to raise the potential of such a condenser from zero to unity would require 250 electrostatic units; equivalent to that required by a simple sphere 5 meters in diameter.

490. The Leyden Jar.—We owe the discovery of the phenomenon of condensation to Kleist of Camin (1745) and to Muschenbroek of Leyden (1746). Cuneus, the pupil of the latter, in endeavoring to electrify water in a bottle which he held in his hand, received a distinct shock when he attempted to remove with the other hand the chain leading to the machine. Franklin's condenser consisted of a plate of glass, its central portions coated on both sides with tinfoil. A more common form of con-

denser consists of a glass jar coated upon both the inside and the outside with tinfoil to near the top (Fig. 244) and provided with a wire communicating with the inner coating and terminating exteriorly in a knob. From the place of its origin such a condenser is ordinarily called a Leyden jar. Its capacity may generally be calculated with a sufficient approximation from the formula $C = S/4\pi e$, in which S is the area of one coating and e the thickness of the glass itself. The charge required to raise the potential of the jar to V units is of course V times this; whence $Q = SV/4\pi e$. The charge is directly proportional to S, the area of the coated surface.

FIG. 244.

EXPERIMENTS.—(1) Place a Leyden jar on a plate of glass and connect the knob to a source of electrification. Only a very small charge will enter the jar, so long as its outer coating is insulated from the earth.

(2) When sparks have ceased passing between the knob of the jar and the machine, hold a metal ball near the outer coating; a spark will pass between the coating and the ball, and at the same time another spark between the machine and the knob. By continuing to take sparks in this way, the jar may be charged.

(3) Test the electrification of the interior by means of the proof-plane, the jar standing on the table. Place the jar on a plate of glass, touch the knob for an instant with the finger, and then test the electrification of the exterior. Observe that the electrifications are unlike.

(4) Connect the outside coating with the knob by means of the discharging-rod, a curved wire with balls at its ends, supported on a glass handle. Just before the knob is reached a bright sharp spark passes and the jar is discharged.

(5) Insulate the jar on a plate of glass and touch the two coatings alternately with the finger or with a metal ball. Each contact transfers electrification and so ultimately reduces the difference of potential to zero.

491. Electric Battery.—A number of jars connected so as to act together constitute an **electric battery**. They may be so arranged that all the outside coatings are joined together and all the inside coatings are similarly

joined. The battery then acts simply like one large jar, its capacity being the sum of the capacities of the separate jars; or, if the jars are all equal, being the product of the capacity of a single jar by the number of jars. Or, secondly, the jars may be connected alternately, the outside coating of the first with the inside coating of the second, and so on. In this case the capacity of the battery is obtained by dividing the capacity of one jar by the number of jars.

492. Energy expended in charging a Condenser.—It has already been shown (481) that the work done in charging a simple conductor, and therefore the energy which is stored up in such a conductor when charged, is represented by $\frac{1}{2}QV$; or, since $Q = CV$, by $\frac{1}{2}CV^2$. The energy in a single jar or in a number of jars arranged in multiple, as in the first case above mentioned, is directly proportional to the capacity of the jar or battery. If this energy be transformed into heat during the discharge, the quantity of heat will be $H = CV^2/2J$; in which J is Joule's equivalent. For a given charge the energy in a battery arranged in multiple is inversely as the number of jars; while for a given potential the energy is directly as the number of jars. For a number of jars arranged in series, as in the second method above given, the reverse is true; the energy being, for a given charge, directly, and for a given potential inversely, as the number of jars. Since a given charge will raise the potential of a single jar higher than it will that of two jars, it is evident that when the charge of a single jar is divided between two, there is a fall of potential and consequently a loss of energy.

C.—DIELECTRIC POLARIZATION.

493. Electrification resident in the Dielectric.—In 1748, Franklin showed that "the whole force of the bottle and power of giving a shock, is in the glass itself; the non-electrics in contact with the two surfaces serving only to give and receive to and from the several parts of glass; that is, to give on one side and take away on the

other." Whenever an electrified body is entirely enclosed within a conducting surface, an opposite electrification is produced upon this surface precisely equal in amount; and this no matter how distant from the body this surface may be. The two conducting surfaces may therefore be regarded simply as the boundaries of the intervening dielectric; this dielectric being oppositely electrified on its inner and outer faces. The term "**field**" refers to the space between the conducting surfaces, "**medium**" to the substance occupying this space. The term "**insulating medium**" refers to the property of retaining the charge possessed by the medium, and the term "**dielectric medium**" to its property of transmitting induction.

EXPERIMENTS.—To repeat Franklin's experiment, provide a bottle, having thin lead-foil on the bottom and half-way up on the outside, and containing water, with which a wire connects through the cork. Charge this bottle, place it on glass, remove the cork and wire, and touching the outside with one hand, bring a finger of the other near the water. A smart shock will be felt. Charge the bottle again, remove the cork and wire and pour the water into a second bottle. No charge will be found in this second bottle, showing that the electrification is not in the water. Fill up the first bottle with ordinary water and it will be found charged. Hence the electrification exists on the glass.

The following methods of exploring an electric field in which air is the dielectric are due to Maxwell:

(1) Place a gilt pith ball hung by a white silk thread in the field. The ball if positively electrified will move from the surface of higher toward the surface of lower potential. If it be negatively electrified, it will move toward the surface of higher potential. The mechanical force urging it is the product of its charge by the electromotive force of the field at the point its center occupies.

(2) Take two small disks of thin metal, provided with insulating handles, and place them in contact, with their planes perpendicular to the electromotive force of the electric field. Then separate them, remove them, and test their electrification. Both will be found equally charged, the one toward the higher potential negatively, the other positively; thus showing a transference of electrification within the dielectric itself.

(3) Place one of the disks with its surface in contact with that of one of the conductors bounding the field. On removing it, it will be found to be charged, the charge being approximately that of the

portion of the electrified surface covered by it. This disk corresponds therefore to Coulomb's proof-plane; and its charge is proportional to the electric density at the point touched.

(4) Suspend in an electric field a short piece of fine cotton or linen thread by means of a silk fiber. Its two ends becoming oppositely electrified, the thread will place itself parallel to the direction of the electromotive force at the place it occupies.

(5) Hang in the field two metal balls, by means of silk threads, and place a fine insulated wire so as to touch them both. On examining the balls they will be found equally and oppositely charged, and since by the contact of the wire positive electrification will have been transferred from the one of higher to the one of lower potential the ball at the former point will be charged negatively and the one at the latter point positively; the charges being proportional to the difference of potential between the points. Evidently, therefore if the balls should be found uncharged, it would indicate simply that they had been placed in positions having the same potential; so that in this way the equipotential surfaces in the region may be mapped out.

(6) Or, using only one ball, place its center at a given point in the field, and touch it for an instant with a wire connected with earth. Its charge will evidently be proportional to the potential at the point; although if the potential there be positive, the charge will be negative.

(7) Connect a small metal ball permanently with one electrode of the gold-leaf electroscope by means of a fine wire, the other electrode being connected to earth. Place the ball in the field and connect the electrodes together for an instant; thus reducing the difference of potential between them to zero. Move the sphere now through the field, taking care that the gold leaves are not deflected. The path of the ball will lie on an equipotential surface.

(8) A better method is to connect the electrified system—a large conductor of irregular form, for example—with the exploring sphere, and after having placed this sphere at a given point, to connect the electrodes for an instant. If now the sphere be so moved in the field that the gold leaves are undisturbed, the path of the sphere must lie on an equipotential surface.

494. Lines and Tubes of Induction.—These experiments clearly prove: 1st, that the dielectric is the seat of the electric action; 2d, that it is traversed by an electromotive force in the direction in which the potential varies most rapidly; 3d, that it may be cut by planes perpendicular at all points to this direction which, since the potential is constant throughout their extent, are called equipotential surfaces; and 4th, that a transfer of

electrification may take place within it, positive electrification being transferred from a point of higher to one of lower potential and negative electrification in the reverse direction. Since the lines along which electromotive force acts are also the lines along which induction takes place, these lines may be called **lines of induction.** Every such line begins at a point on a positively electrified surface and terminates at a corresponding point on a negatively electrified surface. Since the surface of an electrified conductor is an equipotential surface, these lines are perpendicular to such a surface, whether they issue from it or abut upon it. Consider now a limited positive area of such a surface, and from each point of the line which bounds it conceive a line of induction to be drawn, terminating, as it must do, upon the corresponding point of the corresponding negative area on the surface of some other body. These lines taken together will form a **tube of induction,** the two ends of this tube representing electrifications similar in quantity although opposite in sign. Evidently by selecting the area so that its electrification is unity, the tube of induction will be a unit tube; and the electrification of any surface will be proportional to the number of such tubes which issue from or abut upon it. In the same way the induction through any part of an electric field may be represented by the number of unit tubes of induction which traverse a surface extended through this part. Moreover, the field is traversed by equipotential surfaces at such distances that unit of work is done in carrying unit electrification from one such surface to the next. Consequently these surfaces cut the tubes of induction into unit cells. If the potential of the one electrified surface is V_1 and that of the other V_2, then there are $V_1 - V_2$ equipotential surfaces in the field. So that if q represent the number of units charge on one surface, there will be q tubes of induction and $q(V_1 - V_2)$ cells. But as $-q$ is the charge on the second surface, we may also write $qV_1 + qV_2$ for the total number of cells in the field. Since the total energy of the system is $\frac{1}{2}\Sigma(qV)$, it follows

that the number of cells is twice the total energy contained in the electrified system. Hence each cell is that portion of the dielectric in which one half a unit of energy is stored up.

495. Electric Tension.—In the second experiment mentioned in (493), the two disks, after being in contact, tend to separate from each other and to approach the oppositely electrified surfaces bounding the field. The mechanical force acting upon each disk is proportional to the joint product of the electrification of the disk and the electromotive force of the field; but since the electrification on the disk is itself proportional to the electromotive force, the force tending to separate the disks is proportional not only to the area of the disks, but also to the square of the electromotive force in the field. This result is accounted for by supposing that at every point of the dielectric where there is electromotive force, a tension exists like that of a stretched elastic cord acting in the direction of the electromotive force and proportional to the square of this electromotive force. To this force, thus acting within the dielectric mass, the name **electric tension** is given. It represents a tension of so many dynes per square centimeter exerted by the dielectric medium in the direction of the electromotive force. This condition of stress in the dielectric medium Faraday fully recognized. He not only endowed his lines of force with elasticity, capable of shortening when stretched, but he supposed them self-repulsive, thus producing a pressure in the field perpendicular to the lines of force.

496. Dielectric Constant.—We owe to Cavendish (1771–81) the discovery of the fact that the amount of inductive effect which takes place through a dielectric is different for different dielectrics and is therefore a function of the dielectric medium itself. It follows, consequently, that the capacity of a condenser depends not only upon its form and size, but also upon the medium intervening between its plates. Hitherto we have assumed air as the dielectric employed: but if we make

use of other dielectrics in a condenser, such as glass, ebonite, shellac, sulphur, paraffin, etc., we shall notice an increase in the capacity of the condenser produced thereby. The ratio of the capacity of a condenser having any given substance as its dielectric to the capacity of a second and similar condenser having air as its dielectric, was called by Faraday the **specific inductive capacity** of the given substance; and by Maxwell the **dielectric constant.** If both condensers are electrified from the same source, they will both be electrified to the same potential, and therefore the charges which they will receive will be directly proportional to their capacities. Hence by determining the ratio of the charges, that of their capacities is ascertained.

Cavendish measured the capacities of various condensers by comparison with certain other standard condensers, called by him trial-plates. The capacities of these trial-plates were themselves determined in absolute measure by comparison with that of a globe 31·25 cm. in diameter suspended in the middle of his laboratory. One of the coatings of a trial-plate and one coating of the condenser to be compared were connected with the knob of a positively charged Leyden jar, the other coatings being connected to earth. After charging for two seconds, the upper coating of the trial-plate was connected to earth, while the lower coating was connected by a wire with the upper coating of the condenser; thus reversing them with reference to each other. If their charges were equal, both were completely discharged; and the pith-ball electroscope attached to the connecting wire showed no divergence. If the charge was greater on the condenser, the pith balls were positively electrified; if smaller, negatively. Another trial-plate was then selected and the experiment repeated, until two plates were found, one of a little less capacity, the other of a little greater capacity, than the condenser; the mean of these capacities being taken as the true capacity.

Faraday (1837) used two exactly similar spherical

condensers placed concentrically, the interior sphere being 5·8 cm. in diameter and the outer sphere 9 cm.; thus leaving 1·6 cm. between them. The inner sphere was attached by a stem of shellac to the outer one, which was divided at its equator. One of these condensers was filled with air, the other with the material to be examined. The outsides of both were connected to earth, one of them was charged, its potential measured, its interior connected with that of the other, and its potential again measured. If the capacities of the two are equal, the charge will be equally divided between the two condensers and the potential will fall to one half its value. If they are not equal, the charge will divide between them in the direct ratio of their capacities; so that if the second condenser has the greater capacity, it will take more than half the charge and the potential of the first will fall to less than half its initial value. If its capacity is less, the potential will be greater than before. Calling C the capacity of the first condenser, V its potential, and Q its charge, we have $Q = VC$. After connecting it with the second condenser, whose capacity we may call CK, its potential falls to V' and we have $Q = V'C + V'CK$. Hence $VC = V'C + V'CK$ or $V = V'(1 + K)$ and $K = (V - V')/V'$, in which K is the specific inductive capacity of the dielectric in the second condenser. The capacity of a condenser containing a dielectric whose constant is K is $KS/4\pi e$.

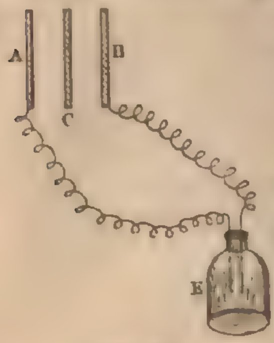


FIG. 245.

EXPERIMENT.—Provide a gold-leaf electroscope E having its leaves a, b supported on separate rods and insulated from each other (Fig. 245). To these two leaves connect the insulated metal plates A and B. To the plate C midway between them, communicate a positive charge. Connect A and B to earth for an instant, and then insulate them; they will both be equally negative and the gold leaves will remain vertical. If A be moved toward C, the gold leaves will move toward each other, a being positive and

b negative. If now, with the plates as at first, a dielectric plate be introduced between *A* and *C*, the gold leaves will again attract each other, and on testing them, *a* will be found positive and *b* negative as before. Evidently, therefore, the effect of introducing a dielectric plate is to increase the induction, whenever the specific inductive capacity of the plate is greater than that of air.

TABLE OF DIELECTRIC CONSTANTS.

Substance.	Dielectric constant.	Observer.
Beeswax	3·67	Cavendish
Shellac	2·00	Faraday
Resin	2·48	Boltzmann
Paraffin	1·96	Wüllner
Flint glass, extra dense	10·10	Hopkinson
Sulphur	2·58	Gordon
Ebonite	2·284	"
Carbon disulphide	1·81	"
Turpentine	2·15	Silow
Hydrogen	0·999674	Boltzmann
Carbon dioxide	1·000356	"
" "	1·0008	Ayrton and Perry

497. Polarization of the Dielectric.—"Induction," says Faraday, "appears to be essentially an action of contiguous particles through the intermediation of which the electric force, originating or appearing at a certain place, is propagated to or sustained at a distance, appearing there as a force of the same kind, exactly equal in amount but opposite in its direction and tendencies." And again: "Induction appears to consist in a certain polarized state of the particles, into which they are thrown by the electrified body sustaining the action, the particles assuming positive and negative points or parts which are symmetrically arranged with respect to each other and the inducing surfaces or particles." In proof of this polarization of the particles of the dielectric, Faraday placed filaments of silk 2 or 3 mm. long in turpentine contained in a rectangular glass vessel provided with two pointed metal rods, entering the opposite ends of the vessel horizontally. On connecting

one of these rods with a source of electrification and the other to earth, the silk filaments were seen to arrange themselves parallel to the line joining the points and to aggregate together there, forming a mass of considerable tenacity. He says: "The particles of silk, therefore, represent to me the condition of the molecules of the dielectric itself, which I assume to be polar just as that of the silk is." Matteucci established the fact that the dielectric is polarized throughout, by forming a condenser out of a number of thin plates of mica pressed between two plates of metal (Fig. 246). After charging the con-

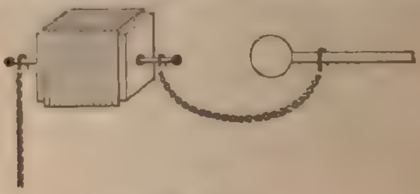


FIG. 246.

denser and insulating it, the metal plates were removed and the mica plates examined. Each one of these plates was found polarized in the direction of the electromotive force applied, the face which was turned toward the positive metal plate being positively and that which was turned toward the negative plate being negatively electrified.

498. Electric Displacement.—Mathematical investigation led Maxwell to the same conclusion which Faraday had reached experimentally. "At every point of the medium," he says, "there is a state of stress such that there is tension along the lines of force and pressure in all directions at right angles to these lines, the numerical magnitude of the pressure being equal to that of the tension and both varying as the square of the resultant force at the point." It was to account for the phenomena thus exhibited by dielectrics that Maxwell proposed his theory of electric displacement. This theory supposes that when an electromotive force acts on a dielectric, as is the case when a medium is sub-

mitted to induction, it causes electricity to be displaced within this medium in its own direction, the amount of the displacement being proportional to the electromotive force but depending also upon the nature of the dielectric. The ratio of the displacement in any dielectric to the displacement in a vacuum due to the same electromotive force is the dielectric constant. Since the displacement through any surface is the quantity of electricity which traverses it, the displacement through unit of surface into a vacuum, if the quantity of electricity traversing it be σ, will be $F/4\pi$; and into a medium whose dielectric constant is K, the displacement will be $KF/4\pi$. Moreover, to produce electric displacement in a dielectric, an amount of work must be expended equal to the product of half the electromotive force into the displacement; i.e., $KF/4\pi \times F/2$ or $KF^2/8\pi$; and this work, which is stored up within unit volume of the dielectric and is the source of the energy of the electrified system, represents numerically the value of the electrostatic tension and pressure at every point in it.

In a conductor, electrical displacement takes place without opposition. But in an insulator, acting as a dielectric, electric displacement calls into action an internal electromotive force acting in a direction opposite to that of the displacement, which Maxwell by analogy has called the **electric elasticity** of the medium. Applying the term coefficient of electric elasticity to the ratio of the electromotive force divided by the displacement produced by it, this coefficient is $F/(KF/4\pi)$ or $4\pi/K$; from which it appears that the dielectric constant of any insulating medium is inversely proportional to its coefficient of electrical elasticity. On charging a condenser, a displacement of electricity takes place from the positive to the negative coating, due to the acting electromotive force. On discharging it, the internal counter-electromotive force, called the electrical elasticity of the medium, reverses the action and effects an equal displacement in the inverse direction. Moreover, the

charges on the coatings, which are simply the charges on the corresponding surfaces of the dielectric in contact with them, correspond to the quantity of the electricity displaced; so that if any change takes place in the charge, a corresponding change takes place in the displacement. Whenever, therefore, a quantity of electricity is transferred from one coating to the other through a conducting wire connecting them, an equal quantity crosses every section of the dielectric from the second coating to the first. We may regard the dielectric and the wire, therefore, as forming a closed circuit through which a current passes whenever electric changes are taking place in the system.

The polarization of the medium is a necessary result of electric displacement. Since throughout a tube of induction the displacement is constant, the same quantity of electricity crossing every normal section, and since this displacement is from the conductor toward the dielectric at one end of the tube and toward the conductor from the dielectric at the other, the density at the first-mentioned end will be σ and that at the other $-\sigma$, and the first will be charged positively, the latter negatively. Within the tube itself there will be no apparent electrification; since the positive electrification on one side of any assumed normal plane is neutralized by the negative electrification on the other side. But if this plane be made a terminal plane, its electrification becomes at once apparent, showing that the dielectric is polarized throughout. "Hence," says Maxwell, "all electrification is the residual effect of the polarization of the dielectric."

499. Nature of Electricity.—Although the idea that electricity is an incompressible fluid is found in the writings of Cavendish, yet it was not until Maxwell investigated the subject that this idea became a part of current scientific belief. We have seen that when any conductor is positively electrified, a definite quantity of electricity is squeezed into the surrounding dielectric, causing a similar transfer of electricity across every sur-

face drawn in the dielectric so as to surround the conductor, the displacement continuing until some external conductor is reached through which the circuit is completed. The dielectric is the only portion of the system really affected by the charge, and it is in this dielectric that the whole energy of the charged body resides. But this squeezing of a quantity of electricity into a dielectric does not imply a condensation of the electricity, but a strain in the dielectric on account of the displacement of the electricity which cannot move without distorting the cells of which the dielectric is conceived to be made up. This fact that electricity always and under all circumstances flows in a closed circuit, the same quantity crossing every section of that circuit so that it is not possible to exhaust it from one region of space and condense it in another, is capable of proof in many ways, but especially by what is known as the famous Cavendish experiment (491) as repeated by Maxwell. "When we thus find that it is impossible to charge a body absolutely with electricity, that though you can move it from place to place it always and instantly refills the body from which you take it so that no portion of space can be more or less filled with it than it already is, that it is impossible by any rise of potential to squeeze a trace of electricity into the interior of a cavity, and that if a charge be introduced a precisely equal quantity at once passes through the walls to the outside; it is natural to express the phenomenon by saying that electricity behaves itself like a perfectly incompressible substance or fluid of which all space is completely full." (Lodge.)

If it be granted that electricity is such an incompressible medium diffused in space, the suggestion at once occurs, is it not identical with the æther the periodic disturbances in which we have shown to be identical with radiation? Maxwell wrote in 1862: "According to our theory, the particles which form the partitions between the cells constitute the matter of electricity. The motion of these particles constitutes an electric current; the tangential force with which the particles are

pressed by the matter of the cells is electromotive force, and the pressure of the particles on each other corresponds to the tension or potential of the electricity. Assuming that induction may be transmitted by an absolute vacuum, this may be regarded as equivalent to the opinion that a certain fraction of the matter called æther is the matter called electricity. Lodge offers a provisional hypothesis that the æther consists "of electricity in a state of entanglement similar to that of water in a jelly." But subsequently he says: "We now proceed a step farther and analyze the æther into two constituents—two equal opposite constituents—each endowed with inertia and each connected to the other by elastic ties, ties which the presence of gross matter in general weakens and in some cases dissolves. The two constituents are called positive and negative electricity respectively, and of these two electricities we imagine the æther to be composed." This, we believe, is the latest opinion on the subject.

Section IV.—Electrostatic Instruments.

A.—HIGH POTENTIAL GENERATORS.

(a) *Friction Machines.*

500. The Electrical Machine.—For producing continuous electrification Otto von Guericke (1671) used a globe of sulphur mounted on an axis and rubbed with the hand. Hawksbee (1709) mounted a globe of glass in the same way and used it similarly. Franklin (1747) also employed a glass globe, but seems to have preferred for producing electrification, tubes of green glass 70 to 75 centimeters long and as large as could be conveniently grasped; these tubes being rubbed with buckskin and kept perfectly clean. The modern so-called friction machine is an aggregation of devices by various experimenters. Bose (1742) added to the revolving glass globe a metal collector, having a bundle of linen threads on the end toward the globe. Winkler (1744) added a

leather cushion for a rubber. Canton (1753) spread on the rubber an amalgam of mercury and tin. Wilson (1752) replaced the globe by a cylinder and furnished the collector with points. And Ramsden (1760) substituted a glass plate for the cylinder. One of the most effective machines of this type is that of Winter (1856) shown in Figure 247. The circular glass plate, which is made to

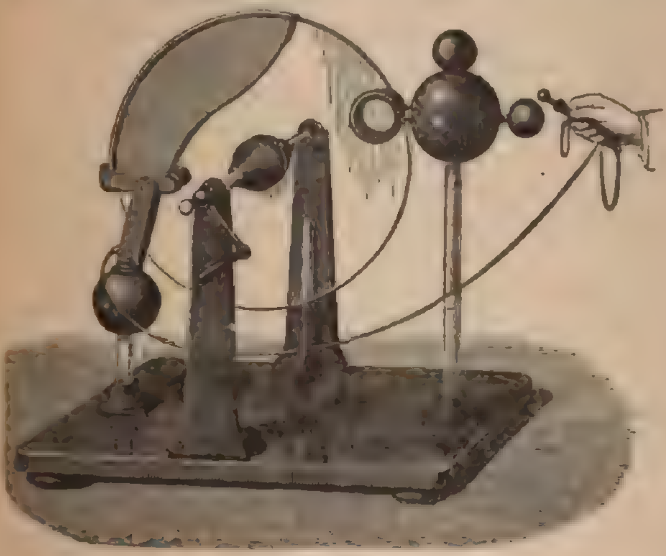


FIG. 247

revolve by means of a crank, is rubbed by the pair of cushions seen on the left which are attached to an insulated metal ball. Being thus electrified, the plate moves over toward the insulated spherical collector upon the right, passing between two rings fastened to the sphere and studded with points on their inner surfaces. By the contact of the rubber and the glass, the glass becomes positively and the rubber negatively electrified. By means of a silk flap which extends up from the rubber, the positive electrification is retained on the glass until it reaches the points on the collector. There it induces negative electrification upon these points; but as the

density is there so great, the negative electrification escapes to the plate and neutralizes its positive electrification; thus making the plate neutral and leaving the collector positively charged. As this operation continues, the opposite charges on the rubbing and the collecting spheres increase until they raise the potential of these spheres to the point of discharge. Sparks and brushes fly off from the spheres, mainly across the plate from one to the other, and the machine has reached its maximum difference of potential. By connecting the left-hand sphere with earth, its potential becomes zero and that of the right-hand or collecting sphere rises proportionally. If the right-hand sphere be grounded, the left-hand sphere falls proportionally in potential; the difference between the two being always constant. The largest machine of this class was made by Ritchie in 1858. It had two plates of glass 1·8 meters in diameter. A similar machine constructed by Cuthbertson for von Marum was furnished with two plates 1·65 meters in diameter. It gave three hundred sparks 61 centimeters long per minute, and the brush discharge was in the form of an aigrette 38 centimeters in diameter.

(b) Induction Machines.

501. The Electrophorus.—The more modern machines produce a high potential by means of induction. Of these the earliest, as it is the simplest, is the electrophorus of Volta (1775). It consists (Fig. 248) of a disk *B* usually of resin or ebonite, upon which rests a somewhat smaller circular plate of metal *A* furnished with an insulating handle. Having electrified the disk by rubbing it with cat's fur, place the metal plate upon it by means of its insulating handle. The negative electrification of the disk induces positive electrification upon

FIG. 248.

the lower surface of the metal plate and negative electrification on the upper surface. On approaching the finger to the plate, a negative spark passes, and all electrification disappears. To raise the plate from the disk, however, work must be done upon it, since the attraction of the two for each other must be overcome; and this work increases constantly the electric potential energy of the system; i.e., the potential of the plate. So that, when raised only a short distance from the disk, a strong spark may be taken from the plate. Since the disk loses none of its electrification in the process, the operation may be repeated indefinitely. To avoid the necessity of touching the plate with the finger at every charge, a strip of tinfoil may be laid across the disk, or a metallic pin may be passed through it, its upper end flush with the surface, both tinfoil and pin being connected to earth.

502. Continuous Electrophori.—Naturally, as the operation of charging by means of the electrophorus is intermittent, attempts were early made to produce continuous electrophori. But the charge of the disk gradually diminishes and must be renewed from time to time. To avoid this necessity Bennet (1787) contrived his "doubler," which consisted of three metallic plates, *A*, *B*, and *C* (Fig. 249). The first, *A*, is varnished only on

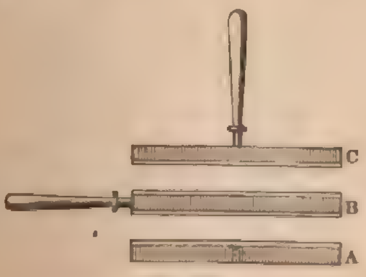


FIG. 249.

its upper surface; the second, *B*, which is provided with an insulating handle, is varnished on both surfaces; and the third, *C*, is varnished only on the under surface. If a small charge be given to *A*, the plate *B* laid upon it

and touched, B becomes electrified oppositely by induction. If B be then lifted, the plate C placed on B and touched, C is electrified by induction oppositely to B but similarly to A. The plate C is then placed beneath and in contact with A, and B is placed upon it and touched. Under the influence of a double inducing charge, that on A plus that on C, a double charge is induced on B; and by placing C again on B, after its removal from A, and touching it, a doubled charge will appear on C. By a repetition of these operations the charge on A may be indefinitely multiplied.

A year later (1788) Nicholson contrived a revolving doubler, consisting of two insulated and vertical fixed field-plates A and C and a third and movable plate B revolving at the end of an insulating arm attached to an axis. Contacts were so arranged that when B was opposite the field-plate A, it was grounded and at the same time A and C were put in communication with each other; while when B was opposite the field-plate C, C was grounded. If A be slightly electrified and B be brought opposite it, B will be electrified oppositely by induction, and on revolving it 180°, will come opposite C and induce in it the electrification given to A. When B comes opposite to A a second time, A and C being in communication, a double inducing charge exists on A and a double charge will be induced on B; which will result, when B is brought again opposite to C, in inducing a double charge on C. Thus the potential on C, and that on A also, which is in contact with C once every revolution, continually rises until at last sparks pass between A and B whenever it comes opposite to it. This action is clearly the same as that of the doubler of Bennet.

EXPERIMENTS. — Construct the reciprocal-influence machine of Belli (1831) as follows (Fig. 250): Bend up into a U form two rectangular pieces of tin, about 10 by 18 centimeters, to serve as the field-plates. Attach two disks of tin about 5 centimeters in diameter to the ends of a glass tube 20 centimeters long and connect this tube at its middle point with a glass or metal rod perpendicular

to its length, to form an axis. Mount it on a bearing attached to a base and fix to it a crank. Attach to the field-plates glass tubes to serve as supports, the sides being vertical. Insert the ends of these tubes into holes in the base, at such distance apart that the disks as they revolve may pass through the space enclosed in the field-

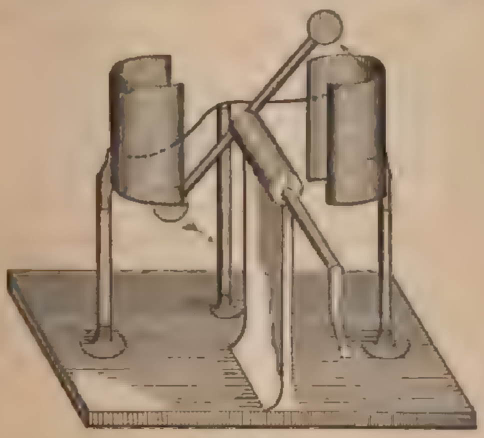


FIG. 250.

plates without touching them. By means of a rod inserted in the base support a wire so curved that its ends simultaneously touch the disks when they are midway within the field-plates; and attach a small spring to each of the field-plates, near the top of one and the bottom of the other, so that they shall make contact with the disks just as they are entering the field-plate space. Communicate a feeble positive charge to one of the field-plates, and rotate the disks. Observe that the disk within this charged field-plate becomes electrified by induction and that its positive charge passes off through the curved wire, leaving it negative. It then passes to the other field-plate, and touching its spring, communicates to it its negative charge, passing out from this field plate positively charged and electrifying the first field-plate positively as it again enters it. As the other disk acts similarly, the two being oppositely electrified by induction from the oppositely electrified field-plates, the process is continuous and reciprocal, and continues until the difference of potential between the field-plates is a maximum. Varley's machine (1860), while similar in principle, is of a much more advanced type. It has a revolving glass disk (Fig. 251) carrying six sectors, passing between two pairs of field-plates.

503. Holtz Influence Machine.—Holtz of Berlin (1864), however, first brought the influence machine into prominence by the many and important devices with which he provided it. The Holtz machine (Fig. 252)

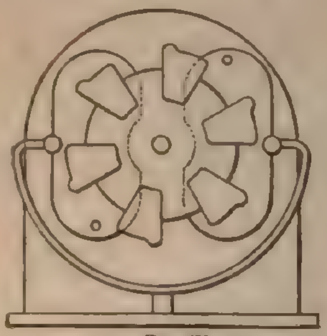


FIG. 251.

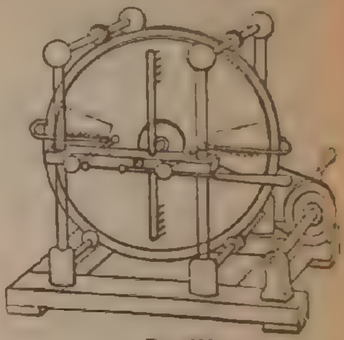


FIG. 252.

consists of a rotating thin glass plate placed vertically, close to and in front of a fixed thin glass plate of slightly larger size, through which are two sector-shaped openings or windows at opposite ends of a diameter. The field-plates, which are sector-shaped, are made of varnished paper and are attached to the rear surface of the fixed glass plate, covering an arc of 90° or more. One edge of each sector is close to a window, and has a pointed paper tongue projecting into the opening. Two pairs of metallic combs or sets of points face the rotating plate. One pair is at the ends of a metallic rod placed diagonally with its axis parallel to the upper edge of one of the paper sectors and the lower edge of the other. This rod with its combs acts as a neutralizing circuit and maintains the charge of the field-plates when the discharging circuit is open. The other pair of combs constitute the discharging circuit. They are placed horizontally, are carried on insulating supports, and are connected with knobs through which slide electrodes provided with ebonite handles. The movable plate is rotated by means of a multiplying wheel.

To put the machine in action, an electrified body,

such for example as an excited strip of ebonite, is placed in contact with one of the field-plates and the movable disk is rotated from five to ten times a second in a direction opposite to that in which the tongues of paper point. The two field-plates, as well as the two sides of the discharging circuit, become oppositely electrified, the work required to keep up the motion increases, the electrifications rapidly rise to a maximum and on separating the electrodes discharges in the form of sparks and brushes take place between them. By the addition of a Leyden jar on each side of the discharging circuit, or even on one side only, to act as a condenser, the amount of electrification at each discharge is increased and the brilliancy of the spark heightened, though the discharges are less frequent.

The theory of the operation of the Holtz machine is simple. The diagram Figure 253 shows the revolving

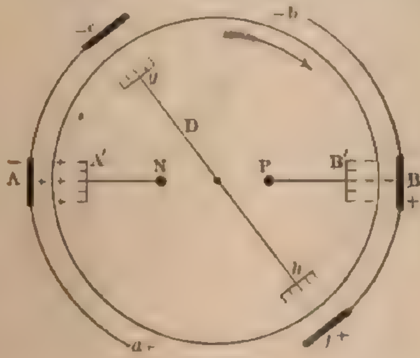


FIG. 253.

plate, the field-plates A and B, the discharging circuit $A'B'$, and the diagonal conductor D. The field-plate A, which is here shown outside of the revolving plate, being feebly electrified negatively by contact with an excited plate or rod of ebonite, acts at e, through the air and the glass plates, upon the upper end g of the diagonal metal rod D of the neutralizing circuit, developing positive electrification there by induction. This

electrification produces a discharge of positively electrified air from the sharp points of the comb upon the front surface of the movable plate; while at the same time the repelled negative electrification accumulates at the other end h of the diagonal conductor and produces a discharge of negatively electrified air from the comb upon the face of the plate opposite to it. Both of these charges thus produced upon the face of the movable plate, react inductively through the glass plates upon the field-plates A and B, the dissimilar electrifications in both cases being attracted to the nearer ends e and f of these field-plates, and the similar electrifications being repelled to the remoter ends a and b. But at these remoter ends are placed the pointed paper tongues which project into the windows; and the repelled electrification is slowly discharged from these paper points upon the rear surface of the movable plate. If now this plate be turned, these electrifications upon the back are carried onward, from left to right above and from right to left below, until they come opposite to the tongues upon the field-plates, which are dissimilarly electrified. These tongues at once discharge their electrification upon the revolving plate, thus neutralizing its electrification and at the same time increasing that of the field plates; the difference of potential between them rising continuously as the plate is rotated, until the gain by duplication is balanced by the loss arising from leakage and spark-discharge.

Thus far the only object attained is the maximum dissimilar electrification of the field-plates; and this has been accomplished solely by means of the electrification upon the rear surface of the revolving plate. But meanwhile the electrifications upon the front surface of this plate have been brought by the rotation opposite to the combs of the discharging circuit; and, acting inductively upon the metallic mass, attract the opposite electrifications to the nearer ends of this circuit, producing a discharge of oppositely electrified air upon the plate; thus neutralizing its electrification and at the same time leaving the

electrodes of the discharging circuit highly electrified, the one positively, the other negatively. This electrification it is which is the cause of the sparks between the electrodes. This part of the operation of the Holtz machine is quite analogous to that of the ordinary friction machine; differing only in the fact that in the Holtz machine the action is duplex.

504. Self-exciting Machines.—The Holtz machine is not self-exciting. It had been observed, however, that machines of the carrier type, like that of Varley already mentioned, having a number of metallic sectors or carriers arranged on a revolving disk, would produce a spark from so feeble an initial electrification, that no outside electrification was required; the normal difference of potential between dissimilar substances being quite sufficient. The Töpler machine (1880) is of this type. It consists of a revolving glass plate (Fig. 254) upon which are six equidistant tinfoil sectors or carriers having a low brass button on each. The field-plates are of varnished paper attached to the rear surface of a vertical sheet of glass. Two brushes of tinsel connected with these plates are supported so as to touch the buttons on the sectors as they revolve, these brushes being at opposite ends of a diameter of the disk, inclined about 30° to the vertical. A second pair of brushes is placed horizontally and forms a part of the discharging circuit.

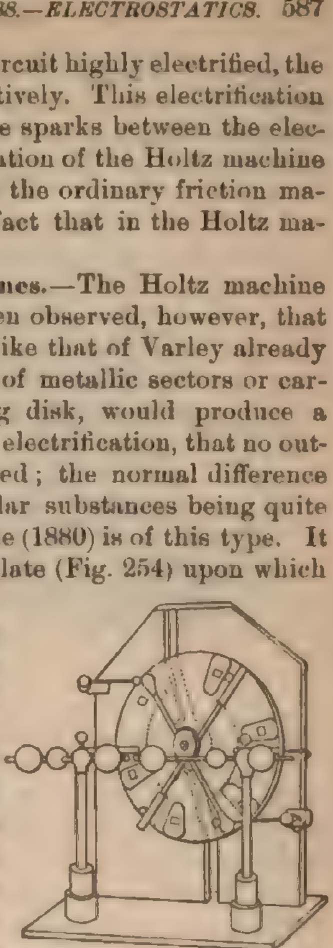


FIG. 254.

The same year Voss brought out a machine on the same principle, in which the Holtz construction was adopted. A revolving glass plate e (Fig. 255) having six tinfoil disks equidistantly placed near its edge moves in front of a somewhat larger circular glass plate e', upon the back of which are two paper quadrants c and c' acting as the field-plates, and reinforced by two tinfoil disks near their ends

connected by a tinfoil strip. At the top of one quadrant and at the bottom of the other, metallic arms a, a', each carrying a brush, pass round the edge of the revolving plate, so that the buttons upon the tinfoil disks or car-

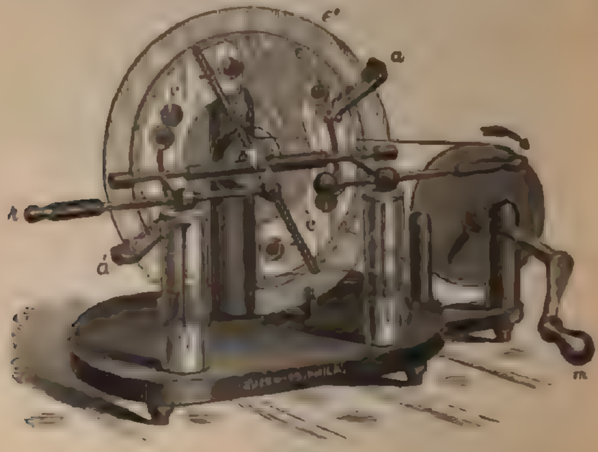


FIG. 255

riers just touch them as they pass. A diagonal conductor with its pair of combs acts as a neutralizing circuit as in the Holtz machine. A second pair of combs in the discharging circuit has the central points removed and replaced by a tinsel brush. Two Leyden jars i, i', are in communication with the discharging circuit.

The operation of the machine may be explained by the aid of the diagram Figure 256, given by Thompson. The circle of sectors represents the revolving plate, and outside of this are segments representing the field-plates. Arrows pointing outward represent positive, those pointing inward negative, charges. The diagonal neutralizing conductor D and the discharging circuit $A'B'$ are also shown. The upper sector at n is under induction from the left-hand positive quadrant; and therefore a displacement takes place from higher to lower potential along the diagonal conductor through the brush in contact with this sector, giving this sector a negative charge and

charging the diagonally opposite sector positively. As these sectors move on, work must be done to separate the attracting bodies; and the charges on the sectors, before held by the opposite electrifications of the field-plates, become free. While thus charged these sectors come into contact with the brushes at the entering ends of the two opposite field-plates; each sector giving up to or receiving from the field-plate a part of its charge,

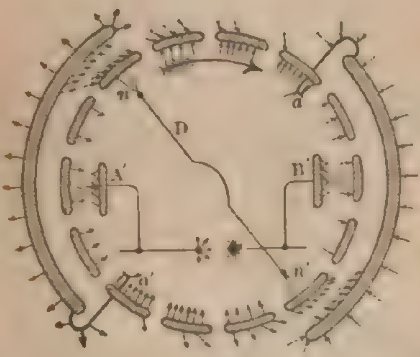


FIG. 256

thus increasing the electrification of the plate. The sectors then pass on, with a potential equal to that of the field-plate with which they have just been in contact, and touch the brushes of the discharging circuit, giving up to them or receiving from them a portion of their charge whenever the sectors are at different potentials. As each successive sector performs these functions, the electrification of the field-plates and that of the discharging circuit reaches a high value in a very short time; and this entirely without the giving of any initial charge to either field-plate. In the most recent form of Voss machine, the revolving plate is doubled.

505. Wimshurst Machine.—Another notable influence machine is that of Wimshurst (1881). Its construction is quite simple (Fig. 257), consisting only of two equal glass disks, each provided with numerous sectors on one side and rotating in opposite directions. These sectors

are on the outer sides of the revolving plates, and act not only as carriers but also, when opposite each other, as field-plates or inductors. Each plate has a diagonal conductor facing it, the two ends of which are provided

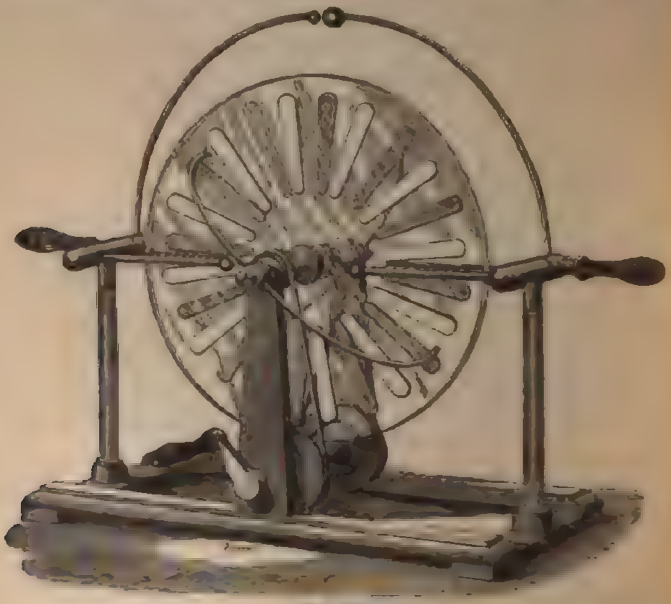


FIG. 257.

with brushes which touch lightly the sectors as they pass. These two diagonal conductors are at right angles to each other. There is also a horizontal discharging circuit provided with a double set of combs, one set facing each plate. It is usually provided with a condenser.

A diagram by Thompson (Fig. 258) may make clear the operation of this machine. Let the sectors on the back plate be represented by the outer row, those of the front plate by the inner row of segments, the two revolving in opposite directions. The two diagonal conductors *ab* and *cd* are perpendicular to each other, both being inclined about 45° to the vertical. The carriers from which lines of force radiate, as shown by the arrows, are charged positively, the others negatively. At the top of the dia-

gram the back carriers are represented as positively, the front carriers as negatively charged; at the bottom the reverse is the case. The maximum charge upon a sector is represented as six units. Consider a front sector at *a* for example. As it passes into the position shown, it comes under the influence of the positively electrified sector opposite to it on the back plate. But at this instant it touches the brush of the diagonal conductor, and a displacement takes place along this conductor from *a* to *b*,

FIG. 258.

leaving the sector negatively charged. Simultaneously the sector at *b* is electrified positively in the same way. So that the sectors on the front plate move onward from the points of contact with the brushes, the upper set negatively and the lower set positively electrified. The diagonal conductor of the back plate, being at right angles to the other one, produces the same results for its sectors but intermediately. Thus the back sector at *c* comes under induction from the negative front sector opposite to it, at the same time that it touches its diagonal conductor and is thus charged positively, while the back sector at *d* is negatively charged in the same way. The inductive action as well as the carrying function of

the sectors is therefore clearly apparent, and the sectors of both plates come to the combs of the discharging circuit similarly charged, positively on the left-hand side and negatively on the right-hand side. The inductive action of these sectors upon the discharging circuit electrifies its two sides oppositely and produces the sparks and brushes which pass between its electrodes.

506. Thomson Electrostatic Generators.—Among the various forms which these carrier machines have assumed, we may mention three contrived by Professor Sir Wm. Thomson* and called the "replenisher," the "mouse-mill," and the "water-dropping machine," respectively. The replenisher was constructed for the purpose of maintaining the charge of the Leyden jar in the quadrant electrometer. It has two metal carriers revolving within two field-plates or inductors which are segments of cylinders. The two carriers while under opposite induction from the field-plates are touched by two springs forming the ends of a metallic circuit, and then touch two springs just as they pass within the field-plate on the opposite side. The mouse-mill was used for electrifying the ink-vessel of the siphon-recorder. The apparatus is cylindrical, the carriers being segments of cylinders ten in number, whence its name. Two semi-cylindrical field-plates or inductors are placed outside of the revolving cylinder. Each carrier is connected to a pin near the axis; and by means of these pins the two carriers entering the two field-plates are put in contact with these plates, while, at the same time, the two carriers perpendicular to these are put in communication with each other. The water-dropping machine was first described in 1867. To stems connected with the inside coatings of two Leyden jars *A* and *B* (Fig. 259) hollow cylinders of metal are connected, which are called respectively inductors *i* and receivers *r*. Each stem supports an

* In consequence of his eminent services to science, Professor Sir William Thomson has recently been raised to the peerage and has assumed the title of Lord Kelvin. By this name, therefore, we shall in future refer to him.

inductor and a receiver, the inductor of the first jar being arranged vertically over the receiver of the second and *vice versa*. The receiver differs from the inductor in containing a funnel placed slightly above the middle of the cylinder with its narrow end opening downward. Two fine vertical streams of water are arranged to break into drops, one in the center of each inductor. These drops fall along the axis of the inductor into the funnel of the receiver and thence to waste. Suppose a small positive charge be given to one of the jars. Its inductor electrifies negatively each drop of water breaking away in its axis from the jet, and these drops communicate their negative charges to the funnel below, which is connected to the other jar. This reacts to increase the positive electrification of the first jar and so on; so that commencing with a feeble charge in one of the jars, only discoverable by a delicate electrometer, a rapid succession of sparks is seen after a few minutes passing in some part of the apparatus, or the drops of water are seen scattered about over the edges of the receivers.

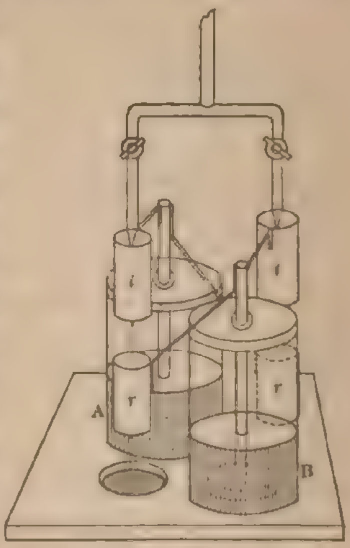


FIG. 259.

B.—MEASURING-INSTRUMENTS.

507. Electrostatic Units of Measurement.—Electrostatics requires the measurement of (1) Quantity, (2) Potential difference, and (3) Capacity. Since these are connected together by the equation $Q = CV$, either of them can be calculated when the other two are known.

I. The **absolute unit of quantity** is that quantity of electrification which, placed at unit distance from an equal quantity of the same kind, repels it with a unit force. In the C. G. S. system, it is the quantity which repels an equal and like quantity at one centimeter distance, with the force of one dyne. The dimensions of the electrostatic unit of quantity, therefore, are $[Q^2/L^2] = [F] = [ML/T^2]$; whence $[Q] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$.

II. The **absolute unit of potential difference** is that difference of potential between two points which requires a unit of work to be done in carrying a unit of quantity from one to the other. In C. G. S. units, a unit difference of potential exists between two points when an erg of work is expended in carrying an absolute unit of electrification from one to the other. The dimensions of the electrostatic unit of potential difference are $[W/Q]$ or $[ML^2/T^2] \div [M^{\frac{1}{2}}L^{\frac{3}{2}}/T]$; hence $[V_1 - V_2] = [M^{\frac{1}{2}}L^{\frac{1}{2}}/T]$.

III. The **absolute unit of capacity** is the capacity of a conductor which requires unit charge to raise its potential by unity. The dimensions of the electrostatic unit of capacity consequently are $[Q/V]$ or $[M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [M^{\frac{1}{2}}L^{\frac{1}{2}}/T]$ or $[L]$; whence $[C] = [L]$ and the capacities of similar conductors are proportional to their linear dimensions.

IV. The force experienced by an electrified body in an electrical field being proportional (1) to the charge upon the body and (2) to the strength of the field, is jointly proportional to the product of these quantities. But since a field of unit strength is produced at unit distance from unit charge, the force experienced by unit quantity in such a field will also be unity at unit distance from unit charge. Whence the dimensions of the unit of electric force, or electromotive intensity (Maxwell), are $[Q/L^2] = [M^{\frac{1}{2}}L^{\frac{3}{2}}/T] \div [L^2] = [M^{\frac{1}{2}}/L^{\frac{1}{2}}T]$.

In practice certain multiples and sub-multiples of the absolute C. G. S. units have been adopted, mainly for convenience in reproducing concrete standards. These units are named from eminent electricians, and are as follows, their values being given in absolute electrostatic units :

Denomination.	Name.	Origin of Name.	Symbol.	Value E. S. Units.
Quantity of electrification	Coulomb	Coulomb	Q	3×10^9
Potential-difference	Volt	Volta	E	$\frac{1}{3} \times 10^{-2}$
Capacity	Farad	Faraday	C	9×10^{11}
Current	Ampere	Ampère	I	3×10^9
Resistance	Ohm	Ohm	R	$\frac{1}{9} \times 10^{-11}$
Work (Volt-coulomb)	Joule	Joule	—	10^7 ergs
Activity (Volt ampere)	Watt	Watt	—	10^7 ergs per sec.

508. Potential Difference.—Instruments for measuring differences of electrical potential by electrostatic action are called **electrometers**. As the capacity of such instruments remains constant, the charges which they receive and which they indicate must vary as the potentials of the bodies with which they are in contact. Lord Kelvin divides electrometers into three classes: Repulsion electrometers, attracted-disk electrometers, and symmetrical electrometers. To the first class the gold-leaf electroscope and Coulomb's torsion-balance belong. The former may be used to indicate equality (1) in the potential of two bodies, electrified similarly or oppositely, by equality of divergence in the gold leaves; or (2) in that of two bodies dissimilarly electrified, by bringing them in contact and observing zero divergence. But the different divergences are not proportional to any simple function of the potential. The Coulomb torsion-balance has already been described (468).

509. Thomson Electrometers.—The electrometers of Lord Kelvin belong to the second and third classes. The Absolute electrometer, the Portable electrometer, the Standard electrometer, and the Long-range electrometer are all Attracted-disk instruments, while the Quadrant electrometer belongs to the class of Symmetrical instruments. Moreover, these electrometers may be used **idiostatically**, when the electrification to be tested is the only one employed; or **heterostatically**, when an auxiliary electrification is used. In the former case, for small values, the indication of the electrometer is proportional to the square of the potential difference, while in the latter it is proportional to this difference itself. Moreover,

heterostatic instruments indicate in addition the sign of the electrification.

The absolute electrometer consists of a light aluminum disk a supported by springs in a circular opening in a large brass plate G, called the guard-plate (Fig. 260).

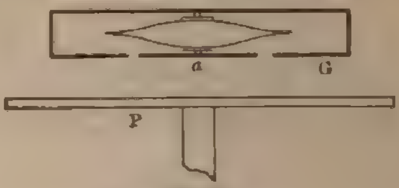


FIG. 260.

Beneath this is a second plate of brass P, adjustable by a micrometer screw and called the attracting plate. The whole is contained in a glass cylinder coated on the outside and the inside with tinfoil and acting as a Leyden jar. The guard-plate is permanently connected with the inner coating and the jar is kept charged by the replenisher, to a constant value determined by a gauge. To calibrate the instrument weights are placed on the aluminum disk and the depression noted. They are then removed, the outside of the jar connected with the attracting plate, and this plate raised by means of its micrometer screw until the same depression of the aluminum disk is produced. The reading of the micrometer is called the earth reading, the outside of the jar being to earth. The body whose potential is to be measured is now put in contact with the attracting plate, and this plate is moved by its micrometer until the aluminum disk is again brought into the plane of the guard-ring. The difference of the two readings substituted in the formula gives the difference of potentials between that of the body examined and the outside of the jar of the electrometer; or since this may be connected to earth, the difference between the body and the earth; i.e., the absolute potential of the body.

The calculation required is as follows: The capacity of a pair of parallel plates, where e is the distance between

the plates and S the surface, is $C = S/4\pi e$, as we have already seen (489); and hence the energy of electrification W, which equals $\frac{1}{2}V^2C$ (481), is equal to $V^2S/8\pi e$. But the force between the plates $F = W/e$; and hence $F = V^2S/8\pi e^2$. From which we obtain

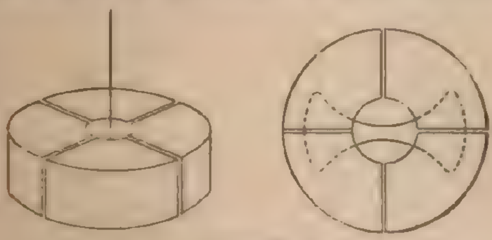
$$V = e\sqrt{8\pi F/S}. \qquad [66]$$

So that if V be the potential difference between the guard-ring and the earth, and V' that between the guard-ring and the body examined, and if e and e' be the corresponding readings of the micrometer, $e' - e$ will be the distance through which the attracting plate was moved between the two readings, and the formula becomes

$$V' - V = (e' - e)\sqrt{8\pi F/S}. \qquad [67]$$

If the area of the aluminum disk be measured in square centimeters, the distance $e' - e$ in linear centimeters, and the force F in dynes, the potential of the body will be obtained in absolute C. G. S. units.

The Thomson quadrant electrometer is so called from the form of its principal acting part. This is a flat cylindrical box (Fig. 261, A) made of brass and divided into four quadrants, each supported upon an insulating glass



A FIG. 261. B

stem. Within this box an aluminum needle shaped like the figure 8 moves about a vertical axis, suspended by a pair of silk fibers, as is seen in the figure (Fig. 261, B). The whole is enclosed in a jar containing sulphuric acid, to keep the air dry within it and to act as the inner coating of a Leyden jar, tinfoil forming the outer coating.

By means of a fine wire of platinum hanging in the acid and connected to the needle, this needle is kept charged; the potential being maintained constant by means of a replenisher and gauge. By two metal electrodes communication is made with the quadrants, the alternate pairs of which are connected together. When these two electrodes are united by a wire, the four quadrants are at the same potential and the needle is unaffected, provided it be placed symmetrically, with its central line coinciding with the plane between the quadrants. If, however, one pair of quadrants be put to earth, and the diagonally opposite pair be connected with an electrified body; or what is substantially the same thing, if the two electrodes be connected to points at different potentials, the needle will move to the right or left according as the one side or the other is at a higher potential than itself. Upon the rod supporting the needle is placed a mirror, so that by means of a telescope and scale the amount of the deflection may be read off. Since the scale is placed at about a meter distance, and since the angular motion of the image is twice that of the mirror, the potentials may be considered as approximately proportional to the deflections. The theory of the instrument shows that

$$D = c(V_1 - V_2)(V - \tfrac{1}{2}(V_1 + V_2)), \qquad [68]$$

where D is the deflection, V, V_1, and V_2 are the potentials respectively of the needle and the two pairs of quadrants, and c a constant depending on the conditions of the apparatus. The deflection is therefore closely proportional to the difference of potentials between the quadrants multiplied by the difference between the potential of the needle and the mean of the potentials of the quadrants. Since V is large in comparison with V_1 and V_2, we may write

$$V_1 - V_2 = C'D,$$

in which the reciprocal of C' represents $c(V - \tfrac{1}{2}(V_1 + V_2))$; whence, since C' is practically constant, the difference of

potentials is proportional to the deflection. These deflections may be reduced to absolute measure by determining the value of the constant C', by empirically calibrating the instrument with a battery whose difference of potential has been determined by means of an absolute electrometer. By using a different number of cells in each experiment, giving different deflections, a mean value can be obtained true through its entire range.

A commercial instrument based on the same principle and called an electrostatic voltmeter has more recently been devised by Lord Kelvin. In this instrument, a vertical figure-of-8-shaped plate, corresponding to the needle of the quadrant electrometer, is supported on knife-edges coinciding with the axis of a fixed plate, which consists of two opposite metallic sectors, each double, the movable plate passing between the two sides. Hence the instrument is practically an air-condenser with one plate movable, the motion of this plate altering its electrostatic capacity. When a difference of potential is established between the fixed and the movable plates, the latter moves so as to increase this capacity, the couple being, as in all idiostatic electrometers, proportional to the square of this difference. A small weight hung on the lower end of the movable plate balances this couple; and by having three different weights, three different degrees of sensibility may be given to the instrument.

510. Lippmann Capillary Electrometer.—The capillary electrometer of Lippmann is an instrument of extraordinary delicacy, based upon changes in the surface-tension between two liquids produced by a difference of electric potential between them. Since mercury and dilute sulphuric acid, being heterogeneous substances, are electrified oppositely by contact, and since, though apparently in contact, they are really separated by approximately half a micromillimeter, the whole acts like a condenser, in which the difference of potential, the electrostatic capacity, and the surface-tension are, by

the law of the Conservation of Electricity, mutually dependent.

EXPERIMENT.—Place a globule of mercury in dilute sulphuric acid containing a trace of chromic acid, and adjust an iron wire within the acid so that it just touches the mercury surface laterally. Observe that the globule increases its convexity and so draws itself away from the wire, breaking the contact. As the globule again flattens by the action of gravity, contact is again made and so a continuous vibration is maintained. The difference of potential developed by contact, aided perhaps by a polarization of the mercury, increases the surface-tension of this liquid and causes it to become more spherical, thus drawing itself away from the iron wire.

This change of surface-tension it is which is made use of in the capillary electrometer. In Lippmann's form of the instrument (Fig. 262) a vertical glass tube *A* drawn out to a fine point at its lower end and filled with mercury is immersed in dilute sulphuric acid contained in the vessel *B*. Electrical contact is made at the points *p* and *p'*. The pressure of the mercury in the tube is sustained by the convex mercury meniscus at the point, the position of this meniscus being observed by means of a microscope. Evidently an increase in the potential-difference between the mercury in *A* and the acid in *B* will cause a rise of the meniscus and a diminution will cause a fall in it. So that by using known differences of potential to calibrate the electrometer, the value corresponding to any given change in the position of the meniscus can be determined. It is preferred, however, to connect the top of the glass tube with a tube of rubber connected to a flexible reservoir containing air which can be compressed; the amount of pressure being recorded on a water manometer. By means of

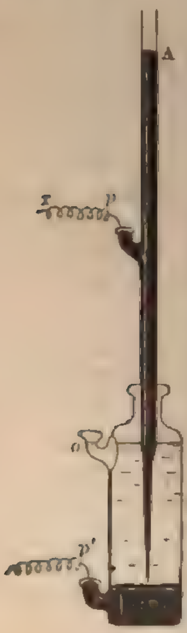


FIG. 262

this, pressure is exerted on the top of the mercury column until the meniscus is returned to the zero position; then the difference of potential is read off on the manometer. This difference is found to be proportional to the pressure up to nearly a volt, although it is not used for potential differences of more than half this value. Its sensibility, however, is so great that it will measure one ten-thousandth of a volt (507). In a modified form of this instrument proposed by Dewar, the capillarity tube is horizontal and contains in its center a drop of acid, mercury filling the rest of the tube on both sides. The ends of the tube terminate in mercury contained in glass jars. A difference of 0·003 of a volt causes the drop of acid to move perceptibly along the tube.

511. Electrical Quantity.—In experiment eight (485), Maxwell's method of adding together a number of equal quantities of electrification was described. By reversing the order of operations, a quantity of electrification may in the same way be successively diminished by equal amounts. This principle of determining a total charge by the number of partial charges required, either to produce it or to reduce it to zero, has been variously applied. Lane (1767) devised a discharging electrometer based upon the fact that the charge on a conductor and the length of spark given by it on discharge, are proportional to the potential; and hence under the same conditions, if the length of spark remains constant the quantity of the discharge will be constant also. Thus, for example, let the outer coating of a Leyden jar be connected with a ball, adjustable micrometrically to any given distance from the knob which is connected with the inner coating. On charging, it is clear that the potential of the jar will rise until the resistance of the air between the knobs is overcome, and then spark-discharge will occur. Since, with the exception of a slight residual charge, the discharge of the jar is complete, it is evident that the quantity discharged by each spark, being the value of its capacity for the given potential, will be constant. Harris subsequently adopted

the principle in the construction of his unit jar (Fig. 263) by means of which equal quantities of electrification can be transferred in succession, from a given source to a condenser. For this purpose the unit jar is insulated, its interior A is connected to the source, and its exterior B to one side of the condenser, the other side of which is connected to earth. When the potential of the unit jar has risen to a degree corresponding to the distance between the knobs A' and B', a distance which is adjustable, it will be discharged, a spark passing across between these knobs; and so on indefinitely, so long as the operation is continued. But the unit jar is insulated, and hence in charging it, precisely the same quantity passes to the condenser from its outside coating that passes to its inside coating from the source. Hence at each discharge of the unit jar, a quantity equal to its capacity for the given potential enters the condenser, the whole quantity finally contained in this condenser being the product of the quantity in each spark by the number of sparks.

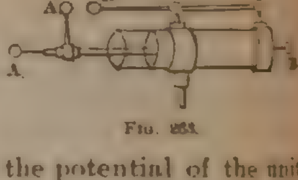


FIG. 263.

For the transference of smaller quantities, Gaugain has used a discharging gold-leaf electroscope in much the same way, calling it an electroscopic gauge. A metal ball is placed within the instrument, supported on a rod rising from the base and therefore connected with the ground. It is adjustable horizontally and is so placed that one of the gold leaves on divergence comes in contact with it (Fig. 264). To determine the charge of a conductor, it is connected with the electroscopic gauge by means of a slightly moistened cotton thread thirty or forty centimeters long. As the electrification is transferred gradually along the thread, the gold leaves

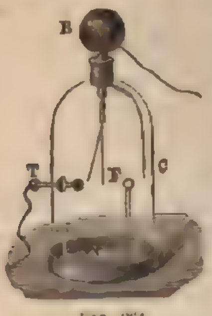


FIG. 264.

diverge until one of them touches the ball, when the electroscope is at once discharged and the leaves fall. A second transference of electrification takes place, and so on, until the conductor is discharged. As the quantity escaping at each contact is the same, the total quantity upon the conductor is proportional, as above, to the number of contacts. Finally a divergence of the gold leaves results, less than is required to bring them in contact with the ball, and there is left in the apparatus a small residual charge. To determine its value for the electroscope, repeat the experiment just given, recharging the conductor to the same potential without previously discharging the electroscope. If n be the number of contacts on the first discharge and n' the number on the second, $n'-n$ will be the residual charge in terms of the quantity discharged at each contact. To determine it for the conductor, put this conductor and the electroscope in connection with a second conductor of greater capacity by means of a cotton thread, and count the contacts. Then having insulated the first conductor and the electroscope without discharging them, charge the second conductor to the same potential as before and discharge it again through the electroscope, noting the number of contacts. If m be the number on first discharge and m' the number on the second, $m'-m$ is the residual charge on the first conductor and the electroscope taken together; whence, since $n'-n$ is the residue on the electroscope alone, the residue on the conductor must be $(m'-m)-(n'-n)$. By varying the position of the ball, the sensibility of the gauge may be varied.

By uniting with such an electroscopic gauge a hollow insulated metal cylinder such as is shown in the figure (Fig. 265) by means of a cotton thread, quantities of electrification on non-conductors may be compared. For if an electrified body be introduced into the cylinder, the similar electrification repelled through the electroscope will be exactly equal to the charge on the body; and may therefore be measured by counting the contacts. In this way opposite electrifications, as well as similar

ones may be compared. Moreover, the division of a charge between two conductors of any form may thus be studied. Charge one of them, bring them in contact for an instant, and then determine the charge of each by the number of contacts. If the total charge be first

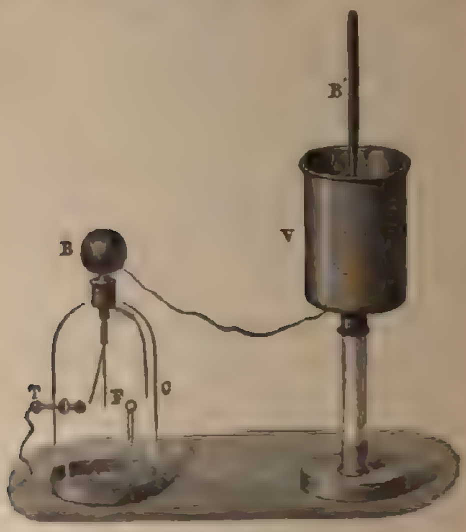


FIG. 265.

determined, the difference between this and the sum of the separate charges represents the loss in the experiment. Since the rate of discharge depends upon the difference of the potentials at the ends of the thread, the number of contacts in a unit of time will give the means of determining potential difference.

512. Thermometric Measurement of Electrical Quantity.—A second method of determining electrical quantity is a thermometric one. Whenever two conductors at different potentials are connected together, electrification passes from the one at higher to the one at lower potential and a loss of electrical energy to the system results. If no mechanical work is done in the operation, this energy thus lost appears as heat-energy in the circuit. If the discharge takes place in the air, a

spark is produced; if through a fine and imperfectly conducting wire, this wire is heated. The heat produced in these cases may be used to determine the electrical energy expended; and since electrical energy is proportional to QV, by making V constant, the heat may be used to determine quantity.

Kinnersley (1761) contrived an electrical air-thermometer, by means of which the heating effect of the spark-discharge in air could be utilized to measure electrifications. A modern form of his instrument is shown in Fig. 266. Whenever the spark passes between the balls within the larger tube, the air therein is expanded and the liquid column in the smaller tube rises, the amount of the rise indicating approximately the quantity of the charge. Harris (1826) described a thermo-electrometer consisting of a glass bulb 12 cm. in diameter, across the interior of which one or more fine wires were stretched. This bulb was connected below with the shorter leg of a glass tube of 2·5 mm. bore, bent twice at right angles and containing a colored liquid, the longer leg rising vertically about 60 cm. and provided with a graduated scale. Riess (1837) modified this apparatus, first by using a spiral of fine platinum wire, whereby the sensitiveness was increased, and second by placing the capillary tube horizontal, but capable of being inclined at any desired angle by means of a hinge attached to the board to which it was fastened (Fig. 267). The liquid used was a mixture of sulphuric acid and alcohol, to which a little cochineal was added to give it color. If a condenser, such as a Leyden jar or a battery of such jars, be discharged through this fine platinum spiral, the heat generated in the wire causes an expansion of the air in the globe, the amount of which may be read off on the capillary tube by the displacement of the liquid column. Since the energy of a charged conductor is $\frac{1}{2}QV$ or $Q^2/2C$, the total heat-energy produced by the discharge is proportional to the square of the quantity of electrification concerned.

FIG. 266.

Mascart (1873) made use of the expansion of the air caused by the heating of a fine wire by the electric dis-

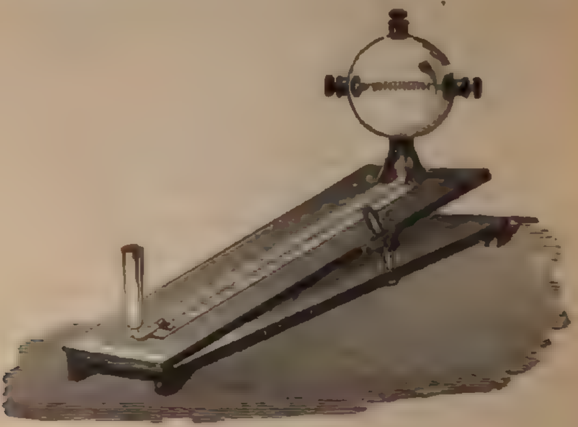


FIG. 267.

charge (Fig. 268) to operate a Marey tambour and thus to obtain a record of the phenomenon. The tambour is a metallic capsule covered with a sheet-rubber membrane, on which rests a plate attached to a lever articulated to the frame supporting the tambour; so that an increase of pressure within the capsule causes the plate to rise and move the lever upward. The end of this lever is pointed and rests against the surface of smoked paper on a chronographic cylinder. The expansion of the air in the cylindrical thermometer itself is communicated through a flexible tube to the tambour, the rise of the lever and hence the height of the curve being proportional thereto.

FIG. 268.

EXPERIMENTS.—1. Place fine wires of the same length and diameter but of different metals across the thermometer-bulb, and

discharge the same battery, equally charged, through these in succession. Observe that the heating effect is greatest for lead and least for silver. 2. Using in the thermometer a platinum wire 0·2 mm. in diameter, coiled in a spiral, charge a battery by means of the unit jar as above described, giving it say 10 unit charges in one case and 20 in another. Observe that the heating effect is four times as much in the second case, being proportional to the square of the charge.

Section V.—Electrostatic Phenomena.

A.—DISCHARGE BY CONDUCTION.

513. Conductive Discharge.—A discharge is called **conductive** when it traverses a conductor, a metallic wire for example; and **disruptive** when it passes through an insulating medium such as air or glass (Faraday). When no other work is done in the circuit, the entire energy of the charge is expended in heat; this principle, as we have already seen (512), being utilized in the electric thermometer. From the experiments of Riess, it appears that the heat produced is proportional directly to the square of the charge and inversely to the number of jars in the battery; i.e., to the charged surface. That is to say, $H = aQ^2/n = aQ^2/S$. But since $Q/S = \sigma$, the surface-density, $H = a\sigma Q$; or the heat produced is proportional to the product of the surface-density by the charge. Moreover, other things being equal, the capacity of a condenser is proportional to its surface; so that we may write $H = aQ^2/C$. Whence, as $C = Q/V$, we have $H = aQV$; which becomes, when $a = \frac{1}{2}J$ and $HJ = W$, $W = \frac{1}{2}QV$; i.e., the electrostatic law that the energy of a charged conductor is measured by half the product of the charge by the potential. The expenditure of this amount of heat-energy upon a given wire raises its temperature by an amount inversely proportional to its mass and to its specific heat; i.e., $T = H/mc$, in which T is the rise in temperature, H the heat produced by the discharge, m the mass of the wire, and c its specific heat. Since $m = sl\delta$, where s is the cross-section, l the length, and δ the density of the wire, this may be

written $T = H/sl\delta c$. Evidently, however, the wire will not reach this temperature owing to the loss of heat by radiation and convection from its surface. Experiment has shown that the rate of loss of heat from these causes from each square centimeter of an unpolished surface of metal is about one four-thousandth of a unit of heat per second for each degree of excess of temperature of the wire above the surrounding medium. Whence if h be the total development of heat per square centimeter of surface per second and x the actual temperature attained, $x/4000 = h$. If wires of different materials be used, the fall of potential in each, as compared with the rest of the circuit, will be different; and hence the heat developed in each, which is proportional to this fall, will be different. The fall of potential in the different portions of a circuit is inversely proportional to the conductivity of those portions; and hence the fall, and the consequent heat developed, is greatest in materials like platinum and carbon and least in silver. Again, it has been assumed that all the energy of the discharge is expended in the wire itself. But in effecting the discharge the spark passes before contact is established, and this spark is a part of the circuit and a part of the energy is expended in it. This energy is less, however, in proportion as the conductivity of the wire is less. So that by reducing it sufficiently in comparison with the rest of the circuit practically the whole of the energy may be expended upon the wire itself. (Riess.)

514. Fusion and Volatilization of Wires.—Evidently if the wire be sufficiently small or the charge sufficiently great, the metal may be fused and even vaporized. For this purpose the jars may be arranged either in multiple or in cascade, according to the dimensions of the wire. With four jars of about two liters capacity arranged in cascade, a brass wire 50 cm. long and 0·2 mm. in diameter, or an iron wire 80 cm. long and 0·33 mm. in diameter, may be volatilized, with a sound like a pistol-shot. The fused iron is projected in all directions, the liquid globules scintillating brilliantly in

the air. If the wire be placed under water, the sound is increased and the shock of the discharge is made more violent.

EXPERIMENTS.—1. Place a white card vertically so as to rest against a fine wire between the points of the discharger and volatilize this wire by the discharge of a battery. Observe that the card is covered with a layer of finely divided material, the color of which depends upon the metal used, silver giving a greenish stain, gold a purple one, etc.

2. Place a sheet of gold-leaf upon a card, through which a design has been cut, a portrait of Franklin for example. Lay the whole upon a sheet of white paper, connecting the two opposite edges of the gold leaf with two strips of tinfoil to serve as electrodes, and enclose it in a wooden screw-press. Upon passing the discharge through it, the gold will be volatilized and the vapors passing through the openings in the card will reproduce the design upon the paper, colored in purple. In Franklin's own experiment the gold-leaf was placed between two glass plates; and the vapors appeared to penetrate the glass so that the deposits could not be removed even by aqua regia.

3. Repeat experiment 1, placing the wire beneath the surface of water. When the discharge takes place, the water will be scattered in all directions and frequently the glass vessel containing the liquid will itself be shattered.

515. Speed of Propagation of the Discharge.—Wheatstone (1834) made the first attempt to determine the time taken by an electric discharge to traverse a metallic conductor. His apparatus is represented diagrammatically in the figure (Fig. 269). The circuit consisted of copper wire 1·7 mm. in diameter, divided into two equal halves, F, F', each 365 meters long, arranged in ten parallel lines, the ends terminating in four balls, 2, 3, 4, 5, upon a spark-board S. Two other balls, 1, 6, forming the ends of the discharging circuit, were also arranged on this board, thus forming three pairs, the distance between each pair being 0·25 cm. The jar L was so placed that when the revolving steel mirror M, 2·5 cm. in diameter, was in the proper position as shown, it was discharged through the wire and of course produced three sparks at the breaks upon the board. When the mirror was slowly rotated on its shaft by means of the

pulley, the three sparks were seen in the mirror as three points of light all in the same straight line; but as the speed of rotation increased, two distinct results were produced: 1st, the points were drawn out into lines, showing an appreciable duration to the spark itself; and 2d, the ends of these lines were not displaced equally, showing that they did not occur at the same instant.

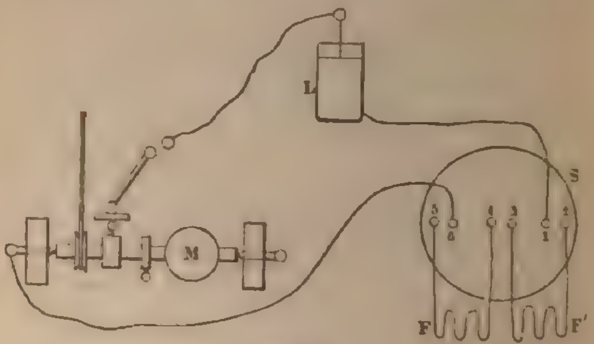


FIG. 269.

The difference in angular displacement observed by Wheatstone when the mirror made 800 turns a second was half a degree, which corresponded to one quarter of a degree displacement of the mirror. Since the mirror made 800 turns a second, it would make one turn in $\frac{1}{800}$ of a second and would turn through $\frac{1}{4}$ of a degree or $\frac{1}{1440}$ of one turn in $\frac{1}{1152000}$ of a second. But in this time the discharge had passed through one of the coils of wire or 365 meters.

While therefore we may accept this experimental result, we are not justified in concluding as Wheatstone did that in an entire second the discharge would have travelled 1152000 times as far through the same wire; or in other words, that the speed of the electric discharge through such a copper wire is at the rate of 420000 kilometers per second. Because, as Faraday pointed out, the speed of propagation even in a given metallic wire depends not only on the electrical pressure employed, but also upon the capacity of the condenser from which

the discharge comes. Moreover, the time of propagation in a wire varies directly with the resistance of the wire and with its electrostatic capacity; and since both these values are functions of the length of the wire, the speed of transmission must vary inversely as the square of the length of the conductor. So that in fact it follows that the length of Wheatstone's wire which the discharge would traverse in a second is only 390 kilometers in place of 420000 as Wheatstone supposed. Again, Beccaria observed early in the century that a charge required two or three seconds to traverse a hempen cord 152 meters long. Faraday (1833) used a wet thread about a meter long to connect his battery to the galvanometer; and found that two or three seconds was required to transfer the battery-charge through it. And Gaugain showed that while eleven seconds was required for the transmission of a charge through a cotton thread 165 cm. long, forty-four seconds was required when the thread was 330 cm. long. Finally, it should be remembered that the time of transmission through a wire depends upon the attainment at the remote end of the wire of a charge sufficient to produce an indication of some sort; and therefore that this time of transmission will be different according to the sensitiveness of the instrument employed to detect the charge. Hence it is evident that the term "velocity of electric transmission," so often used, has no definite meaning.

EXPERIMENT.—Fasten two wires upon a block leaving a small space between their ends, and fill this space with gunpowder. On discharging a jar through the gunpowder it will be blown away without being inflamed. Repeat the experiment intercalating in the circuit 8 or 10 cm. of cotton thread well moistened with water. The gunpowder will now be inflamed, since the heat of the spark is slowly produced and the gunpowder has time to be melted.

B.—SPARK-DISCHARGE.

516. Disruptive Discharge.—That form of discharge which traverses an insulating medium suddenly, breaking through it in the form of a spark, has been called by

Faraday **disruptive**. To produce it an insulating dielectric must be interposed between two conducting surfaces in opposite electrical states; so that as the electrification increases, rupture of the dielectric occurs, a spark passes and the electrification is destroyed. As we have seen (499), the dielectric in a charged condenser is in a condition of strain; and since every charged conductor is one surface of an air-condenser, there is an outward pressure upon its surface against the dielectric in contact with it, which has the value $2\pi\sigma^2$; i.e., varies as the square of the electric density. The limiting value of this pressure for air has been shown by Lord Kelvin to be about 68 grams weight per square decimeter of surface (474). So that when the ordinary atmospheric pressure, which is 103330 grams per square decimeter, is electrically relieved on two very slightly convex metallic surfaces by even the small amount of 68 grams, the air between them is cracked, and, provided the distance between them is not greater than one and a quarter millimeters, a spark passes.

517. Duration of Spark.—In Wheatstone's experiment above described the images of the sparks in the rotating mirror were not points, but lines; showing that they had an appreciable duration. The maximum elongation observed was 24°, corresponding to a rotation of the mirror through 12°. Since this is $\frac{1}{30}$ of the circumference, and since the mirror revolved 800 times a second, the duration of the spark was about $\frac{1}{24000}$ of a second, i.e., $4{\cdot}2 \times 10^{-5}$ second. Feddersen (1864), using a revolving concave mirror, observed that the image of the spark was drawn out by the rotation into a band 20 to 30 mm. long, consisting of a yellowish-white portion, shading into a greenish-white portion, and this again into a red tail; the duration of the first being 3×10^{-5}, that of the second 4×10^{-5}, and that of the third 6×10^{-5} second; thus giving a total duration of $1{\cdot}30 \times 10^{-4}$ for the entire spark. Moreover, he found that the duration was increased either by increasing the length of the spark, the capacity of the condenser, or the resistance of

the circuit. Rood (1869), using a coated surface of 738 sq. cm. and a spark a millimeter long, obtained with a plane rotating mirror a total duration of about $2{\cdot}3 \times 10^{-5}$ second. But he observed that the main illuminating power of the spark was concentrated in the first explosive act, and he showed that this portion of a spark two millimeters long between platinum points occupied about $1{\cdot}75 \times 10^{-7}$ second. Using a coated surface of only 71 sq. cm. and a discharge one millimeter long between platinum points, the duration of the first and brilliant portion of the spark was found to be 4×10^{-8} second. Cazin and Lucas (1872) employed a disk of blackened mica divided on its edge into 180 parts and rotating before a fixed disk of silvered glass having six divisions on its edge, forming a vernier with the first, the spark being viewed through the disks. With a speed of rotation of 400 per second, 400×180 or 72000 lines cross the field in one second, and hence the interval between two lines is $\frac{1}{72000}$ of a second. Since one sixth of this interval is appreciable on the vernier, the interval between two successive vernier coincidences is $2{\cdot}3 \times 10^{-6}$ second. If the discharge takes place at the instant of a coincidence and lasts exactly the above fraction of a second, two bright lines will be seen, owing to the persistence of vision. But if the spark occurs between two coincidences only one line will be seen. Using two Leyden jars in the circuit and a spark five millimeters long, the duration of the spark was found to be $2{\cdot}6 \times 10^{-5}$ second; while with four jars it was $4{\cdot}1 \times 10^{-5}$ and with eight jars $4{\cdot}7 \times 10^{-5}$ second.

518. Oscillatory Character of the Discharge.—Henry (1842), observing that needles magnetized by the electric discharge were not uniform in their polarity, suggested the existence of oscillations in the discharge. "The phenomenon requires us to admit the existence of a principal discharge in one direction, and then several reflex actions backward and forward, each more feeble than the preceding, until the equilibrium is attained." These oscillations Feddersen (1864) succeeded in photo-

graphing by the aid of the revolving mirror (Fig. 270). He observed that the oscillations are most distinct when the resistance of the circuit is low, the capacity of the condensers is considerable, and the sparks are taken between metallic balls, preferably of iron. If the resistance of the circuit be increased, as by introducing liquids into it, the oscillations are damped, and the discharge appears continuous. If it be still further increased, the terminals discharge faster than the circuit can supply the electrification, and the discharge becomes intermittent. Using two jars, each having 0·2 sq. meter of coated surface, and

FIG. 270.

putting 1300 meters of copper wire in the circuit, the total duration of the discharge was increased three- or four-fold (that of a single jar being $1·3 \times 10^{-4}$ second). It was oscillatory, and gave a band on the sensitive plate at the focus of the mirror 23 cm. long. Neither the length of the spark, nor the potential of the charge appeared to have any effect on the duration of a single oscillation; ten jars on a short metallic circuit giving for a 4-mm. spark $3·04 \times 10^{-6}$, and for an 8-mm. spark $3·05 \times 10^{-6}$ second. With 16 jars and a long circuit a 1·5-mm. spark occupied $5·11 \times 10^{-5}$, and a 9-mm. spark $5·14 \times 10^{-5}$ second.

519. Conditions required to produce a Spark.—In disruptive discharge a spark passes when the resistance of the dielectric is broken down by the electrostatic pressure existing upon the bounding conductors. Whatever therefore tends to increase this pressure or to diminish this resistance favors disruptive discharge. As to the electrostatic pressure, it is a function of the density.

nd has the value $2\pi\sigma^2$. But the density upon a given surface is proportional to the charge, and this is itself proportional to the potential. Careful measurements have been made both by Lord Kelvin and by De la Rue to determine the difference of electrostatic potential required to produce a spark in air at the ordinary pressure, the results of which agree well together. From these results it appears that the resulting electrostatic force in air near the surface of a conductor required to produce a spark one centimeter long tends toward 130 electrostatic units (507) as a constant value; although the unexpected result was obtained, that a greater electromotive force per unit length of air is required to produce a spark at short distances than at long ones. Since the pressure in dynes is equal to the square of the electrostatic force divided by 8π,* the value of the pressure required to produce a centimeter spark in air is $130^2/8\pi$, or 672 absolute C. G. S. units of force per square centimeter; i.e., 672 dynes, corresponding to 68 grams weight per square decimeter (Lord Kelvin).

Moreover, the density and hence the electrostatic pressure depends upon the form of the conducting surface and upon the sign of the electrification. Experiment shows that only one half the difference of potential is required to produce a spark between plates 7 or 8 cm. in diameter, that is necessary to give a spark of the same length between balls 3 cm. in diameter. And Faraday proved that the spark between two balls is longer for the same difference of potential, when the smaller one is made positive, than when it is negative. Thus the spark between two balls, one 7·5 cm. and the other 1·25 cm. in diameter, was 2·5 to 3·75 cm. when the larger ball was positively charged, and 25 to 30 cm. when this ball was made the negative electrode. This

* If the distance between two surfaces is a and their difference of potential V, the mean force between them, F, is V/a. The resultant force, F', close to a surface on which the electric density is σ, is $4\pi\sigma$. Since in this case $F = F'$, $V/a = 4\pi\sigma$ and $\sigma = V/4\pi a$. Hence p, which equals $2\pi\sigma^2$, is equal to $V^2/8\pi a^2$; or, since $F = V/a$, to $F^2/8\pi$.

property has been utilized by Gaugain and others in the construction of what are termed electrical valves; the electrical flow being readily transmitted in one direction, but not in the opposite. Again, the small ball gave a spark twice as long when charged positively by induction as it did when charged direct.

The resistance to disruptive discharge is diminished by diminishing the pressure of the gaseous medium in which it occurs. It is a well-known fact that the same difference of potential will produce a longer spark in rarefied air than in air at normal pressure. Wiedemann has shown, however, that the decrease of potential required for a given length of spark is rather less rapid than the decrease of the pressure, the ratio tending toward a limiting constant value. Conversely, when the air is compressed the resistance is increased. Cailletet found that the same difference of potential which would produce in ordinary air a spark 30 cm. long would cause a discharge through only half a millimeter of space in air under a pressure of 40 or 50 atmospheres. As to the rarefaction giving the minimum resistance, Schultz has pointed out that this limiting pressure varies with the size of the tube employed.

The resistance, after decreasing with the rarefaction to a minimum value, afterward increases again; so that in a high vacuum a spark of half a millimeter cannot be produced by a potential difference which would give a spark of 20 centimeters in ordinary air.

Again, the nature of the gaseous medium influences the length of the spark. Faraday arranges gases according to their resistance to disruptive discharge, as follows: hydrogen chloride, ethylene, air, carbon dioxide, nitrogen, oxygen, illuminating-gas, hydrogen. With a given potential difference De la Rue found the spark in hydrogen to be twice as long as in air, both at normal pressure. Morren, using the same tube, has shown that the minimum resistance is obtained for different gases at quite different pressures. Thus for hydrogen the pressure of 2·8 millimeters gives the minimum resistance; for carbon

monoxide 1·6 millimeters; for oxygen 0·6 millimeter; for nitrogen 0·1 millimeter; and for carbon dioxide 0·08 millimeter.

520. Characteristics of the Electric Spark.—The electric disruptive discharge takes different forms, according to the conditions under which it is produced. These are classified by Faraday as the spark-discharge, the brush-discharge, the glow-discharge, and the dark discharge; although the distinction is not absolute, one of these forms frequently running into another.

(1) The spark discharge is the brilliant line of light which marks the breaking down of the dielectric and the total transfer of the electrification through it. When the thickness of the dielectric is not great, the spark is straight; but as the distance increases, its path becomes irregular, due to compression of the air in its front. The volume of the spark increases with the quantity of electrification transferred, the spark of a condenser being the more brilliant and the louder in proportion as the surface increases. The condensed spark in air at ordinary pressure is nearly white; but when the quantity is small and the length of the spark great the color is a bluish purple; becoming reddish purple in rarefied air, and having characteristic colors in different gases.

(2) As the distance between the electrodes increases, the spark branches at the angles, throwing off fine ramifications in the direction of the negative side; until finally the discharge takes the form of a tuft or brush formed of fine parallel sparks, attached to the positive electrode by a single stem, and not quite reaching the negative electrode. In air this brush is purple in color, and is accompanied by a dull snap quite unlike the sharp crack of the spark-discharge. Moreover, the discharge effected by the brush is incomplete. Its intermittence is established by the musical note given when the phenomenon is persistent, which rises in pitch as the electrodes are approached. Under ordinary circumstances the brush-discharge, according to Faraday, takes place between a conductor and *the air*, *only* a single electrode, appa-

rently, being concerned. In this case the appearance is practically the same with a metallic point, whether the electrification be positive or negative. When both electrodes are immediately concerned, however, the negative brush is poorer and smaller than the positive.

(3) On reducing the area of the electrified surface, increasing its electrification, or diminishing the air-pressure upon it, the spark passes into the glow, the discharge appearing like a phosphorescent lambent flame, covering more or less of the electrode and extending a small distance into the air. It appears to depend on a rapid and continuous charging or discharging of the air in contact with the conductor. It is unaccompanied by sound, and is apparently not intermittent. It is more readily produced in air at ordinary pressures on surfaces positively electrified, although in rarefied air a negative glow is easily obtained. With a brass ball 6·25 centimeters in diameter, in air under a pressure of only 110 millimeters of mercury, the positive glow covers its entire surface. And using a ball of half this size, in a higher vacuum, the ball being positively electrified by induction, a glow gradually comes over its surface, increasing constantly in brightness; until finally it becomes very luminous, standing up like a low flame a centimeter or more in height. On touching the walls of the glass vessel it assumes a ring form, appears flexible, and revolves slowly on its axis (Faraday). In ordinary air the glow is accompanied by a current of air either directly from or directly toward the glowing portion of the conductor, this current in the former case being connected with the charge and in the latter with the discharge of the air, so that if the access of air to the conductor be prevented, the glow disappears. And by producing a current of air at the end of a conductor, a brush may be converted into a glow.

(4) As the electrodes diminish in size and the distance between them is increased, a continuous glow is observed on the negative terminal, while the positive terminal and the intervening space remain entirely dark. Further

increase of distance develops a purple discharge on the positive electrode; but this, however, never joins the negative terminal, a dark space remaining between them always. This discharge, therefore, appears to take place between the air-particles themselves, and this, too, without rendering them luminous. Wright (1870) has studied the dark discharge obtained with a Holtz machine having a 50-centimeter rotating plate, the terminals being separated from ten to fifteen and even twenty centimeters. Under these circumstances the positive pole is covered with a diffuse glow, so thin as to appear illuminated by a light shining from the direction of the opposite pole. A single jet appears upon the negative ball, although the interpolar space is dark and the discharge is silent. Very curiously, however, when an object, such as a paper grating for example, is interposed between the electrodes it interrupts the glow, and its outlines are seen sharply defined upon the positive ball, closely resembling a light-shadow, moving as the object moves, and varying in size with the distance. If the jet issues from the side of the negative ball, the lines along which the transfer is effected are curved lines, issuing normally from the one terminal and abutting normally upon the other, the electric shadow being on the side of the positive ball. The form of these curves is easily determined.

Moreover, the electric discharge effects chemical changes. Schönbein (1840) showed that the peculiar

FIG. 271.

odor observed after such a discharge is due to ozone; and that by means of the dark or silent discharge fifteen per cent of oxygen may be converted into this modifica-

tion. Siemens's ozone tube (Fig. 271) consists of two concentric glass tubes, the inner surface of the inner tube and the outer surface of the outer one being covered with tinfoil. Between these surfaces a slow current of cold and dry oxygen passes, being submitted in its transit to the action of the spark.

521. Nature of the Spark.—If the light of the spark be examined with the spectroscope, it will be found to give lines characteristic of the metals forming the electrodes. In this way Masson (1851) mapped the spectra of cadmium, antimony, bismuth, lead, tin, iron, zinc, and copper, and showed that the constitution of these electric spectra is independent of the source of the electrification and in part also of the surrounding medium. The color of the spark is due to the material of the electrodes volatilized and transferred by the discharge: this material forming a bridge over which the electrification travels. With a strong spark in ordinary air, the electrode lines are very bright and are the only ones readily detected. As the discharge is weakened, either by increasing its length or by rarefying the air through which it passes, its color becomes purplish, and other lines make their appearance in the spectrum which are due to the nitrogen and the oxygen of the atmosphere. So that in general the discharge in a rarefied gas gives a light characteristic of the gas employed, and therefore a characteristic spectrum. But even in this case the lines of the electrode metals are not absent. Wright (1877) has shown that even in a vacuum of only 1·5 to 2 mm. of mercury the transfer of matter from the electrodes, though principally from the negative side, is sufficient to completely coat a glass surface of four or five square centimeters in fifteen minutes. The metal employed influences the result, bismuth, gold, and platinum being among the metals most readily deposited in films in this way, and aluminum and magnesium among those deposited with the greatest difficulty. In vacuum tubes, therefore, the wires from which the discharge takes place are usually made of aluminum within the tube and

of platinum without it. Even then there is a trace of metallic discharge within a vacuum-tube contrived expressly to furnish only gaseous spectra. To eliminate this the capillary portion of the tube containing the gas to be examined is lighted by induction, the electrodes being entirely inclosed within bulbs of glass. In rarefied hydrogen the spark is crimson, in nitrogen it is purplish red, in oxygen a greenish white, in carbon dioxide a green, in silicon fluoride a blue, etc.

C.—ELECTRIC PHENOMENA OF THE ATMOSPHERE.

522. Atmospheric Electricity.—The phenomena of disruptive discharge are exhibited on a grand scale in nature in the thunder-storm. In 1749 Franklin enumerated the points of agreement between "the electrical fluid" and lightning as follows: "1. Giving light. 2. Color of the light. 3. Crooked direction. 4. Swift motion. 5. Being conducted by metals. 6. Crack or noise in exploding. 7. Subsisting in water or ice. 8. Rending bodies it passes through. 9. Destroying animals. 10. Melting metals. 11. Firing inflammable substances. 12. Sulphureous smell." To this he adds: "The electric fluid is attracted by points. We do not know whether this property is in lightning. But since they agree in all the particulars wherein we can already compare them, is it not probable that they agree likewise in this? Let the experiment be made." The following year he suggested the placing of a kind of sentry-box on the top of some high tower or steeple, this box being provided with an electrical stand, from which an iron rod rises to a height of six or eight meters, pointed very sharp at the end; so that on the passage of low clouds sparks could be drawn from the rod. In May 1752 this experiment was made by D'Alibard at Marly, and the sparks were obtained. In October of the same year Franklin himself obtained the same result by means of the famous kite experiment. "And when the rain," he says, "has wet the kite and twine, so that it can

conduct the electric fire freely, you will find it stream out plentifully from the key on the approach of your knuckle. At this key the phial may be charged; and from electric fire thus obtained spirits may be kindled and all the other electrical experiments be performed which are usually done by the help of a rubbed glass globe or tube, and thereby the sameness of the electric matter with that of lightning completely demonstrated."

A lightning flash, like the spark of a Leyden jar, is simply a disruptive discharge between opposite surfaces highly electrified. These surfaces may belong to two clouds or to a cloud and the earth; so that the lightning may pass between clouds or from a cloud to the earth. Like the discharge of a jar, the lightning flash is oscillatory, although there is a preponderance in one direction, apparently. The photograph of lightning closely resembles that of a long spark, being irregular in direction and containing no retreating portions. Such a photograph is shown in Figure 272. It was taken by

FIG. 272.

Jennings in Philadelphia in 1892, and represents a horizontal flash passing from one cloud to another. The duration of the flash is very considerable. Rood states

that he has measured flashes which endured for an entire second; and that these discharges were multiple in character, three to six distinct components having been counted, these components lasting from the one-thousandth to the one-twentieth of a second. The spectrum of lightning, as observed by Holden and others, consists of bright lines apparently agreeing with those of the spark taken in rarefied air. Herschel observed the blue line of nitrogen and the red line of hydrogen on the background of a continuous spectrum. Kundt, from an observation of fifty flashes, classifies lightning spectra as line spectra and band spectra; the difference being due to the mode of discharge, whether between the earth and a cloud or between two clouds. In the former case the tension is high, and a forked flash darts to the ground, developing great heat, and raising the oxygen, nitrogen, watery vapor, and carbon dioxide of the air to vivid incandescence. When, however, the discharge takes place between two clouds, it occurs usually in the form of a brush; and the spectrum of the brush-discharge is always a banded spectrum.

We owe to Lord Kelvin the most complete investigation of the electricity of the atmosphere. He finds that the whole surface of the earth is electrified, and that it is electrified negatively, as a rule; though in time of rain it may become locally positive. Moreover, the density of the earth's electrification varies greatly at different times and in different localities. In Arran, for example, he has found it to vary from a given value to double this value and back in one minute: a local result, due probably to electrified atmospheric masses moving along within a few miles of the observer. The suddenness of these changes shows that their origin cannot be at a great distance. In general, however, and even in continued fair weather, the earth's electrification is influenced very largely, as it would seem, by external electrified matter somewhere,—probably at a distance of not many radii from its surface. "We must suppose that there is always *essentially* in the higher aërial regions a

distribution arising from the self relief of the outer highly rarefied air by disruptive discharge. This electric stratum must constitute very nearly the electropolar complement to all the electricity that exists on the earth's surface and in the lower strata of the atmosphere; in other words, the total quantity of electricity reckoned as excess of positive above negative or of negative above positive in any large portion of the atmosphere and on the portion of the earth's surface below it must be very nearly zero."

The origin of this atmospheric electricity is not certainly known. Its amount and distribution may readily be measured by a Thomson quadrant electrometer, suitably arranged. For this purpose one side of the electrometer is put to earth and the other is connected with a burning match or an insulated tube from which a stream of water issues, situated at the point in the air whose potential is desired. If a conductor at the potential of the earth be insulated at a point where the potential is different from this, and if it be made to throw off continuously portions of matter from its surface, the difference of potential between its condition and that of the medium at the point will speedily be reduced to zero; in other words, the conductor will rapidly be brought to the potential of the surrounding air. The rapid transference of electrification by flame-convection is a well-known phenomenon; the air of a room being rapidly electrified by placing a lighted metallic spirit-lamp on the positive or negative side of an electrical machine in active operation. For the same reason, passing an electrified body through a flame completely discharges it; probably the only method of entirely freeing a non-conductor from electrification. Thomson's water-dropping collector consists of a vessel of water carefully insulated, and provided with a tap and narrow tube, through which the water may be made to issue in fine drops. The vessel is placed within the house with the tube projecting outside at a suitable height above the ground. On connecting it to the electrometer, an in-

creasing difference of potential is observed between the pairs of quadrants, which when the maximum is reached can be read off in absolute electrostatic units. In the experiments at Arran, the difference of potential ordinarily observed between the earth and the air three meters above it was from 0·75 to 1·5 electrostatic units; or from one quarter to one half of an electrostatic unit per meter of height. In fair weather with an easterly wind, the difference of potential reached sometimes six to ten times the above maximum value; or from nine to fifteen electrostatic units.

The function of clouds seems to be to collect and to concentrate the diffused electrification of the atmosphere. Moreover, it is easy to see that admitting a charge upon the vapor particles, their simple condensation and aggregation must result in the production of a very high potential. Suppose, for example, that a thousand such spherical particles, each having unit charge, coalesce to form a single drop of water. The diameter of this drop will be ten times that of a single particle, and its capacity will be ten times as great, since the capacity of spheres is proportional to their radii. But the charge upon the drop being the united charges of the thousand particles, each of unit value, will be a thousand units. Whence, since $V = Q/C$, the potential of the drop will be 1000/10, or 100 times that of the particle. It is evident therefore that, since the vapor-particles are exceedingly minute, we may have in their condensation into water-drops a cause competent to produce the high potential observed in lightning discharges, even when the flash is one or two kilometers in length. This potential, moreover, need not be as high as is sometimes supposed. Lord Kelvin has shown that the difference of potential per centimeter necessary to produce a spark in air tends toward a limiting value of 130 electrostatic units. Whence it follows that to produce a flash of lightning a kilometer long would require a difference of potential of only about thirteen million electrostatic units.

523. Protection from Lightning.—In 1749 Franklin first suggested the use of pointed rods of iron, placed on the highest parts of buildings and connected with the ground, as a protection from lightning. Until recently, it was supposed that the most perfect protection attainable was secured (1) by having the conductor project considerably above the building and terminate in a sharp point; (2) by having it made of a good conducting material, such as copper, and of a sufficient cross-section; and (3) by having it enter the ground so as to reach permanent moisture with plenty of contact. Now, however, Lodge has called attention to certain new features of the discharge arising from its oscillatory character, which require considerable modification in the means of protection. In the first place, rapidly alternating discharges confine themselves to a thin outer layer of the conductor, —thinner in the case of iron than of copper. Hence surface rather than cross-section is of importance in a lightning conductor, and iron is a better material than copper. Moreover, owing to the exceedingly high potentials involved and the brief duration of the discharge, no conductor can furnish an easy path for lightning. Side flashes will pass off from even a stout copper rod well grounded, to imperfect conductors and even to insulated bodies. If a copper bar 2·5 centimeters thick have a greater length of the finest platinum wire placed with its ends near it, some of the discharge will leave the bar and spark across a minute gap at each end, in order to utilize the hair-like platinum wire. Hence sharp bends and corners should be avoided, and the conductor should proceed to earth by the most direct path. Since the attempt is hopeless to make the lightning-rod so much the easiest path that all others are protected, all possible paths should be separately cared for. Tall pointed rods are not as efficient as a number of smaller ones along the ridge of a roof. It is not always safe to connect metallic surfaces to the conductor, though it is always safe to earth them independently. Water-pipes and gas-pipes should be connected together, but not to the lightning-rod,

except under ground. For an ordinary house the cheapest way to protect it is to run common galvanized-iron telegraph-wire up all the corners, along all the ridges and eaves, and over all the chimneys, taking these wires down to the earth in several places, and at each place burying a load of coke around the wire in order to establish at each point an efficient connection with the ground.

CHAPTER III.

ENERGY OF ÆTHER-VORTICES.—MAGNETISM.

SECTION I.—MAGNETIZATION.

A.—MAGNETS AND MAGNETIC SUBSTANCES.

524. Historical.—Under the name *Μαγνῆς λίθος*, Dioscorides mentions a mineral occurring near the city of Magnesia in Lydia, Asia Minor, which possessed the property of attracting iron. This mineral, known as magnetic iron-ore or magnetite, although abundant, does not always attract iron. When it does so, it constitutes a natural magnet, and is known as **lodestone**. Although as early as the tenth or twelfth century it appears to have been known that a fragment of lodestone suspended by a thread would place its axis in a north and south direction, yet it was not until the appearance of Dr. Gilbert's book "De Magnete" in the year 1600 that magnetism became a recognized branch of science. Gilbert observed that the magnetic effect is concentrated in two opposite regions of the magnet, which he called its **poles**, and that midway between the poles a non-active region exists, which he called the **equator**. To the line connecting the poles he gave the name of **magnetic axis**. Moreover, he discovered that the region surrounding a magnet possesses activity; and by means of iron filings he mapped out this region and studied the effects produced by it. In 1733 Brandt discovered that cobalt was attracted by the magnet, and in 1750 Cronstedt added nickel to the list. Brugmans in 1778 called attention to the fact that bismuth was repelled by the

magnet. Faraday in 1845 confirmed this observation, and extended it to phosphorus, antimony, and copper; thus dividing substances into paramagnetic and diamagnetic bodies,—spheres of the former being attracted and those of the latter repelled by a magnet.

525. Magnetic Substances.—Substances which are affected by the magnet are called **magnetic** substances. They are divided into two classes, called respectively **paramagnetic** and **diamagnetic** substances, according as they are attracted by the magnet, like iron, or are repelled by it, like bismuth. Diamagnetic effects are exceedingly feeble as compared with paramagnetic ones, although paramagnetic substances are much fewer in number than those possessing diamagnetic properties. Among paramagnetic metals are iron, nickel, cobalt, manganese, chromium, and cerium; the last being weakest. Among diamagnetic metals in increasing order are tungsten, iridium, rhodium, uranium, arsenic, gold, copper, silver, lead, mercury, cadmium, tin, zinc, antimony, and bismuth.

526. Magnetic Properties.—Bodies which attract iron are said to possess magnetic properties and are called **magnets**. A fragment of lodestone, for example, is a natural magnet. Moreover, the property is communicable; so that a nail suspended from either pole of the lodestone will itself support another nail at its farther end. The iron nail, however, is only a temporary magnet, since it loses its magnetic property on removing it from the lodestone. If the experiment be repeated with a rod of steel, however, more or less of the magnetic property will be permanently retained, and the steel rod will become a permanent magnet. Steel, however, is not as easily magnetized as iron to the same degree.

527. Characteristics of a Magnet.—On dipping a straight steel magnet into iron filings, the magnetic effects will be observed to reach a maximum at its extremities, and to be equal at these two ends. The magnetic properties of such a bar diminish rapidly toward its center, at which point the magnet will sustain no filings. The

ends of the bar, when it is freely suspended, point toward the poles of the earth; and hence these ends are called **poles**, and the magnet is said to exhibit **polarity**. A distinguishing mark is placed on the end which turns toward the north, and this end is called the **marked** or north-seeking pole. If the bar be indefinitely thin and long, and be uniformly magnetized, the whole magnetic effect may be assumed to reside at its ends, and the magnet may be viewed as a pair of poles of equal magnetic mass united by a bar exerting no action. The line in a bar magnet which places itself in the meridian when the bar is suspended is called the **magnetic axis** of the bar; the magnet may or may not be symmetrical about it. Strictly speaking, it is the extremities of this axis which constitute the poles.

528. Dual Nature of Magnetism.—If two suspended bar magnets be brought near each other, it will be observed that the two north-seeking ends or the two south-seeking ends will repel each other; while the north-seeking end of the one will attract the south-seeking end of the other. Hence the general law that **like magnetic poles repel and unlike magnetic poles attract each other.**

B.—MAGNETIC FIELD.

529. Magnetic Action traverses Space.—Magnetic attraction, like gravitation, is exerted through the surrounding medium, whatever its nature. The space surrounding a magnet, therefore, is traversed by the magnetic forces, so that a magnetic pole placed anywhere in the vicinity of a magnet experiences a force tending to move it. Such a region, in which a magnetic pole tends to move in a fixed direction, or in which a freely suspended magnet takes up a definite position, is called a **magnetic field**. Clearly, therefore, the universal tendency of a magnetic needle to turn upon its pivot so as to place its axis in the north and south

direction, is a proof of the existence of a magnetic field surrounding the earth.

530. Force in a Magnetic Field.—The force acting upon a magnetic pole in a magnetic field is jointly proportional to the magnetic mass of the pole and to the strength of the field. So that, calling m the magnetic mass of the pole and H the strength of the field, both in C. G. S. units, the force in dynes experienced by this pole in this field will be mH. Evidently, therefore, a pole is of unit magnetic mass when, if placed in a field of unit strength, it is acted on by unit force, i.e., by a dyne; and a field is of unit strength when a unit magnetic pole placed in it is acted upon by unit force. This unit magnetic mass of pole is called a **weber**, and this unit strength of field a **gauss**. Since $H = f/m$, the strength of field in gausses is the ratio of the force in dynes experienced in the field by a pole of magnetic mass m to the magnetic mass of the pole itself, expressed in webers; i.e., is the force experienced by unit pole in this field.

The direction in which a pole will tend to move if placed in a magnetic field depends upon the character of the pole, an unmarked pole experiencing a force in the opposite direction to a marked one. It is agreed to call the direction in which a marked or north-seeking pole would move in a field the **positive** direction. A magnetic field is said to be completely determined when we know (1) the force experienced by a unit pole placed in this field, and (2) the direction in which the resultant force acts through it.

531. Action on Two Poles.—Moments.—Couple.—An actual magnet, as we have seen, has two poles. When placed in a magnetic field, these two poles being of opposite name, experience forces in opposite directions, the marked pole tending to move in the positive direction and the unmarked pole in the negative direction. The total force acting upon the magnet is then of the nature of a couple, tending to produce rotation. The arm of the couple is the distance between the poles; so that if

m be the magnetic mass of either pole and l the distance between the poles, the **moment** of the magnet, which is the product of either pole by the distance between them, will be ml; and the moment of the couple, being the product of the force mH acting on either pole by the distance l between the poles, will be mlH. Or, calling M the product ml, which is the moment of the magnet, and G the moment of the couple, $G = MH$. Obviously, since the arm of a couple is always perpendicular to the direction of its forces, this supposes the axis of the magnet to be perpendicular to the direction of the force of the field. If it is parallel to this direction the couple is zero, and the magnet experiences no rotating force. The force is a maximum, consequently, when the angle which the magnet makes with the direction of the field is a maximum, i.e., is 90°; it is therefore a function of the sine of that angle. In the figure (Fig. 273) the magnet is inclined to the direction of the field by an angle α. Hence the arm of the couple is $l \sin \alpha$, and the moment of the couple is $mlH \sin \alpha$ or $MH \sin \alpha$.

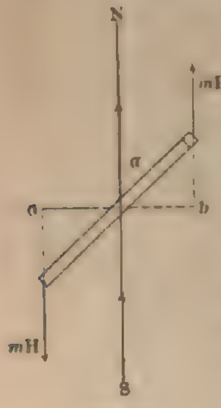


FIG. 273

532. Intensity of Magnetization.—The effectiveness of a magnet is measured, not by its magnetic moment alone, but by the ratio of its magnetic moment to its volume; so that the **mean intensity of magnetization**, as this ratio is called, is expressed in terms of a magnet of unit volume and of unit magnetic moment; i.e., in the C. G. S. system, of a magnet one cubic centimeter in volume having unit magnetic poles upon its ends. Thus calling ml the moment of a magnet and s its cross-section, its volume v will be sl; whence the intensity of its magnetization I, or the magnetic moment per unit of volume, is ml/sl, or m/s. If we call the moment of the magnet M, we have $M = Isl$; whence it appears that the moment of a magnet is a direct function not only of the

length of the magnet, but also of the intensity of the magnetization, and of the cross-section. Since sl is the volume of the magnet, however, this is equivalent to saying that when the magnetization is uniform, the magnetic moment is proportional to the volume.

533. Magnetic Density.—By analogy with electric density we may speak of magnetic density as the amount of superficial magnetization at a pole divided by the surface there; i.e., the amount of magnetization upon unit of surface. Calling m the magnetic mass of the pole and s the surface, we have $\sigma = m/s$. But we have just seen that $I = m/s$, whence $\sigma = I$; or the superficial magnetic density of the bar is equal to the intensity of its magnetization. Since $m = sI$, the magnetic mass of a pole is the product of the intensity of magnetization by the cross-section of the magnet.

534. Lines of Force.—In a magnetic field a magnet free to move will place its axis parallel to the resultant force if this resultant is a straight line, or tangent to it if it is a curve. Faraday proposed to represent the direction of this resultant force at various parts of a magnetic field by means of imaginary lines drawn through it. These lines he called **lines of force.** Maxwell extended this suggestion to include the strength of the magnetic force in the field as well as its direction, by supposing to be drawn through a surface perpendicular to this direction as many lines per unit of area as corresponded to the strength of the field. Thus a field of a strength of 12 gausses would have 12 lines passing through each square centimeter. Moreover, each line of force represents one unit of force, i.e., a dyne. So that the strength of a field, H, is represented in dynes by the number of lines of force which pass through each square centimeter of a surface perpendicular to the resultant force of the field. The force thus traversing a surface is sometimes represented as a flow of force, after the analogy of a fluid, and is represented by Φ. Since the total flow through a surface is evidently the product of the flow through unit area by the number of units of

area, i.e., the total surface, we have $\Phi = HS$. Whence if we divide the total number of lines of force crossing an area by the area in centimeters, we have the strength of the field in dynes.

The magnetic actions which take place in the field, and which are due to the condition of strain in the medium, are well illustrated by supposing with Faraday that each line of force, like a stretched elastic thread, is endowed with a tendency to shorten along its length, while at the same time these lines of force are self-repellent. So that as a result there is a tension developed in the medium along these lines and a pressure at right angles to them. The direction of these lines of force, being the same as the direction of the force in the field, i.e., the direction in which a marked pole would move in it, is evidently from a marked to an unmarked pole. So that the lines of force of a magnet issue from the marked end and enter the unmarked end, just as the lines of the field would do if the magnet were placed in it in equilibrium. Clearly if the magnet be deflected, the lines of force will be distorted; and tending to shorten, will exert a stress upon the magnet of the nature of a couple tending to return it to its first position. So, again, if the marked end of one magnet be placed opposite the unmarked end of another one and in line with it, the lines of force issuing from the former will enter the latter; and, tending to shorten, will produce attraction. While if the similar ends of the two be opposed, the lines of force will be turned away from each pole in all directions; and thus becoming parallel will repel each other, and so will repel the magnetic poles connected with them. Lines of force, although non-existent in fact, yet, like the parallels and meridians of the earth, are useful for mapping out a region.

535. Magnetic Phantoms.—By means of iron filings the field in the vicinity of a magnet may be studied. This was first done in the sixteenth century by Dr Gilbert, who called the figure produced the **magnetic phantom**.

EXPERIMENTS.—1. Dust fine filings of wrought-iron uniformly over a glass plate by means of a sieve, and place the plate above the poles of a vertical U-magnet. By gently tapping the plate the filings will arrange themselves in the form of curves connecting the marked and the unmarked poles. The same result will follow if the unlike poles of two different magnets be used, provided they be of the same strength.

2. Place a short magnet, such as a piece of magnetized sewing-needle, suspended by a silk fiber, just above the filings, and move it about within the field. The magnet will place itself parallel always to a tangent to the curves, and its marked end will point along the curves always in the positive direction.

3. If the phantom of the field due to a single pole be obtained in the manner described, its lines of force will be seen to be straight lines, radiating from the pole as a center, in all directions.

4. Repeat the first experiment, using two like poles instead of two unlike ones. Each pole will give its set of radiating lines, but the lines of one will push away those of the other and become finally parallel with them; thus indicating repulsion.

An excellent representation of the form of magnetic phantom which is given by a bar magnet is shown in

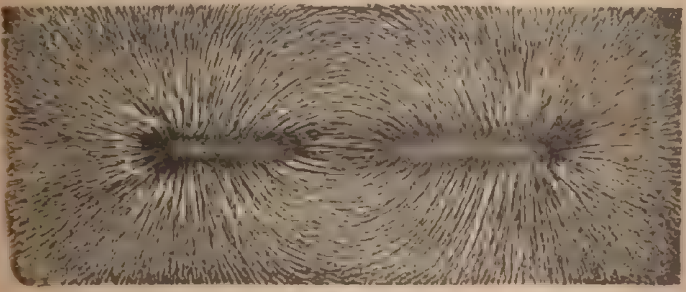


FIG. 274.

Figure 274, taken from a plate prepared by Mayer. Since the lines of force are symmetrical about the magnetic axis, such a phantom may be obtained in any plane containing this axis.

536. Uniform and Variable Fields.—A field is said to be **uniform**, when its direction and strength are the same in every part. In such a field the lines of force are straight lines, and they are equidistant. A **variable** field is one in which either the direction of the force

or its magnitude varies from one point to another. The lines of force in such a field are curved or radiating, and not parallel. The field of the earth is a practically uniform field; while the fields produced by artificial means, such as magnets and the like, are in general variable ones. Such variable fields are shown in the magnetic phantoms.

537. Astatic System.—Since the force acting upon a magnetic needle in a magnetic field is a couple, it is obvious that by using two equal needles reversed in position we may obtain a system upon which the resultant action of the field will be zero; since the couple acting on one of the needles will be equal and opposite to that acting on the other. Such a system is called an astatic system, since it may have no directive tendency and may remain at rest in any position in which it is placed. If, however, the needles are not in exactly the same plane, this result will not follow. Calling OA (Fig. 275) the moment of one needle and OB that of the other, the diagonal OR of the parallelogram will represent the moment of the system. Let δ be the angle AOB between OA and OB, and α be the angle AOR, between the resultant moment and that of one of the needles; then $BOR = \delta - \alpha$. Whence, calling m the moment OA, m' the moment OB, and M the moment OR, the triangle OBR gives $m' : m : M = \sin \alpha : \sin \delta - \alpha : \sin \delta$, and $\tan \alpha = \dfrac{m' \sin \delta}{m + m' \cos \delta}$. In the case supposed, the moments are opposite in sign, and δ is nearly 180°; so that we may write δ for $\sin \delta$, and -1 for $\cos \delta$; and the formula becomes $\tan \alpha = m'\delta/(m - m')$. Hence, if m and m' are equal, $\tan \alpha = \infty$ and $\alpha = 90°$. In other words, the system as a whole places itself at right angles to the lines of force of the field. Such an astatic system, therefore, in the field of the earth will place itself more nearly

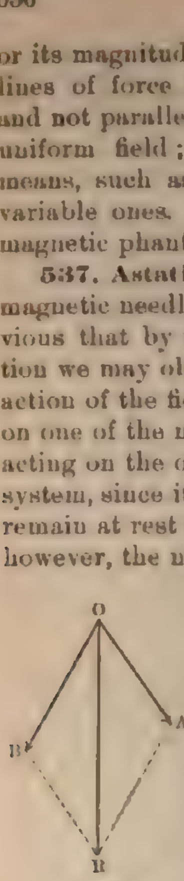


FIG. 275.

east and west, in proportion as the magnetic moments of the two needles approach equality.

C.—METHODS OF MAGNETIZATION.

538. Magnetization by Contact.—To magnetize any magnetic substance, it is necessary only to bring it in contact with a magnet. The magnet itself suffers no loss of magnetic properties in the process, those acquired by the substance being developed within it. The amount of magnetization thus developed in a substance depends jointly upon the nature of the substance and upon the strength of the magnet used. With a given magnet, iron is capable of receiving the greatest amount of magnetization, and next to it comes steel. Different kinds of iron and steel differ from one another in this particular, and steel varies also according to its hardness. As the magnetizing force increases, the magnetization produced increases, at first rapidly and then more slowly. So that the point at which the magnetization ceases to increase, which is called the **saturation point**, is a function of the force of magnetization. A sort of magnetic inertia appears to exist, which acts to resist magnetization at the outset, and to retain it afterward. This action, to which the name **coercitive force** has been given, is most strongly marked in the case of steel. It is not only more difficult to magnetize steel than iron, but the steel retains its magnetization permanently. The latter property Lamont has called **retentivity**.

539. Single and Double Touch.—Various modes of applying magnets have been proposed, in order to obtain magnetic bars of maximum strength. The simplest of these, known as the method of **single touch**, consists in moving one of the poles of a magnet over the bar to be magnetized from end to end, and always in the same direction; the operation being repeated until the effect ceases to increase. The end of the bar last touched by the magnet is of opposite polarity to that of the pole touching it. In the method of **double touch**, the two op-

posite poles of two equal bar magnets are placed in the middle of the bar to be magnetized, the magnets being inclined about 30° to the horizontal, and the two poles are then separated in opposite directions and moved to the extremities of the fixed bar. The effect is increased by placing the ends of this bar upon the ends of two magnets in line with it, whose polarity corresponds with that of the magnets moving over the corresponding ends. A modification of this method consists in placing a small block of wood between the poles of the moving magnets, and moving the poles, thus kept close together, alternately toward the two ends, taking care that the number of passes is the same for each of the two halves of the bar.

540. Magnetization by Magnetic Field.—Magnetic Induction.—A magnetic substance placed in a magnetic field becomes a magnet by induction; and if it be paramagnetic, its magnetic axis coincides with the lines of force of the field. Evidently the magnetization produced is stronger as the field is more intense. And inasmuch as the magnetic fields produced by electric currents are far more intense than those given by magnets, this method of magnetization has displaced all others. We shall defer the consideration of electromagnetism until we have studied the laws of currents.

541. Results of Magnetization.—It has been observed that the magnetization is at first superficial and then progressively advances into the interior of the mass. So that a hollow steel tube may be made to give nearly as strong a magnet as a solid rod of the same diameter. If a bundle of steel wires be magnetized, the interior wires are often found to be unaffected. And in some cases the magnetism of a bar has been entirely removed by dissolving away the outer portions by means of an acid. Indeed, by subjecting a thin steel rod to the discharge of a Leyden jar, Carhart has proved that the magnetism thus produced in it is arranged in alternate and opposite layers; so that on acting upon the bar with acid, alternating polarities to the number of three or

four may be developed as the exterior layers are successively dissolved.

Moreover, magnetization changes the length but not the volume of the bar magnetized. Bars of iron and of cobalt become longer and smaller when they are magnetized, while those of steel and of nickel become shorter and thicker. Again, the magnetization may not be uniform throughout a bar. The total magnetization, algebraically speaking, must be zero; but a pair of similar poles may exist at the center, so that both the ends are of the same name. Such a magnet is said to have **consequent poles**. In magnetizing a bar this result is of course to be avoided.

542. Magnetic Circuit.—Since each portion of the end of a magnet repels every other, and tends to develop in these portions opposite polarities, it is evident that when the poles of the magnet are free, the total effect of these actions is to weaken the magnet as a whole. Consequently it has been found necessary, in order to preserve the strength of a magnet, to connect its poles with a piece of iron called a keeper. If the bar be in the U form, the keeper is straight and rests on the ends. If it be straight, the bar is placed near a similar bar in the reversed position, and two keepers are placed at the ends of the system. In this way a closed circuit of magnetic material is formed, which is called a **magnetic circuit**. An iron ring constitutes such a closed magnetic circuit.

543. Compound Magnets.—By magnetizing a number of magnets separately to saturation, and then combining them to form a compound magnet, a magnet superior in strength to a simple magnet of the same mass is obtained; but the strength of such a compound magnet is by no means the sum of the strengths of the separate magnets which compose it. Evidently, if two equal bar magnets be placed with their opposite poles together, there will be complete neutralization of the one by the other, and there will be no external magnetic field. The magnetism is said to be bound, and not free.

So if the two magnets be united with their similar poles together, each pole will tend to weaken the other by developing in it the opposite polarity; so that the combined strength of all the poles will not equal the sum of their strengths taken separately. The advantage of compound magnets lies in the fact that thin plates can be easily magnetized throughout their mass; the intensity of the magnetization of the resulting compound magnet, which is the ratio of its magnetic moment to its volume, being therefore much greater than if the magnet were made of a single mass. Thus Jamin found that if six plates each of which lifted 18 kilograms were combined into a compound magnet, the magnet itself lifted only 64. On separating the plates again, each now lifted less than 10 kilograms; owing to the weakening action above mentioned. Jamin himself has constructed laminated magnets composed of thin plates of steel separately magnetized and then combined, which are of remarkable power, lifting sixteen times their own weight immediately after construction, and thirteen and a half times after attaining their permanent condition.

544. Lifting-power of Magnets.—Portative Force.—The lifting-power of a magnet, or the force required to overcome the attraction between a magnet and its keeper, is proportional to the area of the surface in contact and to the square of the intensity of magnetization; or calling F the required force, I the intensity of magnetization, and S the surface, $F = 2\pi I^2 S$. Since $m = IS$, we may write this equation $F = 2\pi m I$; or the force is proportional directly to the magnetic mass of the pole multiplied by the intensity of magnetization. The maximum attraction which it has been found possible to obtain between a magnet and its keeper is 4000 grams or $3 \cdot 92 \times 10^6$ dynes per square centimeter. This corresponds to a value for I of 791 C. G. S. units. The **portative force** of a magnet is measured by the maximum weight which it will sustain. From his experiments on steel magnets Häcker has deduced the empirical formula $P = aW^{\frac{2}{3}}$; in which W is the mass of the magnet and

a a constant varying from 12·6 in Häcker's experiments to 19·5 or 23 in those of Logemann and Wetteren. It follows from this that the portative force of a magnet relative to its own mass rapidly diminishes as the size of the magnet increases: two magnets, for example, of masses 1 and 125, sustaining weights of 1 and 25; the former sustaining five times as much as the latter per unit of mass. Logemann succeeded in making steel magnets which would sustain 28 times their own weight. The largest steel magnet yet constructed is a Jamin magnet weighing 50 kilograms, which is able to support permanently 500 kilograms.

545. Effect of Temperature on Magnetization.—Gilbert (1600) states the fact that at a red heat iron is incapable of being magnetized. Subsequent investigations by Rowland and others have determined the law of variation of magnetization with temperature. Ledeboer finds that up to 680° iron preserves its magnetic properties almost unchanged; but that a rapid alteration in them takes place at about this temperature, so that at 750° the magnetic power is much weakened and at 770° it disappears entirely. Hopkinson has shown that iron which at 700° had a permeability of 11000, had a permeability of zero at 737°. For nickel the magnetic coefficient increases slightly up to 200°; it then decreases to 340°, at which it becomes zero. The coefficient of cobalt increases from 0° to 325°.

SECTION II.—MAGNETIC POTENTIAL.

A.—ATTRACTION AND REPULSION.

546. Theoretical Magnet.—Since in no actual magnet is the magnetization uniform throughout, no single point in such a magnet can be considered as a pole. Hence for purposes of discussion a theoretical magnet is assumed, long and indefinitely thin, and uniformly magnetized; so that the entire effect produced by it may be assumed to reside at its ends, which thus act as centers of force and may be considered as its poles.

In order to study the action of the poles separately, however, the magnet must be so long that the magnetization of the center does not interfere with that of the ends. Thus, for example, to secure this result with a rod one centimeter in diameter, it would require to be five or six meters long. The magnetic mass at any point of such a magnet is of course the product of its transverse section at the point by the intensity of the magnetization there; and the intensity of the magnetization is the moment of an element of volume at the point divided by the volume (532); i.e., is the moment of one cubic centimeter.

547. Laws of Magnetic Action.—The laws of magnetic attraction and repulsion were originally studied by Coulomb (1785). In his first set of experiments he employed the method of oscillations, using a magnetic needle 2·5 centimeters long suspended by a silk fiber, and a bar magnet 62·5 centimeters long and about three millimeters in diameter, the latter placed vertically in the plane of the meridian, with its lower end opposite the pole of the needle of unlike polarity. In the earth's field alone, the needle made 15 vibrations in a minute. When the magnet was in position and at 10 centimeters distance, the needle made 41 vibrations. At 20 centimeters the vibrations were reduced to 24 in number, and at 40 centimeters to 17. Since the forces are as the squares of the number of oscillations, those due to the bar alone are $41^2 - 15^2$, $24^2 - 15^2$, and $17^2 - 15^2$, or 1456, 351, and 64; or, corrected for the action of the upper pole at the greater distance, 1456, 331, 79. The distances being 10 : 20 : 40, the forces are approximately as 4^2 : 2^2 : 1^2; i.e., are inversely as the squares of the distances.

In a subsequent set of experiments, Coulomb made use of his torsion balance (468). Within the glass case a bar magnet 60 centimeters long and 3 millimeters in diameter was suspended by a fine copper wire free from torsion; and opposite to its marked pole the marked pole of a similar bar magnet was placed. The mutual repulsion produced a torsion of 24°. In order to reduce the devi-

ation to 17°, a rotation of the torsion-head of $3 \times 360°$ was required; and to reduce it to 12°, a rotation of $8 \times 360°$. Without the second bar, the earth's force alone acting, a rotation of $2 \times 360°$ was required to produce a deviation of 20°. Hence the repulsion between the magnets, which produced a deviation of 24°, was equivalent to a torsion of 864°, that which produced a deviation of 17° to a torsion of 1692°, and that of 12° to one of 3312°. But since the force of torsion is proportional to the angle of torsion, these angles represent forces of 864 : 1692 : 3312. And these forces are approximately in the inverse ratio of the squares of the distances, $24^2 : 17^2 : 12^2$; considering the chord as equal to the arc.

Since the moments of two magnets oscillating in the earth's field at a given place are directly proportional to the squares of the number of vibrations made in a unit of time, the ratio of the moments of the two bars used in the above experiment may be determined. Whence, as the moment of a magnet $M = ml$, the ratio of the magnetic masses of the two poles $m : m'$ may be obtained. Repeating the torsion experiments with poles of different magnetic masses, it was found that the force exerted between such poles at the same distance is always directly proportional to the product of these masses. So that the magnetic laws established by Coulomb may be thus stated:

1st. The force exerted between two magnetic poles at the same distance is directly proportional to the product of the magnetic masses of these poles.

2d. **The force exerted between two magnetic poles of the same magnetic mass but at different distances, is inversely proportional to the squares of the distances.**

Both these laws may be summarized in the expression

$$F = \pm\, mm'/r^2, \qquad [69]$$

in which F is the force acting, m and m' the magnetic masses of the poles, and r the distance separating them all in absolute measure. Evidently, since poles of opposite name attract and poles of the same name repel each other, the upper or positive sign will represent repulsion and the lower or negative sign will represent attraction.

548. Unit Magnetic Pole.—If in the above equation we make m, m', and r equal to unity, F will also be equal to unity. So that a unit pole will exert unit force on a similar pole at unit distance. The absolute C. G. S unit magnetic pole, therefore, is a pole of such magnetic mass that, when placed a centimeter distant from another unit pole, the force between them shall be one dyne.

EXPERIMENTS.—1. Suspend a long, thin, uniformly magnetized magnet—a rod 50 or 60 centimeters long and 2 or 3 millimeters in diameter, for example—by a fiber free from torsion or by a fine metallic wire from a graduated torsion-head. Place another similar magnet vertically, with its lower end on the same level with the opposite pole of the suspended magnet, and nearly in the plane of the meridian with it. Since the two poles will attract each other, they are prevented from contact by interposing a glass plate of known thickness. The torsion-head being at zero when the vertical magnet is absent, rotate it in the direction to separate the poles until the end of the movable magnet just touches the plate, and again read the torsion. Repeat the experiment with plates of thickness 2, 3, 4, etc., times the first one, and observe that the forces will vary approximately as the values 1, $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, etc.

2. Repeat the experiment, using the opposite ends of both magnets, and notice that the forces of torsion are the same for the same distances; showing that for uniformly magnetized bars the two poles exert equal forces and are equal in magnetic mass.

3. Make another experiment, using a second vertical magnet, and noting the torsion required for a given separation. Then use both vertical magnets together and make a reading. The torsion will be found to be the algebraic sum of the separate torsions; showing that the force acting is directly proportional to the magnetic mass of the magnetic poles.

549. Field produced by a Magnetic Pole.—Since a unit magnetic pole exerts unit force upon a similar pole at unit distance, it follows that the intensity of the magnetic field produced by a magnet at unit distance from

it must be unity; and hence that a unit pole placed there will be acted on by unit force; i.e., a dyne. The unit pole just now defined is the same as that already mentioned (520). It is called a **weber** and the unit field which it produces at unit distance is called a **gauss**.

550. Lines of Force of a Unit Pole.—Lines of force radiate in all directions from a magnetic pole. If we draw a sphere of unit radius—i.e., a centimeter—about a unit pole, the surface of such a sphere will be evidently 4π square centimeters. Now since the field at unit distance from a unit pole is unity, the surface of the sphere is in such a unit field; and hence there must be a line of force passing through each square centimeter of it; or 4π lines of force through the entire surface. But all the lines of force radiating from the pole pass through this surface; and therefore the total number of lines of force which radiate from a unit pole is 4π. In general the number which radiate from a pole of magnetic mass m is $4\pi m$. The flow of force from any magnetic pole, therefore, is 4π times the magnetic mass of the pole; and the total flow of force through a surface enclosing a magnetic pole of magnetic mass m is $4\pi m$. Consequently the intensity of the field at any part of this surface is $4\pi m/S$; which is equal to H.

551. Field due to Two Magnetic Poles.—The field surrounding a magnet is evidently due to the joint action of both of its poles. If, for example, a north-seeking pole be placed at n (Fig. 276) in the field of the bar magnet NS, it will be acted on with a repulsive force in the direction Nn and with an attractive force in the direction nS. The resultant of these two forces will obviously be the diagonal na of the parallelogram constructed on these forces as sides; and the pole n will tend to move along this diagonal. By constructing the direction of this resultant for various parts of the field

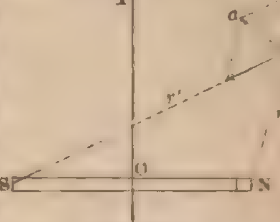


Fig. 276.

this field may be mapped out; and it will be found that a series of curves will be obtained corresponding precisely to those characterizing the magnetic phantom, thus proving that the lines of force of a field are simply the lines of resultant magnetic action in that field. The resultant due to several poles may be compounded in the same way; but if the polygon form a closed curve, the resultant is zero, and there is no resulting magnetic force in the field.

552. Law of Action of Two Poles on a Single One.—If both poles of a magnet act on a single pole, the force exerted varies inversely as the cube of the distance.

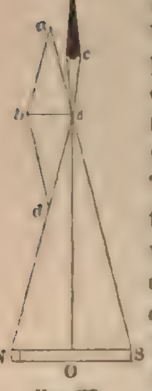


FIG. 277.

Suppose a bar magnet NS (Fig. 277) to be placed so that its middle point is in line with the axis of a magnetic needle ns. The pole S will repel the similar pole s in the direction sa; so that if m be the strength of the pole S, that of s being unity, and if r be the distance Ss, the value of sa will be m/r^2. The dissimilar pole N attracts s, and with the same force, since S and N are equal; whence sd represents in magnitude and direction the attracting force. The resultant of these two forces is sb. But as the triangles SsN and sab are isosceles and similar, we have $Ss : sa :: SN : sb$. Letting $Ss = r$, $sa = m/r^2$, $SN = l$, and $sb = F$, and substituting, we have $F = ml/r^3$. If the distance from the magnet to the needle is considerable, we may take sO or even cO for r; and then say that the total action of a magnet upon a single distant pole varies inversely as the cube of the distance. Since $ml = M$, we may write $F = M/r^3$; or if the needle is deflected through an angle ϕ, and its moment is M', we have $F = (MM' \cos \phi)/r^3$ as the moment of the couple tending to increase the deflection of the needle.

553. Mutual Action of the Field due to the Magnet and that due to the Earth.—The needle under these conditions continues to be deflected until the magnetic

deflecting force is balanced by the antagonizing force of the earth's field. We have seen above (531) that a magnetic needle of moment M' placed in a magnetic field of strength H experiences a couple of moment $M'H \sin \phi$ tending to bring its axis to coincide with the meridian. Equating these two equal forces, we have $M'H \sin \phi = (MM' \cos \phi)/r^3$; whence we have $M = Hr^3 \tan \phi$; r being the distance between the centers of magnet and needle.

The position of the magnet above given, with reference to the needle, is called "broadside on." If the magnet be so placed that its axis while perpendicular to the meridian and to the needle passes through the center of the needle (Fig. 278), it is said to be "end on." In

FIG. 278.

this case the resultant force is twice as great, and $F = 2ml/r^3$. The moment of the couple causing the deflection of the needle is $(2MM' \cos \phi')/r^3$, and the moment of the magnet is now $\frac{1}{2}Hr^3 \tan \phi'$; hence $\tan \phi' = 2 \tan \phi$.

554. Deflection-Methods.—Magnetometer. — The calculation of the moment of a magnet from the formula $M = ml$ is not possible, except in the case of long, thin, uniformly magnetized magnets, and then only approximately. The equations given above, however, enable us to obtain this moment experimentally, by observing the deflection which the given magnet produces. Since $M = Hr^3 \tan \phi$, the absolute value of the moment in C. G. S. units is obtained by multiplying together the tangent of the deflection-angle, the cube of the distance between the center of the magnet and that of the needle expressed in centimeters, and the value of the horizontal force of the earth's field in dynes. The relative moments of two

magnets are evidently as the tangents of the deflection angles, other things being equal; since $M_1 : M_2 :: \tan\phi_1 : \tan\phi_2$. A needle so arranged that the deflections can be readily observed, is called a **magnetometer.** A convenient form of it is a plane silvered mirror having several light magnets on the back, and suspended by a silk fiber so as to oscillate about a diameter; the whole enclosed in a suitable case. The deflections are read by means of a telescope and scale placed at a convenient distance.

B.—MAGNETIC WORK AND ENERGY.

555. Magnetic Potential at a Point.—To bring a magnetic pole to a given point in a magnetic field it is evident (1) that work must be done by or upon it, and (2) that this work when done upon it must be stored up in it as potential energy. The magnetic potential at a point in a magnetic field is a condition at the point such that to bring a unit pole there will require the expenditure of positive or negative work upon it. The potential of a pole at the point is measured by the potential energy it has there; in other words, by the work which has been done by or upon it to bring it there. Magnetic potential is therefore essentially analogous to electrostatic potential, and the difference of magnetic potential between two points is measured by the amount of work required to move a unit pole from the one to the other. If this work is one erg, then the difference of magnetic potential between the two points is unity.

Thus let a and b be two points in a magnetic field, V_a be the magnetic potential at a, and V_b that at b. The work required to carry a unit magnetic pole from a to b against the magnetic forces is evidently $V_a \sim V_b$. But this work is also equal to Fl, in which F is the mean force between the points and l their distance; hence $F = (V_a \sim V_b)/l$. Now by the law of magnetic action $F = mm'/l^2$. Equating these two values of F, we have $V_a \sim V_b = mm'/l$; and supposing m' to be a unit pole

we may say that the difference of potential between two points in a magnetic field produced by a pole of magnetic mass m, these two points being l units of distance apart, is directly proportional to the magnetic mass of the pole and inversely proportional to the distance separating them. If V_b be zero, $V_a = m/l$; or the potential due to a pole m at a distance l is the ratio of m to l. If m' be not unity, the total work done in bringing this pole from a point where the potential is zero, i.e., from an infinite distance, to a point where the potential is V, is evidently $m'V$. But in a uniform field $F = m'H$; and W, which is equal to Fl, is also equal to $m'Hl$. Equating these two values of W, we have $V = Hl$; or the potential between two points in a magnetic field at a distance l apart is the product of the intensity of the field by this distance.

556. Variation of Potential in Magnetic Field.—A line of force is the direction along which the resultant force acts, and since there cannot be two resultant forces at a point, two lines of force cannot intersect. From the expression $F = (V_a \sim V_b)/l$ it appears that the force in any direction in a magnetic field is simply the rate of variation of the potential in that direction. Whence it follows, that, since the force is a maximum along a line of force, the rate of variation of potential along a line of force is also a maximum; and a line of force may be defined as that direction in which the potential varies most rapidly. Since the positive direction at a point in a magnetic field is the direction in which a marked pole would tend to move if placed there, and since a body having potential energy tends to move in the direction in which the potential diminishes most rapidly, it is evident that the potential along a line of force diminishes most rapidly in the positive direction.

557. Equipotential Magnetic Surfaces.—About a single magnetic pole the variation of potential is equal in all directions, so that the potential is the same at the same distance. If a spherical surface be described about the pole as a center, every part of it will be at the

same distance from the pole, and hence will be at the same potential. Such a surface is called an **equipotential surface**; and the equipotential surfaces due to a single pole are evidently the surfaces of concentric spheres having the pole as their common center. To carry a unit pole from one such surface to the next, work must of course be done; and the distance between these surfaces may be so chosen that this work may be unity. Thus if the magnetic mass of the pole at the center be m units, the potential on the surface of a sphere drawn about it, whose radius is m centimeters, will be unity; and an erg of work will be done in bringing a unit pole from an infinite distance to this surface against repulsion. To do two ergs of work, the surface must be at only half this distance from the pole at the center; since then $V = 2$ and $2 = m/l$; whence $l = \frac{1}{2}m$. So to do three or four times the work, the radius of the equipotential surface must be one third or one fourth the value required when only one erg is done (Fig. 279). The distance between

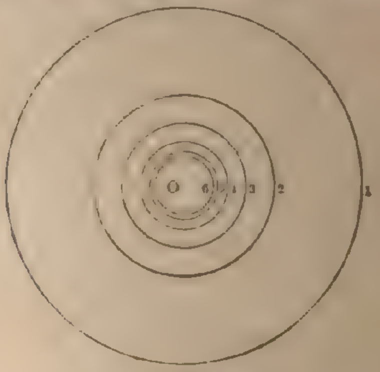


FIG. 279.

each pair of surfaces corresponds to the same work done; and as these distances in the above example are $\frac{1}{2}m$, $\frac{1}{6}m$, $\frac{1}{12}m$, it follows that they are directly proportional to the square of the mean distance (the geometrical mean) and inversely proportional to the magnetic mass of the pole, or what is the same thing, to the product of the extreme

distances divided by the magnetic mass of the pole. If $V = m/l$, and $V' = m/l'$, it is obvious that in order to have $V - V' = 1$, $m/l - m/l'$ or $m(1/l - 1/l') = 1$; whence $l' - l = ll'/m$; or the distance between each of the equipotential surfaces is proportional directly to the product of their radii and inversely to the magnetic mass of the pole. Thus in the above figure the radius is $\frac{1}{2}m$ for the second surface and $\frac{1}{3}m$ for the third; whence the distance between the two is $(\frac{1}{2}m \times \frac{1}{3}m)/m$ or $\frac{1}{6}m$. The mean force acting is of course $(V - V')/\frac{1}{6}m$ or $6(V - V')/m$; and the work done, being the product of this mean force by the distance through which it acts, is $6(V - V')/m \times m/6 = V - V'$; that is to say, the work done is proportional to the difference of potential between the surfaces. Since this difference is unity, the distance between the equipotential surfaces of a system is always such that unit of work is done in carrying unit pole from one to the other against the magnetic forces.

Since an equipotential surface is everywhere perpendicular to the lines of force of the field, the number of such lines which traverse it per unit of area represents the intensity of the field. But we have just seen that the intensity of the field, as measured by the work done in moving a unit pole across it, is the reciprocal of the distance between the equipotential surfaces. Hence the flow of force through unit area of an equipotential surface is the reciprocal of the distance between that surface and the next in order.

If the field be uniform, the lines of force are straight, parallel, and equidistant, and the equipotential surfaces are equidistant planes perpendicular to the lines of force.

558. Tubes of Force.—The strength of a magnetic pole is mentally pictured, according to the method of Faraday, by the number of lines of force which enter or leave it. Each of these lines of force is continuous, passing from the marked pole to the unmarked one outside of the magnet, and from the unmarked to the marked pole within it. The amount of magnetization at the point where a line of force leaves the one pole of a

magnet is exactly equal and of contrary sign to that at the point where it enters the other. Suppose now that two areas be taken on two consecutive equipotential surfaces, and through the boundaries of these areas lines of force be drawn; it is evident that these lines of force will enclose a tubular space, cylindrical if the lines of force be straight and parallel, conical if they be straight and not parallel. Such a system of lines of force is called a **tube of force**; and since the number of lines of force is the same at all parts of the tube, the strength of the magnetization is the same at each of its cross-sections. If the area enclosed represent unit magnetization, the whole equipotential surface may be divided up into such unit tubes of force, and the strength of a pole will be proportional simply to the number of unit tubes which thus abut upon it. If the tube be conical, its two bases will be unequal; although the strength of the magnetization will be the same on each. Hence, since the strength of the magnetization, or the flow of force through these bases, is HS, or the product of the flow through unit of area multiplied by the number of units of area or the total surface, it follows that HS on the first surface must be equal to $H'S'$ on the second; i.e., that this product must be the same at any cross-section. Since $HS = H'S'$, we have $HS - H'S' = 0$, and also $H : H' :: S' : S$; or the intensity of the field is inversely as the cross section of the tube.

559. Equipotential Surfaces due to Two Poles.—Since potentials may be directly added together, the potential at a point due to two magnetic poles is simply the algebraic sum of the potentials produced by the same poles acting separately. If m/l be the potential due to the magnetic pole m at the distance l, and $-m/l'$ be the potential due to the equal and opposite pole $-m$ at the distance l', the potential due to both poles at the point P, distant l from the one pole and l' from the other, will be $m/l - m/l'$ or $m(1/l - 1/l')$. Since this is true of every point in the field, the expression $V = m(1/l - 1/l')$ when V is constant represents the equation of an equi-

potential surface due to both poles, over which the potential has the constant value V. If, for example, two points be taken to represent two opposite poles, and about each of these points as centers spherical equipotential surfaces be drawn as above described, it is evident that at the points where the circles cut one another the potential due to the separate poles must have the same value; this value being of course the algebraic sum of the separate values. By marking these intersections with their values thus obtained, and by

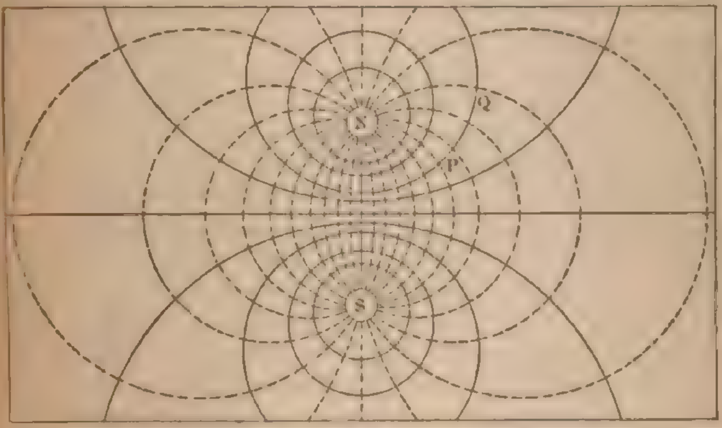


FIG. 280.

drawing curves through all the points at which the value is the same, a series of curves will be obtained which will represent the equipotential surfaces due to both poles taken together (Fig. 280). In the special case here taken these surfaces are spherical in shape and enclose the poles, the ratio NP/SP or NQ/SQ being constant for any given surface. Those about the marked pole correspond to positive values of the potential, and those about the unmarked pole to negative ones. Hence the two regions are separated by a symmetrical plane perpendicular to the line joining the poles, whose potential is zero. The full circles are drawn for successive equal changes of potential; and the dotted circles, which represent lines of force and which cut the first set

at right angles, are so drawn that the amount of magnetic force which flows in a given time through the space bounded by portions of each contiguous pair is the same. (Tait.)

560. Energy of Magnetization.—The work which must be done upon a magnet to reverse its position in the magnetic field—work which is done against the magnetic forces, if its direction coincides with that of the field and the magnet is in stable equilibrium, or by these forces if its direction is opposite to that of the field—is measured by the product of the force acting by the distance through which it acts; i.e., by mH multiplied by l, or mlH; which since ml is the same as M is MH. And this of course represents the change of energy of the magnet due to its position. To rotate the magnet from a position in which it makes an angle α with the earth's lines of force, to its normal position of parallelism with these lines, requires of course the work corresponding to the product of the force by the versed sine of the angle; i.e., to $mH \cdot \frac{1}{2}l(1 - \cos\alpha)$, or to $\frac{1}{2}MH(1 - \cos\alpha)$. And as this work is done by the magnetic forces, the potential energy of the magnet is diminished by this amount.

If the distance is very great with respect to the size of the magnet we may proceed as follows: The potential at P (Fig. 281) due to the very small magnet NS will be of course $m(1/r' - 1/r'')$ or $m(r'' - r')/r'r''$; now we may replace $r'' - r'$ by the approximate value $l\cos\alpha$ and $r'r''$ by r^2. Whence $V = (ml\cos\alpha)/r^2$, or, representing by μ the moment ml of this small magnet, $V = (\mu\cos\alpha)/r^2$. If now a circle be traced about O as a center, perpendicular to the magnetic axis, of such size that its surface represents numerically the value μ of the magnetic moment, and this surface, viewed from the point P, subtends the solid angle ω, this angle, since it is the ratio of the surface which subtends it to the

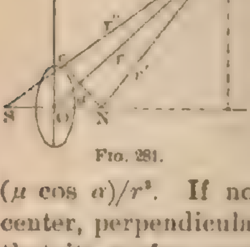


FIG. 281.

square of the radius, will be equal to μ/r^2 seen normally; or to the projection of μ perpendicular to r, divided by r^2 when seen from P; and hence $\omega = (\mu \cos \alpha)/r^2$. But this is exactly the value of V above given. Whence $V = \omega$; or the potential of a minute magnet at a point is measured by the solid angle subtended by a surface equal to the moment of the magnet, as seen from the point, the surface being perpendicular to the axis of the magnet at its middle point (Joubert). But the potential of this small magnet represents evidently the work which must be expended upon a unit positive pole to bring it from infinity to the point P, against the forces of the small magnet; or to bring the little magnet from infinity to its actual position, in presence of a unit magnetic pole at P. But this is evidently the total energy of the system consisting of the small magnet and the unit pole at P. If the pole at P were of strength m, the energy of the system would be $m\omega$.

To carry a magnet of moment M from a field of strength H to infinity requires work MH. To carry a soft iron bar to the same distance requires half this work. The other half $\frac{1}{2}MH$ is the intrinsic energy of the magnetization. According to Joubert, work equivalent to 10000 to 15000 ergs per cubic centimeter must be expended upon soft iron in order to magnetize it; and from 120000 to 200000 ergs per cubic centimeter upon steel.

C.—THEORY OF MAGNETIZATION.

561. Magnetization a Molecular Phenomenon.—If an electrified body while under induction be divided into two parts, both of these parts on separation will be found to be electrified, the one positively, the other negatively. But if a magnet be broken into halves, no such result appears. Each half is found to be a complete magnet, two new poles, of strength nearly equal to that of the original poles, being developed on the two sides of the fracture. So that it is not possible to ob-

tain marked or unmarked magnetization alone. Moreover, as this subdivision of a magnet may be continued indefinitely and with the same results, each particle being always a complete magnet, it follows that the magnetic property must reside in the smallest particle capable of existing by itself; that is, in the molecule.

562. Theories of Poisson and of Weber.—The analogy between magnetic and electric phenomena led Poisson to propose a two-fluid theory of magnetism. But since these magnetic fluids cannot pass from one molecule to the next, the process of magnetization on this theory must consist in separating more or less the fluids contained in each molecule, causing an accumulation of the one fluid at one end of the molecule and of the other fluid at the other.

Weber, on the other hand, propounded the theory that the molecules of a magnetic substance are always magnets; and that the reason why the substance exhibits no magnetic properties is simply because the magnetic axes of the molecules are turned indifferently in every direction. According to this theory, the process of magnetization consists in rotating these axes more or less, so that they shall all point in the same direction. Evidently if the magnetizing force be powerful enough to place these axes parallel to the direction of the field, the maximum amount of magnetization possible will be attained. That such an upper limit of magnetization actually exists, Beetz established by depositing electrolytically a fine filament of iron parallel to the lines of force of a magnetic field. The filament was found to be so strongly magnetized that no further permanent magnetization could be produced in it even by a powerful magnetizing force; while the increase of the temporary magnetization was but slight. Hence it would appear that at the instant of deposition each molecule had its axis placed in the direction of the magnetizing force. These experiments of Beetz would seem to be decisive in favor of the theory of Weber. This theory is known as the theory of **magnetic polarization.**

EXPERIMENTS.—1. Fill a glass tube with steel filings and magnetize them by drawing the marked pole of a strong magnet from the middle point toward one end, and the unmarked pole from this point toward the other end. The tube will act as a magnet and possess polarity, one end attracting and the other repelling the marked end of a suspended magnetic needle.

2. Shake the tube strongly so as to intermingle thoroughly the steel particles. If now the magnetic axes of the filings have been made to point in all directions equally, there will be no directive tendency in the mass as a whole, and both ends of the tube will attract the marked or the unmarked end of a suspended needle. Here it is plain that the act of magnetization consists simply in so arranging the axes of a lot of already magnetized filings that their polarities shall all point in the same direction.

3. Take a piece of watch-spring about ten centimeters long and magnetize it in the ordinary way. Examine the strength of its magnetization by ascertaining how large a nail it will support. Then break the magnet at its middle point: and notice that two complete magnets will result, the two new poles developed at the point of fracture being nearly the equal of the old ones in strength.

4. Bend a watch-spring magnet until the two ends are in contact. The ring thus formed will show no polarity whatever; thus proving the exact equality of the two opposite poles.

563. Electrical Theory of Magnetization.—Ampère's theory supposes that each magnetic molecule is the seat of an electric equatorial current to which the axes are perpendicular. The further consideration of this theory must be postponed until after we have discussed the magnetic properties of the electric current.

SECTION III.—MAGNETIC INDUCTION.

A.—MAGNETIC SUSCEPTIBILITY AND PERMEABILITY.

564. Induced Magnetization.—Whenever a magnetic substance is placed in a magnetic field it acquires magnetic properties and becomes polarized. The production of magnetization in this way is called **magnetic induction.** Paramagnetic substances like iron, when placed with their axes parallel to the lines of force of the field, are magnetized in the positive direction; i.e., so that the end where these lines enter becomes an un-

marked pole and that where they issue a marked one. Diamagnetic substances, on the other hand, are polarized in the inverse direction; the end where the lines of force enter becoming a marked pole and that from which they emerge an unmarked pole. Iron which is soft in the technical sense is soft also in the magnetic sense, and loses its magnetization entirely when removed from the magnetic field. All operations which tend to harden it, such as hammering, rolling, and the like, make it less easy to magnetize and cause it to retain more or less magnetization after removal from the field. Soft steel is magnetized almost as readily as soft iron and loses its magnetization almost as readily. But when hardened by sudden cooling, it becomes the best material known for permanent magnets. The total induced magnetization while in the inducing field is the sum of the temporary and permanent magnetizations.

565. Magnetic Susceptibility.—The amount of induced magnetization which is developed when a magnetic substance is placed in a magnetic field is dependent (1) upon the strength of the field and (2) upon the magnetic properties of the substance on which the induction acts; that is to say, it is proportional to the product of the strength of the field by the intensity of the magnetization which a unit field produces in the substance. Intensity of magnetization I has been defined as the ratio of the magnetic moment to the volume; or in the C. G. S. system, as the magnetic moment of one cubic centimeter of the substance. So that if k represent the intensity of magnetization of a magnetic substance in the form of a long thin bar when placed in a unit magnetic field parallel to the lines of force, the total intensity of magnetization when the substance is placed in a field of strength H will be kH; whence $I = kH$ and $k = I/H$. The relation between the strength of the field and the intensity of magnetization which it produces in a long thin bar of a substance when placed parallel to the lines of force, is called the **magnetic susceptibility** of the substance, the induced magnetization being the

greater in proportion as the susceptibility is greater. In the case of such a bar the ratio of the intensity of the magnetization to that of the field producing it, or the magnetization produced by a field whose strength is unity, which ratio is represented by k, is called the coefficient of induced magnetization or the **coefficient of magnetic susceptibility.**

566. Magnetic Permeability.—Another mode of expressing the same result has been suggested by Lord Kelvin. Since the number of lines of force issuing from a pole of unit magnetic mass, i.e., the flow of force from this pole, is 4π, the number of such lines entering or leaving a pole of magnetic mass k, developed in a unit field, will be $4\pi k$; and the total number of lines traversing the cross-section of the pole assumed to be unity will be $1 + 4\pi k$, or the number due to the field plus the number due to the pole which the field induces. The relation between the total flow of force induced across unit of area of the magnetic substance and the flow across unit section of the inducing field alone, is called by Kelvin the **magnetic permeability** of the substance; and magnetic permeability, or μ, is defined as the ratio of the number of lines of force induced through unit area of a substance to the number traversing unit area of the field. Or, calling B the number of lines traversing unit area of the substance, i.e., the magnetic induction, and H the strength of the inducing field, we have $\mu = B/H$; whence, since $\mu = 1 + 4\pi k$, we have $B = \mu H = H + 4\pi kH = H + 4\pi I$. That is to say, the magnetic induction is equal to the strength of the field increased by 4π times the intensity of the magnetization. If in the expression $\mu = B/H$, the value of H is unity, μ will evidently represent the number of lines of force which traverse unit area of the substance when placed in a field of unit strength perpendicular to the lines of force. This is called the coefficient of permeability or the **coefficient of magnetic induction**. Magnetic permeability may be considered as "the conducting power of a magnetic medium for lines of force" (Faraday); or as "the specific mag-

netic inductive capacity of the medium" (Maxwell). The total flow of force through any surface, therefore, is BS, i.e., is the product of the magnetic induction through unit surface by the number of such units.

567. Lines of Induction.—The directions along which magnetic induction takes place may be called **lines of induction.** These lines coincide with lines of force only in a magnetic field. Within a magnet the lines of induction are not parallel to the lines of force, but have a direction varying with the molecular condition of the substance. All the lines of induction within an induced magnet are directed from the negative pole to the positive, while outside they go from the positive toward the negative pole; the lines of force in both cases being directed from the positive toward the negative pole (Maxwell). The result is that a magnetic pole continually tends to repel its own magnetization, and thus to weaken itself. On the contrary, two opposite magnetic poles strengthen each other by induction although they weaken each other's fields.

568. Curves of Magnetization.—Hysteresis.—When a rod of iron is subjected to a gradually increasing magnetic field, the intensity of magnetization increases, at first approximately with the magnetizing force, and then more slowly tending toward a limit. Since therefore I, or the intensity of magnetization, has still a finite value when H, or the magnetizing force, is infinite, the value of k, the coefficient of susceptibility, must tend toward zero as its limit. This limiting value, toward which the intensity of magnetization tends as a maximum, is called the **point of saturation.** In the case of iron, this term has reference to the maximum value attainable by a bar while under induction. In the case of steel, however, the term is usually employed with reference to permanent magnetization, so that steel magnets are sometimes said to be supersaturated. That is, after magnetization they gradually lose their force at a decreasing rate for days and even weeks frequently; becoming finally normal. This process may be hastened by

any operation which facilitates molecular readjustment, such as heating and cooling, taps with a hammer, and the like.

If the results of magnetization be plotted, making the magnetizing forces the abscissas and the induced intensities of magnetization the ordinates, curves will be obtained (Fig. 282), showing the law of variation for dif-

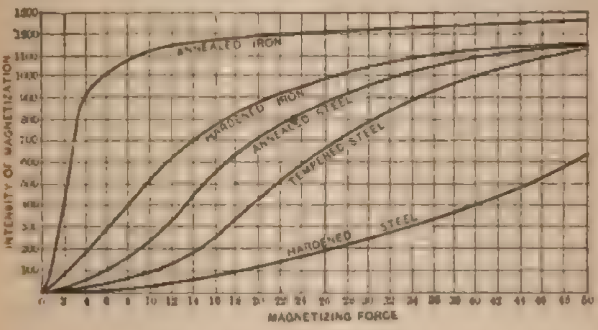


FIG. 282.

ferent samples of metal. In the case of soft wrought-iron, the curve rises rapidly, attaining a value of 900 C. G. S. units for a magnetizing force of 4 C. G. S. units; then it changes its direction and develops only an intensity of 1200 units when the magnetizing force reaches 24 units. Under the action of very intense magnetizing forces, Ewing has obtained a value which for wrought- and for cast-iron, for several kinds of steel, for nickel and for cobalt, is sensibly constant. The field required had a magnetizing force of 2000 C. G. S. units for wrought-iron and for nickel, and of less than 4000 C. G. S. units for cast-iron and for cobalt. The constant value of $4\pi I$ in the equation $B = H + 4\pi I$, he found to be for nickel 5030 to 6470 C. G. S. units, for cast-iron 15580 units, for cobalt 16300 units, and for wrought-iron 21360 units. At these high values the maximum intensity of magnetization of wrought-iron is attained, since it does not sensibly increase when the field varies from 2000 to 20000 C. G. S. units.

If a long thin rod of iron be placed in a magnetic field parallel to the lines of force, and if the strength of the field be raised from zero to a certain value H, the curve of magnetization, as above, will be represented by the line OA (Fig. 283). If now the field be diminished again to zero, the intensity will not follow the same values in decreasing order, but will describe the upper curve AA'; so that when the magnetizing force has become zero, the magnetization has a considerable value, OA'. On increasing again the strength of the field to H, the lower curve $A'A$ is described, and so on; the cycle being continually traversed in the order indicated by the arrows. To secure this result, however, it is necessary that the bar should have a length 300 to 400 times its diameter, and should be carefully protected from vibration; the remanent magnetization being practically zero, when the

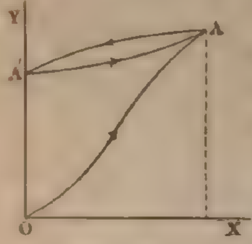


Fig. 283

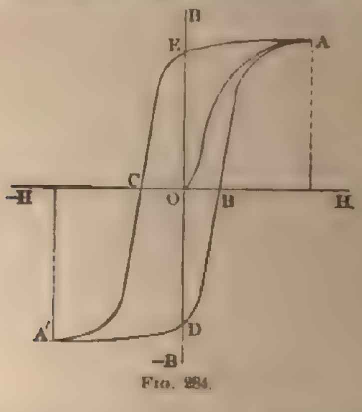


Fig. 284.

bar is subjected to shocks. If, instead of causing the magnetizing force to vary between zero and H, it be made to oscillate between $-H$ and $+H$, the curve described is that given in the figure (Fig. 284). The initial magnetization is represented by the curve OA, the mag-

netizing force increasing from zero to H. As the values of this force decrease from $+H$ to $-H$, the magnetization follows the curve ACA'; and as it increases again from $-H$ to $+H$, the magnetization describes the curve $A'BA$; the cycle being repeated always in the same order. For the same value of the magnetizing force, therefore, the magnetization possesses two values, being greater in the descending than in the ascending order; so that the magnetization of a bar would seem to depend not upon the actual conditions of the experiment alone, but also upon its previous state. There is therefore said to be a retardation of the magnetization in relation to the magnetizing force; and the phenomenon is called **hysteresis** by its discoverer, Ewing. The curves are the more separated in proportion as the coercitive force is greater. There is therefore a dissipation of energy on magnetization, the amount of which is proportional to the area of the cycle; and hence iron is heated by repeated magnetization and demagnetization.

569. Values of Magnetic Constants.—The C. G. S. unit of intensity of magnetization is that of a bar, the magnetic moment of which is one C. G. S. unit and the volume of which is one cubic centimeter. The C. G. S. unit of magnetic moment is the moment of a bar which when placed in a field of one C. G. S. unit strength with its axis perpendicular to the lines of force, is acted on by a C. G. S. unit couple. Since the dimensions of force F are MLT^{-2}, the dimensions of magnetic mass of pole $m = F^{\frac{1}{2}}L$ are $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$, the dimensions of strength of field $H = F/m$ are $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$, the dimensions of magnetic moment $M = ml$ are $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$, and the dimensions of intensity of magnetization $I = M / V$ are $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$, or the same as the dimensions of strength of field.

The maximum intensity of magnetization of which iron is capable appears to be about 1600 C. G. S. units, attained in Bidwell's experiments. Ewing, using a field varying in strength from 0 to 50 C. G. S. units, obtained an intensity of 1200 C. G. S. units as a maximum. The value of I obtained by Rowland for cobalt was 800 C. G. S.

units as a maximum, and for nickel 494. For steel the permanent intensity of magnetization varies with different specimens, from 400 C. G. S. (Weber) and 470 C. G. S. (von Waltenhofen) to 785 (Rowland) and 878 (Hopkinson). Since $k = I/H$, the value of the susceptibility can be obtained, knowing the intensity of the induced magnetization and the strength of the inducing field. Its maximum value is reached when the field is very feeble, when the iron is very soft, and when it is carefully protected from vibration. With a magnetizing force of 2·5 to 3 units, the value of k may rise to 200 or 275. If, however, the rod is jarred during magnetization, it reaches its maximum with a field of a strength of only 0·2 unit, and may then reach a value of 1600. On the other hand, with very intense fields the magnetic susceptibility may fall as low as 0·15. The value of k for iron is given by Barlow as 32·8, by Thalén as 32 to 44, by A. Smith as 80 to 90, by Stoletow 21 to 174, by Rowland in Norway iron 366, and by Ewing in thin soft-iron wires as 1300 to 1400. In the case of steel, k reaches its maximum of from 10 to 35, according to its character, with a magnetizing force of 25 to 40 C. G. S. units.

The maximum magnetic induction of which a magnetic substance is capable is the same quantity as its maximum intensity of magnetization, although expressed in terms of the lines of force traversing unit area of the metal. The magnetic induction B is 4π times the intensity of magnetization plus the strength of the inducing field; $B = 4\pi I + H$; or since $B = \mu H$ and $I = kH$, we have $\mu = 1 + 4\pi k$, which gives the permeability in terms of the susceptibility. Thus, for example, a specimen of iron for which k has the value 250, will have a permeability of above 3000 and be capable of receiving a magnetic induction of about 32000; i.e., of having 32000 lines of force traverse it per square centimeter. The values found for B differ greatly according to the quality of the iron used. Rowland gives 16600 for wrought-iron, Kapp 16740 for wrought-iron, 20460 for sheet-iron, and 23250 for iron wire; and Hopkinson 18250

or wrought-iron and 19840 for mild steel. In a field f 220, the induction reached 11000 for cast-iron; the esidual induction being about 5000. The induction in ray cast-iron with a field of 240 reached 10783; in nottled cast-iron 10546; in malleable cast-iron 12408. Ewing by the use of an enormous magnetizing force has riven the induction in Lowmoor iron up to 31560, and ven to 40000; the permeability being lowered in the ormer case to three and in the latter to two. Hopkin- on has found that a steel containing 12 per cent of man- anese is curiously non-magnetic, the maximum induction eing only 310. If Ewing's values be plotted with the nagnetizing forces as abscissas and the permeability as rdinates, the curves obtained will be equilateral hyper- olas, having $H = 0$ and $\mu = 1$ as asymptotes.

The following table gives the values of the various nagnetic constants as determined by Bidwell:

TABLE OF MAGNETIC CONSTANTS.

H	k	I	μ	B
3·9	151·0	587	1899·1	7390
10·3	89·1	918	1121·4	11550
40·0	30·7	1226	386·4	15460
115·0	11·9	1370	150·7	17330
208·0	7·0	1452	88·8	18470
427·0	3·5	1504	45·3	19330
585·0	2·6	1530	33·9	19820

In the first column the values represent the number f lines of force per square centimeter in the field alone; n the last column, the number in the iron within the ield. Since $k = I/H$ and $\mu = B/H$, the values in the ther columns are readily obtained from these (Thomp- on).

The value of the susceptibility k is always positive or paramagnetic substances such as iron; while for dia- nagnetic substances such as bismuth and copper it has eebly negative values. The permeability μ, however, is lways positive. In the case of paramagnetic substances he permeability never falls below unity. The value of

k for bismuth is $-\cdot0000025$, and the value of μ is 0·9999968584. This metal is the strongest diamagnetic substance known, and its permeability is the smallest of any known substance.

EXAMPLES.—1. Suppose a rod of soft iron a millimeter in diameter and 50 centimeters long to be placed in the earth's field of force parallel to its lines. Since the strength of the earth's field is 0·61 dyne at Philadelphia, the intensity of the induced magnetization in the bar, supposing the susceptibility k to be 40 for this inducing force, will be 0·61 × 40 or 24·4 C. G. S. units.

2. Since the intensity of magnetization is the ratio of the magnetic moment to the volume, the magnetic moment of this bar may be obtained by multiplying the intensity of magnetization above obtained, by the volume. The volume of the rod is $\pi r^2 l$ or 0·4 cubic centimeter; whence the magnetic moment of the magnet thus made is 24·4 × 0·4 or 9·76 C. G. S. units.

3. The strength of the pole of this magnet is the product of the intensity of magnetization by the section of the rod. The section is $\frac{1}{4}\pi d^2$ or ·007854 square centimeter; whence the strength of the pole is 24·4 × ·007854 or 0·192 weber. Such a pole will repel a similar pole placed at a distance of ten centimeters with the force of $\cdot192^2/100$, or 0·000369 dyne.

4. If a magnetized steel bar a centimeter in cross-section be arranged to move about a pivot in a horizontal plane the force acting on each of its poles when in the east and west position, supposing the intensity of its magnetization to be 785 C. G. S. units—and therefore, since $m = IS$, the strength of each pole also 785 C. G. S units—will be mH; or, since the value of H in Philadelphia is 0·195, will be 785 × 0·195 or 153·0 dynes. This is equivalent to 153·0/980 or 0·1561 gram-weight for each pole or 0·3122 gram-weight for both poles; and this is the weight which must be hung on fine cords passing over pulleys in order to keep the magnet in its east and west position.

5. If we suppose this steel bar to be 50 centimeters long, the couple acting on it in its east and west position will have a moment MH or mlH, if $M = ml$. The moment of the couple will be 785 × 50 × 0·195 or 7654 C. G. S. units.

570. Magnetic and Electrostatic Induction.—In magnetic as in electrostatic induction, magnetization by induction precedes attraction. But magnetic induction differs from electrostatic induction in the fact, first, that it acts between molecules and not between masses; second, that the action between magnets and mag-

netic substances is always attractive and never repulsive; and, third, that magnetic attraction is not a general property of matter, but is limited to a few substances.

B.—MAGNETIC SOLENOIDS AND SHELLS.

571. Solenoidal Filament.—For purposes of discussion, two theoretical arrangements of the magnetization with reference to the magnetic axis are employed. In the one the arrangement is linear in the direction of the axis, and the arrangement is called **solenoidal**. In the other it is superficial, in a plane perpendicular to the axis, and the arrangement is called **lamellar**. A series of identical magnetic elements placed along a straight or curved line so that their axes coincide with this line and all point in the same direction, constitutes a **solenoidal filament**. Since the marked pole of each element is in contact with the unmarked pole of the element next to it, the whole constitutes a magnetic filament of indefinitely small and constant cross-section, and of constant intensity of magnetization, which is neutral throughout its length, and is terminated by equal magnetic masses of opposite sign. The attraction of such a filament is reduced simply to that of these two terminal magnetic masses, and depends only on their position; being independent alike of the form and of the length of the filament. If several such straight filaments, all exactly alike and parallel, be suitably associated, they will form a cylinder whose magnetization will be uniform throughout, and will be free only at the terminal faces. Such a cylinder is called a **simple solenoidal magnet**. "A magnetic solenoid is an infinitely thin bar of any form, longitudinally magnetized with an intensity varying inversely as the area of the normal section in different parts" (Kelvin). In the case of such a magnetic solenoid, the filament of magnetic matter is so magnetized that the intensity of magnetization is the same at every cross-section; so that the product of this intensity into the area of a normal section is everywhere constant. This constant

product is called the **magnetic strength** of the solenoid, and the magnetic moment of any portion of it is equal to the product of the magnetic strength of the portion by its length. Since $I = ml/v$, the magnetic strength IS is equal to mlS/v; which, since $lS/v = 1$, is equal to m. In other words, the magnetic strength of a solenoidal magnet is what we have called the magnetic mass of a pole. If a number of magnetic solenoids of different lengths be placed together, so as to produce a single indefinitely thin magnet, this magnet is called a **complex solenoid**; and its strength at any section is the sum of the strengths of all the simple solenoids which pass through that section. "The action of a magnetic solenoid is the same as if a quantity of positive or northern imaginary magnetic matter numerically equal to its magnetic strength were placed at one end and an equal absolute quantity of negative or southern matter at the other end" (Kelvin). The action of a magnetic solenoid is therefore independent of its form, and depends solely on its strength and on the position of its extremities. The ends of a solenoid may consequently be called in the strict sense its poles. The potential due to such a solenoid is evidently the algebraic sum of the potentials of its ends. So that if m and $-m$ be the strength of the poles, and l_1 and l_2 the distances from these poles to a given point, the potential V at the given point will be simply $m/l_1 - m/l_2$; or $V = m(1/l_1 - 1/l_2)$. If the solenoid be closed upon itself so as to form a ring, the potential at every point is zero, and it exerts no magnetic action externally.

572. Magnetic Shells.—If the magnetic molecules, instead of being arranged linearly with their axes in the same line, are arranged side by side, in a plane or curved surface perpendicular to the direction of these axes, an infinitely thin sheet of magnetic matter will be obtained, which is called a **magnetic shell**, the arrangement being evidently a lamellar one. A magnetic shell is therefore defined as an indefinitely thin sheet of a magnetic substance of any form, magnetized in a di-

rection everywhere normal to its surface with an intensity varying inversely as the thickness in different parts (Kelvin). The constant product of the intensity of magnetization at any point, into the thickness at that point, is called **the magnetic strength of the shell** at the point. And the product of the magnetic strength into the area of any plane portion is the **magnetic moment** of that portion. A shell whose strength is everywhere equal is called a **simple magnetic shell**. If the strength varies from one point to another, the shell is called a **complex magnetic shell**; and the result may be viewed as due to the superposition of a number of overlapping simple shells. The potential of a magnetic shell at a point A, whose mean distance is r (Fig. 285), is the quo-

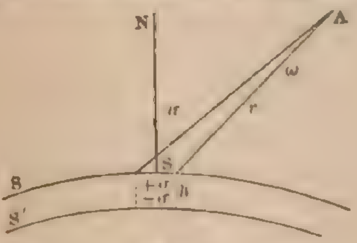


FIG. 285.

tient of the magnetic moment by the square of this distance (560); or $V = (\Phi S \cos \alpha)/r^2$; where $h\sigma$ or Φ is the magnetic strength of the shell, S its area, and α the angle which a line drawn from the point makes with a normal to the positive surface; i.e., with the magnetic axis. But the solid angle subtended by a surface S, as seen from a point, is the ratio of the component normal to the line drawn from the point to the surface, to the square of the mean distance; i.e., $\omega = (S \cos \alpha)/r^2$. Whence $V = \Phi\omega$; or the potential at any point due to a magnetic shell of any form is equal to the solid angle subtended by the shell at that point, multiplied by the magnetic strength of the shell (Gauss). Since the magnetic strength is constant, the equipotential surfaces of such a shell are defined by the condition that from every point on any one surface the

apparent area of the shell is the same. As to the sign of the potential, the above formula shows that it will be positive or negative according as the angle α is acute or obtuse; i.e., according as the point views the positive or the negative surface of the shell. The potential energy of a unit positive pole at a point on the positive side of the shell being the product of the strength of the pole into the potential due to the shell at the given point, is of course $\Phi\omega$; and the potential energy of a positive magnetic pole of strength m, is $m\Phi\omega$. Since such a pole will tend to move in the direction in which its potential energy diminishes most rapidly, this pole will experience a force tending to move it along a line of force from the positive to the negative side of the shell. If, however, the pole be fixed instead of the shell, the shell will tend to move, so as not only to diminish its positive area, but to increase its negative one; i.e., it will tend to rotate on a diameter as an axis. Hence the position of stability of a magnetic shell in a field is the position which corresponds to the minimum potential energy; that is, when the number of lines of force which enter its negative surface is a maximum. And, in general, the potential energy of the shell is the product of the strength of the shell by the number of lines of force which enter its negative face, taken with the contrary sign, $W = -\Phi Q$.

If the point be taken indefinitely near to the surface of the shell, or if the shell be indefinitely large, the solid angle subtended by the shell will differ inappreciably from 180° or 2π; whence the potential at such a point will be $2\pi\Phi$, positive if the point be opposite the positive face, negative if opposite the negative one. To move a positive magnetic pole of strength m, therefore, from such a point on the negative side of the shell to a similar point on the positive side, work must be done upon it equivalent to the difference of potentials between the points, i.e., to $4\pi\Phi m$; so that the potential on the positive surface of the shell exceeds that on the negative surface by $4\pi\Phi$. So, on the other

hand, if such a pole of strength m be placed on the positive side, it will tend to move round to the negative side, and in so moving will do an amount of work precisely the equivalent of the potential energy it loses, $4\pi\Phi m$. If the magnetic shell forms a closed surface, the action it exerts upon an external point, being the algebraic sum of two equal and opposite values ΦS and $-\Phi S$, will of course be zero. The potential outside a closed magnetic shell is everywhere zero, therefore; and the potential in the space within the shell, as the difference on the two sides is $4\pi\Phi$, must be $4\pi\Phi$; positive if the positive face is turned inward, negative if the reverse is true. Since the potential is zero outside the shell and is constant within it, such a shell exerts no action on a pole placed either inside or outside of it.

C.—DISTRIBUTION OF MAGNETIZATION.

573. Distribution within a Magnet.—Magnetization differs from the electrification of conductors in the fact that its distribution is not confined to the surface, but penetrates into the interior; thus resembling the electrification of non-conductors. In consequence the experimental determination of the magnetic distribution in the case of any given magnet is not possible. Whatever the actual distribution, however, it is capable of demonstration that its action as a whole is equivalent to that of two equal superficial layers of contrary signs distributed over the surface according to a definite law. Such a double fictive layer will not be in general a layer of equilibrium and the lines of force will not be normal to the surface. Its total mass will of course be zero, and its component masses will be distributed toward the ends of the magnet and separated by a neutral line. Further, the total magnetism of a magnet is made up of two portions: one that kept in place by the coercitive force and called fixed magnetism; the other produced by the induction of the first portion upon the magnetic substance, and which is induced magnetism. Since these two react

upon each other to diminish the magnetization as a whole, the apparent magnetization of a magnet, which is all that is directly observable, is practically of little use in determining either the intensity or the distribution of the magnetism in the magnet. Calculation shows, however, that when the demagnetizing force is proportional at each point to the rigid magnetization there, the law of distribution is the same as if this secondary inductive effect did not take place. This is the fact in the case of a uniformly magnetized sphere, of an ellipsoid uniformly magnetized along one of the axes, and of an indefinite straight cylinder magnetized perpendicularly to the axis. The demagnetizing force is the least possible with thin plates or for very long cylinders.

574. Experimental Determination of Distribution.—Every actual bar magnet may be looked upon as a complex magnetic solenoid, whose cross-section is constant and whose strength is variable at different parts of its length. A linear arrangement of magnetized particles all exactly equal in strength would produce a magnet with poles at its ends, the only free magnetization being on its end surfaces. If the magnetized particles are unequal in strength and increase from the center toward the ends, there will be a maximum of free magnetization at these ends; but free magnetization will also exist at intermediate points, being zero at the center, then rising to a maximum and decreasing toward the ends, where the second and principal maximum is reached. If as a third case the increase in the magnetization of the particles takes place from the ends toward the center, there will be no effect at the center, and the free magnetization will gradually increase toward the ends, where it will reach its maximum. This condition of things is the one observed in actual magnets. And in fact if a series of short steel cylinders be arranged axially to form a bar and then magnetized, it will be found that the central ones of the chain are more highly magnetized than those at the extremities. In consequence of this presence of free magnetism at points other than the extremities

of the bar, the poles of the magnet, since they are the points through which the resultant force passes, are not at the ends, but are slightly within them.

1. Method of Oscillations.—Coulomb sought to ascertain the arrangement of magnetization along a bar by oscillating a small magnetic needle opposite different points of its length. The needle was 13·5 millimeters long and 6·7 millimeters in diameter, and was suspended by a silk fiber. In order to decrease the rapidity of its oscillations, a piece of copper wire 25 millimeters long and 4·5 millimeters in diameter, with its axis horizontal, was fastened beneath the needle and oscillated with it. The magnetic bar was a wire 73 centimeters long and about 4 millimeters in diameter, fastened to a board and fixed vertically about 18 millimeters from the needle and in the magnetic meridian with it. Since the forces acting are as the square of the number of oscillations, we have $F : f + F :: n^2 : N^2$, where F is the force of the earth's field alone, f that of the magnet, n the number of oscillations per minute for the former, and N for the latter plus the former. Whence $f/F = (N^2 - n^2)/n^2$. Repeating the experiment at a point 13·5 millimeters from the first, we have $f' : F :: (N'^2 - n^2)/n^2$. Combining these equations, $f : f' :: N^2 - n^2 : N'^2 - n^2$. In this way the magnetic forces at different points of the bar may be compared. Coulomb assumed that the perpendicular component of the force thus obtained was proportional to the density of the fictive magnetic layer, except quite close to the end. Here he arbitrarily doubled the oscillation value.

2. Method of Torsion.—Coulomb employed also the torsion method in order to determine the distribution of the free magnetism along a bar. A magnetic needle was suspended by a fine metallic wire so that one of its poles was separated from a similarly magnetized point on the vertical wire magnet by a board 2 or 3 mm. thick. By rotating the torsion-head so as to bring the repelled end of the needle just in contact with the board, when opposite different points on the wire magnet, the torsion-angle measured the free magnetism. Since the

end of the wire acted on one side of the needle only, Coulomb doubled the value of the torsion obtained at this point, as he did in the case of the oscillation method. In the figure (Fig. 286) the torsion-values thus obtained

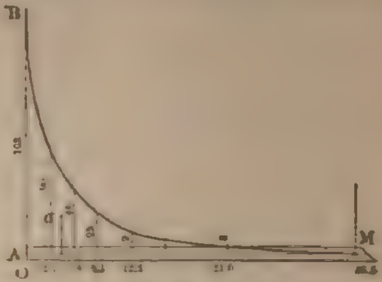


FIG. 286.

are plotted as ordinates and the distances along the bar as abscissas; the resulting curve showing the distribution of the magnetization as ascertained by this method.

3. **Method of Contact.**—If a soft iron needle be employed in the oscillation method above described, the result obtained with it will be proportional to the square of the perpendicular component of the magnetization, since the magnetization of this needle is due solely to the strength of the field. Jamin has made use of a modification of this method, by placing a piece of soft iron in contact with the magnet at different points and measuring with a dynamometer the force necessary to detach it. Since this method assumes the mutual action between the bar and the piece of soft iron to be constant, the results are not as satisfactory as those obtained with magnets.

4. **Method of Induction.**—By moving a wire ring, the ends of which are connected with a galvanometer, through a given distance along the bar at different points, currents are obtained which are proportional strictly to the perpendicular component of the magnetization at these different points. This method, originally proposed by Van Rees (1847), was the one used by Rowland (1875)

in his studies on magnetic distribution. We shall refer again to it after considering the laws of induced currents.

575. Experimental Results.—Coulomb in his experiments sought to determine the distribution of the magnetization in cylindrical magnets. For short magnets, i.e., those whose length is less than fifty times their diameter, he found that the perpendicular force at each point is directly proportional to the distance of this point from the center; so that the curve of distribution

FIG. 287.

is a straight line, making a given angle α with the axis of the bar (Fig. 287); a tolerably approximate result since Coulomb himself proved that for short bars the magnetic moment varies as the cube of the length. For bars whose length is more than fifty times the diameter, no free magnetism is detectible for a certain distance on the two sides of the center, although the distribution may still be represented by a triangle having a base twenty-five times the diameter of the bar (Fig. 288). If

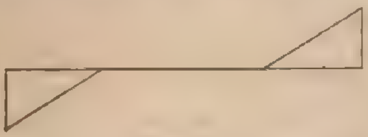


FIG. 288.

successive equal portions of the bar be taken, beginning at the end, the hypothenuses of the resulting triangles will be tangents to a curve, these tangents making equal angles with each other. Hence the magnetic moment tends to become proportional to the length.

576. Depth of Magnetization.—According to Jamin, the magnetization of a bar extends to a greater or less depth, depending upon the strength of the magnetizing force. Indeed by placing the bar successively in op-

posite fields, the latter one weaker than the former, two opposing magnetic layers may be superposed in the same bar; so that even when the apparent magnetization of a magnet is zero, it may be strongly magnetic. By grinding away the superficial layer of such an apparently neutral magnet, or by dissolving it away by an acid, he succeeded in developing the magnetization existing in the subjacent layers. The possibility of thus magnetizing a steel bar in alternating layers of opposite polarity was made use of by Henry in 1842 to explain the fact that the direction of magnetization of steel needles by the Leyden jar discharge was anomalous; the polarity being sometimes in one direction and sometimes in the other. This discharge, he says, "is a principal discharge in one direction and then several reflex actions backward and forward, each more feeble than the preceding, until equilibrium is obtained." This conclusion has been experimentally verified by Carhart (541), who magnetized steel rods 6 cm. long and 1·8 mm. in diameter by the discharge of a Leyden battery, and then determined their magnetic moments by the deflection method, as the successive layers were removed with acid. Plotting the results with moments as ordinates and masses as abscissas, a curve was obtained rising to a maximum as the external shell was removed and then decreasing nearly as the mass. The core was found always free from magnetization.

577. Relation between Magnetization and Electrification. — Magnetization and electrification have many points of resemblance. Excepting in the fact that the magnetization of a magnet is fixed in position and has no tendency to pass from one point to another as electrification has, and consequently that in the case of magnetization the question of equilibrium upon conductors does not enter, the fundamental laws for the one are the same as for the other, and lead to similar consequences. Magnetic potential is defined and determined precisely in the same way as electrical potential. And inasmuch as the unit of magnetic quantity is selected in

the same way as the electrostatic unit of quantity, amount of magnetization is represented by the same numerical value as amount of electrification. Moreover, the two are distributed in the same manner and they give the same strength of field, in the sense that the force at each point is the same in direction and has the same value; and hence the lines of force are the same, the equipotential surfaces are the same, the value of the potential is the same throughout, and is represented in both cases in the C. G. S. system by the number of ergs necessary to carry a positive unit from infinity to the point in question. This resemblance however is numerical only and does not at all imply an identity in the character of the two. Electrification has no action upon magnetic substances as such and magnetization has none upon electrified bodies. And although the two fields have properties depending upon the same medium, yet these properties arise from distinct modifications of this medium, which while coexisting do not act upon or compound with each other. Electrification, as we have seen, is due to æther-strain. Magnetization and magnetic phenomena appear to be due to an æther-motion analogous to that of vortex-rings.

578. Diamagnetism. — A paramagnetic substance has been defined as one having a positive coefficient of susceptibility and consequently a permeability greater than unity. In the case of a diamagnetic substance the coefficient of susceptibility is negative and the permeability is less than unity. The phenomena exhibited by paramagnetic substances as distinguished from diamagnetic ones, result from these definitions. Thus, when a paramagnetic substance is placed in a uniform magnetic field, since its permeability is greater than that of the field more lines of force will flow through it; while if the substance be diamagnetic, its permeability is less than that of the field and the number of lines is diminished. The two figures here given which we owe to Lord Kelvin show the two effects very well; the first (Fig. 289) corresponding to a substance whose permeabil-

ity is 2·8 and the other (Fig. 290) to one whose permeability is 0·48.

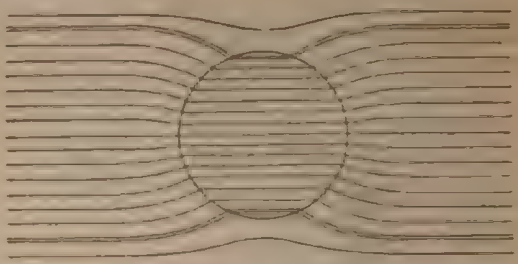


FIG. 289.

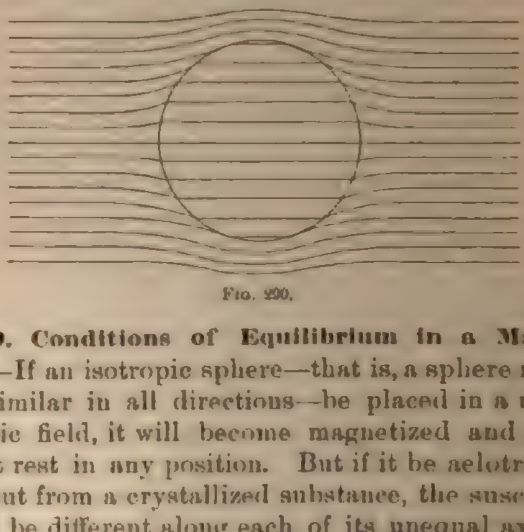


FIG. 290.

579. Conditions of Equilibrium in a Magnetic Field.—If an isotropic sphere—that is, a sphere magnetically similar in all directions—be placed in a uniform magnetic field, it will become magnetized and will remain at rest in any position. But if it be aelotropic, as when cut from a crystallized substance, the susceptibility will be different along each of its unequal axes, and the sphere will rotate until one of these axes becomes parallel to the direction of the field. The equilibrium is stable however only when the energy of the body is a minimum; and this in the case of paramagnetic substances, is only when this axis is the axis of maximum susceptibility; and in the case of diamagnetic media, when it is the axis of minimum susceptibility. If the body be elongated, it will place its longer axis parallel to the lines of force if isotropic, whether paramagnetic

or diamagnetic. But if the substance be crystallized, there is antagonism; if the susceptibility be small, the form becomes subordinate, and the axis of maximum susceptibility places itself parallel to the direction of the field; if the susceptibility be considerable, the form controls the result, and the longest axis is placed along the lines of force.

If the magnetic field be variable, the same conclusions may be extended to a body so small that the field may be considered uniform within the space it occupies. If such a small body, of volume v and susceptibility k, be placed in a field of strength H, with its axis parallel to that of the field, its magnetic moment will be kHv and its energy will be $-\frac{1}{2}kH^2v$. If the field be uniform, this body will be in equilibrium; but if it be variable, it will tend to move in a direction depending on the value of k. If k be positive, i.e., if the substance be magnetic, the body will move toward points where the force is a maximum; i.e., it will be attracted. If k be negative, it will move in the direction in which the force diminishes most rapidly, and will be repelled. In consequence if the body be elongated, it will place itself axially, or parallel to the lines of force, if it be paramagnetic; and equatorially, or at right angles to these lines, if it be diamagnetic. Thus in all cases each of the elements of volume of a paramagnetic body tends to move toward points in the field where the force is a maximum; and of a diamagnetic body toward points where it is a minimum.

Section IV.—Magnetism of the Earth.

A.—General Considerations.

580. The Earth's Magnetic Field.—A magnetic field is recognized by the fact that a freely suspended magnet takes a definite position of equilibrium when placed in it. The fact that the earth is surrounded by such a field so that a magnetic needle takes up an approximately north and south position in consequence, seems to have

been known to the Chinese at least a thousand years before the beginning of the Christian era and two thousand years before this property of the needle was known in Europe. Indeed as early as the third or fourth century "Chinese vessels navigated the Indian Ocean under the direction of floating magnetic needles pointing to the south" (Humboldt). An instrument in which the tendency of a magnetic needle to place itself north and south is made use of to fix directions upon the earth, is called a **compass.** In the mariner's compass, a circular card is placed upon the needle and moves with it. It is divided into 32 equal parts, called points, the axis of the needle coinciding with the line passing through the north and south points. In the best forms of compass several small needles are employed, fastened in the plane of the card. The compass used in surveying has its graduation fixed the needle moving within or above it, its agate cap resting on a steel pivot. Where sensitiveness is required the needle is frequently suspended by a silk or quartz fiber, to avoid friction.

581. Naming of Magnetic Poles.—Since by the law of magnetic action, dissimilar poles attract and similar poles repel, it would seem necessary to call the pole of a needle which turns to the north, the south pole. But since custom has sanctioned calling it a north pole, confusion would result from the attempt to reverse this naming. Various terms have been suggested for designating the poles. Faraday made a mark on the end of the magnet which turned to the north and called this end the **marked** pole. Airy painted red the end which turned to the north and blue the end which turned to south, calling the poles by these colors. Lord Kelvin, following the Chinese and the French practice, calls the end which points to the north, the true south pole and the end which points to the south, the true north pole. For mathematical discussion the north-seeking end of a magnet is called positive and the south-seeking end negative. Maxwell says "the magnetism of the north end of a magnet is Austral, and that of the south end is Bo-

real," meaning by the north end the end which points to the north, and *vice versa*.

582. The Force of the Earth's Field simply Directive.—Within any limited space free from magnetic substances, such as that of an ordinary room for example, the magnetic field of the earth may be considered as uniform and the lines of force as equidistant straight and parallel lines. Since a positive magnetic pole in such a field experiences a force in the positive direction and an equal negative pole an equal force in the negative or opposite direction, the total force acting will be due to the combined action of two equal and opposite forces upon the ends of the magnet; i.e., the force will be of the nature of a couple, and its value when the axis of the magnet is horizontal and perpendicular to the direction of the field will be mlH; or MH, if M represent ml, the moment of the magnet. In other words, the couple experienced by a magnet in the earth's field is equal to the product of the moment of the magnet by the intensity of the field. If the axis of the magnet make an angle α with the lines of force, the couple will be $MH \sin \alpha$. Hence the effect of the earth's field is simply to rotate the magnet into the position where its potential energy is a minimum, and its equilibrium, in consequence, stable. The earth's force, therefore, is directive only.

EXPERIMENTS.—1. Slightly oil a magnetized sewing-needle, and carefully lay it on the surface of water. It will float, and will turn so as to place itself in a north and south direction; after which, if it be not too near the walls of the vessel, it will remain at rest. Thus showing the action of no force tending to produce a motion of translation. The motion is rotatory only.

2. Bring the end of a bar magnet into the vicinity of the floating needle and opposite to one of its poles. The needle will move toward or from the magnet, always keeping parallel to itself. Thus showing an excess of attraction or repulsion by the magnet for that pole of the needle to which it is nearest. If the end of the magnet be not placed in the axis of the needle, there will be a motion of rotation as well as one of translation. Evidently the force of translation exists only where the length of the needle is an appreciable quantity in comparison with the distance of the magnet.

3. Magnetize a second similar sewing-needle, and, holding it vertically, bring one of its ends down near that end of the floating needle which is of opposite polarity. The latter will move toward the second needle and come to rest with a point inside its end opposite to a similar point inside the end of the vertical needle, so that the two needles will intersect at points within their ends. These points are evidently the poles of the magnetic needles.

4. Carefully weigh a steel bar and then magnetize it strongly. It will be found to have the same weight as before; showing that the earth's field exerts no force upon it in the vertical direction.

583. Earth's Field in general not Uniform.—From one point to another, however, upon the earth's surface the earth's magnetic field changes both in direction and in intensity, and this very irregularly. Moreover, even at any given place, these values are not constant, but have periodic variations of greater or less extent, occupying more or less time for their completion. To determine the values of the magnetic field at different places and the variations of these values is the object of governmental magnetic surveys. The methods and results of these surveys will now be considered.

B.—MAGNETIC ELEMENTS.

584. Elements of Terrestrial Field.—A magnetic field is completely determined when the direction and the intensity of the force in it are known at all points. In the case of the earth's field, it is convenient to consider (1) two directions of the magnetic force; (*a*) the direction measured in a horizontal plane which the horizontal component of the force makes with a north and south line also in this plane; and (*b*) the direction which the force itself makes with the horizontal at the place of observation, both measured in the same vertical plane; and (2) two intensities of this force measured in the horizontal and vertical directions respectively. These values for a given place are called generally the **magnetic elements** of the place. Individually they are called **declination, inclination,** and **horizontal** and **vertical intensity.**

585. Magnetic Declination.—If we call the geographical meridian the vertical plane which passes through the place of observation and the geographical poles of

the earth, and the magnetic meridian the vertical plane coinciding in direction with that of the earth's field and containing therefore the axis of the needle, the angle between these planes is called the **magnetic declination**; and the declination is said to be east or west according as the north-seeking end of the magnetic needle lies to the east or west of the geographical meridian. Although the fact that the magnetic needle does not point exactly north and south appears to have been known to, and its amount to have been determined by, the Chinese as early as the twelfth century, yet it was not known in Europe until the thirteenth; and was first distinctly delineated upon Bianco's charts in 1436. Columbus in 1492 discovered a point of no declination in the Atlantic Ocean south of the Azores; and S. Cabot a few years later, a second point to the north of these islands.

580. Measurement of Declination.—If in the figure (Fig. 291) the vertical plane containing OA is the plane of the geographical meridian, and if OM is in the plane containing the needle, i.e., the plane of the magnetic meridian, then α, which is the angle between these planes, is the declination. The instrument used to measure this angle is called a declination-compass or **declinometer**. In general it resembles a theodolite, and consists of two parts—a magnetic needle for fixing the magnetic meridian and a telescope for determining the geographical meridian, both mounted side by side and movable about the same vertical axis, which is provided with a graduated circle. The needle is frequently a tube of steel suspended by a silk fiber, and having a fine glass scale at one of its ends and a lens at the other, the focal length of the lens being the length of the tube, so that the scale is in its principal focus and the rays from it are parallel. When the needle is at rest, the zero point of its scale should

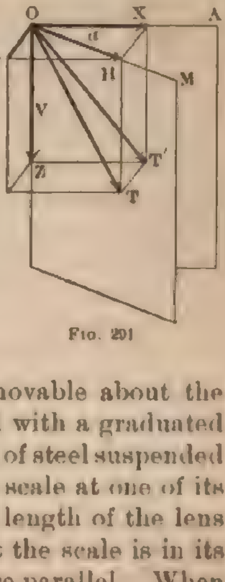


Fig. 291

coincide with the cross-wires of a small reading telescope mounted upon the instrument. The position is then read on the graduated circle, and the apparatus is turned about the vertical till the axis of the large telescope comes into the geographical or astronomical meridian, determined by observing the sun or a given star. The difference between the two readings gives the declination angle. To eliminate errors, readings are made on both sides of the circle, and also with the needle rotated on its axis 180°. Since the needle is balanced, the action of the vertical component of the earth's field is eliminated.

587. Variation of Declination.—The fact that for a given place the declination is not constant, but changes with time, was first observed about 1630. In 1580, Boroughs had found the declination in London to be 11° 18′ east; in 1622, Gunter found it to be only 6° 15′; and in 1634, Gellibrand observed that it did not exceed 4° 4′ 49″. Moreover, the variation is a periodic one. Thus in Paris in 1630 the declination was 4° 30′ east, in 1666 it was zero; then it became west, increasing steadily until 1824, when it reached a maximum of 24°. It is now slowly decreasing, being 15° 52′ west in 1888. Hence the period is about 900 years. In London the declination, which was 11° 18′ east in 1580, became zero about 1663, and the needle pointed true north and south. It then became westerly, and reached a maximum value of 24° 30′ in 1818; since which time it has been steadily diminishing, and in 1889 was 15° 42′ west. Its present annual rate of decrease is about five minutes. In Philadelphia in 1835 the declination was 3° 11′ west; and it has been increasing slowly, until in 1890 it was 6·9° west, and is increasing at the rate of 4·4′ annually. The change of long period is called the **secular variation** of the declination. Besides this there are two other periodic variations, called respectively the **diurnal** and the **annual** variations. The first of these, which depends upon the rotation of the earth, was discovered by Graham in 1722. In the northern hemisphere, the north-seeking

end of the needle begins at sunrise to move from east to west, and attains the extreme western limit of its swing about two hours after noon. It then slowly returns toward the east, reaching its original position about ten o'clock, and remaining practically stationary through the night. The swing of the needle during its daily variation is small in middle latitudes but is 1° or more in high latitudes, and is nearly proportional to the arc described by the sun. In Philadelphia, according to the observations of Bache, the mean arc of the vibration in summer is 10½′ and for winter 5½′; the mean for the entire year being 7½′. It increases toward the magnetic pole. The annual variation of the declination was first observed by Cassini in 1780. It reaches its maximum in Europe about the time of the vernal equinox, decreases steadily until the summer solstice, and then slowly increases during the nine following months. Its value is small, not exceeding a few minutes. Moreover, the variation itself is found to vary periodically. In, 1850 Schwabe announced a periodicity in the occurrence of sun-spots, the period being about 11 years. And almost simultaneously Lamont and Sabine announced a similar period in the range of vibration of the magnetic needle, the maxima and minima of the one set of curves coinciding closely with those of the other. So that it would appear that the condition of the sun's surface has much to do with the magnetic condition of the earth.

Besides these periodic variations of declination there are irregular variations, or perturbations, as they are called. These disturbances are observed at the time of earthquakes or volcanic eruptions, and particularly during a display of the aurora borealis. The disturbance produced by the aurora is so great as sometimes to cause a variation of 1° or 2°. These perturbations are felt over wide areas, and are often called magnetic storms.

588. Isogonic and Agonic Lines.—The astronomer Halley in 1700 proposed to draw a line through all those places on the earth at which the declination is the

same. These lines are called **isogonic lines**, and the particular line for which the declination is zero is called an **agonic** line. From the chart published by the U. S. Coast and Geodetic Survey it appears that for 1890 (Fig. 292, Frontispiece) the isogonic line of + 7° (west declination) enters the United States near Atlantic City, and passes northwesterly near Philadelphia, Bethlehem, Wilkesbarre, Oswego, Ithaca, and Rochester. Going south the westward declination decreases, and the agonic line is found to enter the United States near Charleston, and to pass in a N.N.W. direction, through the mountains of North Carolina and West Virginia, passing near Marietta, Columbus, Sandusky, and Toledo, Ohio, and Ann Arbor, Michigan. The isogonic line of – 7° enters the United States near Galveston, passes in a direction a little to the east of north by Hot Springs, Ark.; Jefferson City, Mo.; Keokuk, Iowa City, and Dubuque, Iowa; and La Crosse and Chippewa Falls, Wis. The isogonic line of – 17° enters the U. S. from the Pacific near San Francisco, and passes in an E.N.E. direction near Virginia City and Salt Lake, through Wyoming, Montana, and North Dakota. It will be observed that all these lines are extremely irregular, the declination being affected largely by local conditions, apparently.

589. Magnetic Inclination.—The magnetic inclination at a given point is the angle which a plane containing the axis of a freely suspended magnetic needle makes with a horizontal plane; i.e., it is the angle by which such a needle is depressed below the horizontal, and hence is sometimes called the **dip**. It was discovered by Hartmann in 1544 and first measured by Norman in 1576. Having carefully balanced the needles of his compasses before magnetizing them, Norman observed that after he had touched "the yrons with the stone, that presentlie the north point thereof would bend or decline downward under the horizon in some quantitie." Exact measurements gave him the value 71° 50′ for the inclination in that year in the city of London.

590. Measurement of Inclination.—The instru-

ment used for measuring the dip is called an inclination compass, or **inclinometer**; or sometimes a dip circle. In principle it consists simply of a magnetic needle mounted with great delicacy on a horizontal axis so as to move only in a vertical plane, and provided with a graduated circle. When the plane of the circle is in the magnetic meridian, only the component of the earth's force in that vertical plane acts upon the needle, and its axis places itself parallel to the lines of force. The angle read off on the circle is the angle of the dip. Evidently, if T (Fig. 293) be the direction of the lines of force, and therefore that of the axis of the needle also, H the horizontal and V the vertical component, the plane of the figure being that of the magnetic meridian, we have $H/T = \cos\delta$ and $V/T = \sin\delta$; whence $V/H = \tan\delta$ and $H/V = \cot\delta$. If, however, the plane of the circle is not that of the meridian, the horizontal component is not H, but $H\cos\beta$, where β is the angle between these planes. Hence $\cot\delta' = H\cos\beta/V = \cot\delta\cos\beta$; in which δ' is the apparent and δ the true dip. Evidently, when $\beta = 0$, $\delta' = \delta$; or the dip shown by the needle is the true one. If, however, $\beta = \frac{1}{2}\pi$, $\cot\delta' = 0$ and $\delta' = \frac{1}{2}\pi$ or $90°$; and hence the needle is vertical. This fact is made use of in setting the instrument. The graduated circle, which is mounted so as to move about a vertical axis, is placed with its plane approximately east and west, and is then adjusted until the needle is vertical. By turning it now through 90°, the plane of the circle will be in the meridian. To eliminate errors of centering, both ends of the needle should be read; to eliminate errors of graduation, the circle should be turned through 180°; to eliminate errors in the magnetic axis, the needle should be turned about this axis 180°; and to eliminate errors in the position of the center of gravity of the needle, its magnetism should be reversed. So that the final result is the mean of sixteen readings.

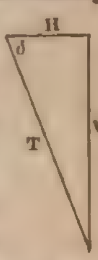


Fig. 293.

591. Variations of Inclination.—The value of the

inclination at different points upon the earth's surface is of course different. Moreover, the value for any given point is not constant, but varies from the same causes that affect the declination. In a general way it may be said that the inclination increases from the equator toward the poles. Thus, for 1885, the inclination at the following places in North America is computed as follows, the latitude being also given :

VARIATION OF DIP WITH LATITUDE.

Place.	Latitude N.	Inclination
Panama	9°	33°
Mexico	19° 30′	44·5°
Key West	24° 30′	54·5°
New Orleans	30°	60°
Charleston	33°	64°
St. Louis	38° 38′	69°
Washington	38° 53′	70·547°
Baltimore	39° 18′	71·16°
Philadelphia	40°	71·38°
New York	40° 43′	71·932°
Chicago	41° 53′	72·41°
Boston	42° 21′	73·116°
Albany	42° 39′	73·995°
Montreal	45° 30′	76°
Winnipeg	49° 53′	80°
Point Barrow	71° 30′	83°

With regard to the secular variation of the inclination, the value observed in London in 1576 by Norman was 71° 50′. It then increased slowly, and in 1720 reached its maximum value of 74° 42′. Since then it has been decreasing, being 67° 25′ in 1888. The earliest recorded observation of dip on this continent was made at Unalashka in 1778; although systematic determinations were not made until about 1820. Loomis, comparing the observations of Sabine and Franklin (1822–25) with others made in 1835, concludes upon a mean value of – 1·8′ as the annual diminution of the dip in the eastern portion of the United States. Schott in 1856 showed

that in the northeastern United States the dip decreased until about 1843, when it became stationary. It then began to increase, reaching in 1856 an annual increase of 2·7′. But curiously enough this wave of increase was of short duration, and reached its maximum about 1859, when the value again began to decrease and has continued to the present time, the annual rate at Philadelphia being about − 5·5′ and at Washington 3·5′. Since the rate of decrease is decreasing toward the south and west, it must soon become zero. And upon the chart of the Coast Survey Schott has given a shaded band skirting the northern coast of Cuba, passing over lower Louisiana and central Texas, through Mexico, crossing the Gulf of California, following the coast northward and passing off to sea near San Francisco, throughout which the rate of the annual variation of the inclination in 1885 was zero, and where therefore the inclination then had a constant value. The dip is less in summer than in winter, and less at night than in the daytime.

592. Isoclinic Lines.—Magnetic Equator.—Lines drawn through points upon the earth having the same value for the inclination are called **isoclinic lines**. Since the value of the dip increases with the latitude, there must be in the vicinity of the equator an **aclinic line**, or line where there is no dip and the needle remains horizontal. This line is called the **magnetic equator**. Beginning, for example, on the east coast of Brazil, in south latitude 16°, it passes eastward and a little to the north toward the coast of Africa, intersecting the geographical equator about 2° east longitude, and entering Africa at the Gulf of Benin; then continuing its slight northerly direction it meets the parallel of 10° north, and moves east to Cochin in Hindostan; then turning south again it cuts the equator in 170° west longitude, continuing on until it meets South America in latitude 7° S. on the coast of Peru, and thus to the point of departure. So again, since the dip increases, it must finally reach a point near the pole where its value is 90° and the needle

is vertical. Such a point is called a **magnetic pole**. Gauss calculated that in 1838 the north magnetic pole was at 73° 35′ N. and 95° 39′ W. Ross in 1831 observed that in latitude 70° 5′ N. and longitude 96° 45′ W. the inclination was 89° 59′, or within one minute of being vertical. Schwatka in 1879 concluded from his observations that the pole had shifted to longitude 99° 35′ W. Gauss's position for the south magnetic pole was latitude 72° 35′ S., longitude 150° 10′ E. This pole has not yet been reached. Ross in 1841 observed in latitude 76° 12′ S., longitude 163° 2′ E., an inclination of 88° 40′; from which he concluded that the magnetic pole was about 256 kilometers distant.

The isoclinic line of 69° (see Frontispiece) passes across the United States from near Norfolk by Richmond, Louisville, St. Louis, Topeka, Cheyenne, Boise City, and Portland. That of 72° passes near Princeton, Harrisburg, Pittsburgh, Fort Wayne, Chicago, Iowa City, Deadwood, Helena, and Vancouver.

593. Magnetic Intensity.—The third magnetic element is magnetic intensity; i.e., the strength of the magnetic field at the point in question. Instead of determining the total force directly, it is found more convenient to determine by experiment its horizontal component; and then to calculate the total force from this; since $T = H/\cos\delta$. Moreover, the value of H, or the horizontal component of the earth's magnetic force, is a constant of importance in certain physical determinations. The first attempt to ascertain the value of the magnetic intensity appears to have been made by Graham in 1723, who measured the oscillations of his dipping-needle, in order to see if they were constant and to compare the value of the magnetic force with that of gravity. In 1769, Mallet compared the oscillations of a dipping-needle at different stations in Europe in order to compare their magnetic intensities. The first accurate observations were those of Humboldt, made about 1800, in South America.

594. Measurement of Intensity.—We owe to Gauss

(1833) the method at present in use for determining the value of the horizontal component of the earth's field in absolute measure. It consists of two distinct operations. By the first, which is a method of oscillations, the product of the horizontal component by the moment of the magnet is obtained. By the second, which is a deflection method, the ratio of the magnetic moment to the horizontal component is obtained; whence, having both the product and the quotient of two quantities, either of the quantities can be calculated.

I. **Method of Oscillations.**—Whenever a body oscillates under the action of a force, the square of the time of a single oscillation is directly proportional to the moment of inertia of the body about the axis of oscillation, and inversely proportional to the directive force. In the case of a magnetic needle the directive force is MH; i.e., is the product of the moment of the magnet by the horizontal component of the earth's magnetism. So that we may write $t = \pi \sqrt{I/MH}$ for the time of a single oscillation. Whence we have $MH = \pi^2 I/t^2$. Since the number of oscillations is the reciprocal of the time, we may also write $MH = n^2 \pi^2 I$.

II. **Method of Deflections.**—The magnet NS is placed with its axis on an east-and-west line passing through the center of a delicate magnetometer needle ns and at a definite distance r from it (Fig. 294). The needle will

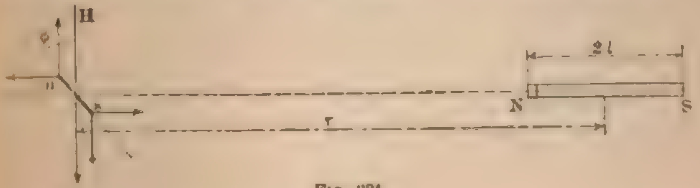


FIG. 294.

be deflected, and will take up a position of equilibrium between the two couples; i.e., the displacing couple due to the magnet $(2MM' \cos \phi)/r^3$ (r being the distance between the centers and ϕ the angle of deflection) and the restoring couple of the earth, $M'H \sin \phi$. Equating

these values, $(2MM' \cos \phi)/r^3 = M'H \sin \phi$; and therefore $M = \frac{1}{2}Hr^3 \tan \phi$ (553); whence $M/H = \frac{1}{2}r^3 \tan \phi$. Errors are eliminated by reversing the magnet, by placing it on the opposite side of the needle, and by repeating the experiment with different values of r; both ends of the needle being always read.

Since for a rod of small diameter, of mass m, and of length $2l$, the moment of inertia is $ml^2/3$, the formula for MH becomes $ml^2\pi^2n^2/3$; so that the product MH is readily calculable by squaring the number of oscillations in a second and substituting this value in this expression. Having, therefore, $MH = A$ and $M/H = B$, we obtain, by multiplying these equations, $M = \sqrt{AB}$, and by dividing them, $H = \sqrt{A/B}$. Whence the value of H is obtained.

595. Variation of Magnetic Intensity.—Like the other magnetic elements, the intensity varies from point to point of the earth's surface when measured at the same instant, and also varies from time to time when measured at the same place. In the Coast and Geodetic Survey tables for 1885 the value of the horizontal component and of the total force are given as follows for the places designated:

Place.	Horizontal.	Total.
New Orleans	·2775 dyne	·5519 dyne
Charleston	·2551 "	·5847 "
San Francisco ...	·2531 "	·5451 "
St. Louis...	·2135 "	·6003 "
Washington.....	·2026 "	·6087 "
Baltimore	·1978 "	·6128 "
Philadelphia ...	·1951 "	·6110 "
Chicago	·1877 "	·6216 "
New York	·1872 "	·6048 "
Boston	·1704 "	·5898 "
Albany	·1695 "	·6156 "
Montreal...	·1474 "	·6119 "

It will be seen that, owing to the increase of the dip with the latitude, the horizontal component steadily

decreases. The total force, however, varies very irregularly from point to point.

During the twenty years prior to 1860, the horizontal component of the earth's force in the eastern United States was slowly decreasing, the annual rate being about one thousandth part of the whole. About this year it reached its minimum limit, and has since been steadily increasing. The total force, on the other hand, has been continually decreasing from the time of the earliest observations. The value of the annual change of the horizontal force in C. G. S. units, as given by Schott in 1885, is at Boston + ·00031, at Philadelphia + ·00023, at Baltimore + ·00009, at Havana − ·00016, at San Francisco − ·00022, and in Washington State − ·00032. In consequence of this change of sign, from increasing to decreasing variation of the horizontal force, there is a belt where the annual change is zero. This belt about the epoch 1885 crossed the United States from St. Augustine, through Nashville, St. Louis, Cheyenne, and Butte City, to British Columbia.

596. Isodynamic Lines, Horizontal and Total.— Lines drawn through those places where the earth's force has the same value, are called **isodynamic** lines. Evidently the character of the isodynamic lines representing the horizontal force will be quite different from those which represent the total force. The former cross the United States in nearly an east and west direction, inclining to the north a little as they go westward. Thus the isodynamic line of ·1844 dyne (see Frontispiece) passes near New York, Cleveland, Ann Arbor, Chicago, Dubuque, Helena, and New Westminster, B. C. The isodynamic lines have therefore a general parallelism with the isoclinic lines. The isodynamic lines for the total force are exceedingly irregular, being greatly influenced by local causes apparently. Thus the line of ·5994 dyne passes through New Brunswick in a direction to the east of south, enters the Atlantic near Eastport, turns westward, crosses Cape Cod, turns south and enters the United States near the Virginia and North Carolina line,

touches the coast again near Wilmington, then turns sharply to the west and north, passing near Charlotte, Chattanooga, Springfield, Mo., and Denver, and entering British Columbia through Idaho and Washington. The line of ·6225 dyne passes E.S.E. through the northern part of Lake Superior and Lake Huron, makes a closed loop in the Province of Ontario, reaching its eastern limit near Ottawa, then passes W.S.W. through the southern end of Lake Huron, turns east again through Lake Erie, then west through northern Ohio, Indiana, and Illinois, and passing through Iowa, Nebraska, and Wyoming, enters British Columbia through Montana.

597. Automatic Records.—In magnetic observations the variations of short period, such as the annual and diurnal changes as well as the disturbances, are registered by means of photography on self-recording instruments. The needle carries a mirror which reflects a beam of light on a strip of sensitive paper moving over a cylinder driven by clockwork. The curves described are those of the declination and of the horizontal and vertical components of the intensity.

G.—THEORIES OF THE EARTH'S MAGNETISM.

598. Theory of Gilbert.—The first precise theory of the earth's magnetism is to be found in the remarkable book of Gilbert, published in 1600, and entitled "De Magnete." A careful study of the terrestrial magnetic phenomena known to him led him to enunciate his theory in the following words: "Magnus magnes ipse est globus terrestris"; i.e., "the earth itself is a great magnet." He assumed that only the solid portions of the earth are magnetic, and thus accounted for changes in the direction of the needle.

599. Subsequent Theories.—Bond, in 1676, supposed the earth to be encompassed by a magnetic sphere, having an axis inclined 8° 30′ to that of the earth, and revolving somewhat more slowly than the earth. Halley in 1692 supposed the earth to have four magnetic poles, two belonging to an outer magnetic shell and two to an

inner one, of which two shells the earth is made up, variation being due to a difference in their rates of revolution. Biot in 1805 supposed the earth to contain a central magnet with its poles close together. Hansteen in 1819 adopted the hypothesis of four poles. He supposed that by the action of the sun or that of its satellites a planet may have magnetic axes developed in it, one more in number than it has moons. The slow motion of the resultant axis accounts for the secular, the earth's rotation for the annual, variation. Gauss, in 1833, however, made a remarkable mathematical investigation on the distribution of the earth's magnetism, and decided on two poles only. In his view the earth does not contain a single great magnet, but is made up of irregularly diffused magnetic elements, which, taken together, have a remote resemblance to the condition of an ordinary magnet. The application of Gauss's formula required the determination of 24 numerical coefficients. He computed the value of the inclination, the horizontal force, and the total intensity for 91 places on the earth's surface, and found the results to agree remarkably with those observed. In 1820 Oersted had observed the production of magnetism by means of an electric current; and in 1822 Seebeck had discovered thermo-electricity. Combining these two discoveries, Grover in 1849 proposed the theory that the magnetism of the earth is due to electrical currents circulating around it, produced by the action of the sun and modified by the earth's motion; a theory which is now generally adopted.

600. Constants of Terrestrial Magnetism.—The phenomena of terrestrial magnetism may be roughly represented by an indefinitely short magnet placed within the earth, and inclined about 15° to the earth's axis of rotation. The lines of the field, as shown in the figure (Fig. 295), are symmetrical with reference to PP', the earth's magnetic axis, and the line EE' perpendicular to this axis represents the magnetic equator. Magnetic parallels may be drawn, on each of which the magnitude and direction of the force is constant. These are there-

fore isomagnetic lines. The inclination at any latitude ϕ is given by the expression $\tan I = 2 \tan \phi$; and the total force by the formula $T^2 = T_e^2 (1 + 3 \sin^2 \phi)$, in which T_e is the total force at the equator. Evidently every great circle through P and P' is a magnetic meridian; so that the angle at which these meridians intercept the geographical meridian of a place is its

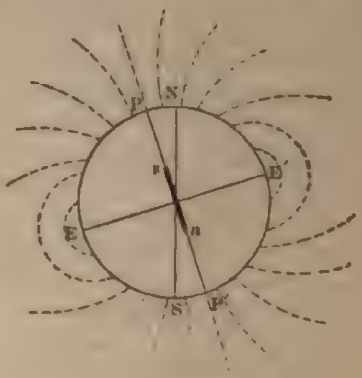


FIG. 295

declination. One of the geographical meridians passes through the magnetic pole and coincides with a magnetic meridian. Evidently for points on this combined meridian the declination is zero, and this meridian constitutes an agonic line.

EXPERIMENTS.—1. Magnetize a steel disk by placing it in a strong field with a diameter parallel to the lines of force. Place the disk thus magnetized in the vertical lantern, and over it place a glass plate on which soft iron filings have been sifted. On slightly jarring the plate, the filings will arrange themselves along the lines of force, forming a magnetic phantom representing in miniature that of a section of the earth.

2. Suspend a minute magnetic needle by a fiber of silk, and carry it about through the field thus shown. Opposite the poles the axis of the needle is radial, and here the inclination is 90°. Opposite the equator the axis is tangential, and the inclination is 0°. At intermediate points the dip varies according to the formula above given.

601. Magnetic Moment of the Earth.—Upon the hypothesis of Gauss above given, the earth's magnetic

moment may be calculated. Since the intensity of magnetization is the ratio of the magnetic moment to the volume, we have $M = IV = T_e R^3$. So that calling T_e or the total force at the equator 0·33, and $R^3 = (8 \times 10^{27})/\pi^3$, we find for the earth's magnetic moment $8{\cdot}5 \times 10^{25}$ C. G. S. units. Consequently we have for the intensity of magnetization of the earth

$$I = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{T_e R^3}{\frac{4}{3}\pi R^3} = \frac{3T_e}{4\pi} = \frac{0{\cdot}99}{12{\cdot}56} = 0{\cdot}079;$$

or about one three-thousandth of that of an ordinary steel magnet. It results from the calculations of Gauss that the total magnetic action of the earth is the same as that which would result if in each cubic meter of its volume there were contained nine magnets of steel each of a mass of half a kilogram, and each saturated.

CHAPTER IV.

ENERGY OF ÆTHER-FLOW.—ELECTRO-KINETICS.

SECTION I.—TRANSFERENCE OF ELECTRIFICATION.

A.—ELECTRIC CURRENTS.

602. Electric Flow.—Whenever two points at different electric potentials are connected by a conductor, a transference of electrification takes place through this conductor, producing an electric current. Since the transference is assumed to take place from the point of higher to the point of lower potential, the direction of the current is the direction in which positive electrification is transferred. This current continues until both ends of the conductor have the same potential. It is therefore temporary if the quantity of electrification is limited; as when, for example, a Leyden jar is discharged. But if by any device, such as a voltaic cell, the difference of potential be maintained constant, the flow of current is continuous, and is the same through every cross-section of the circuit.

603. Speed of Transference.—When electrification is transferred to a conductor, the first result attained is the charging of that conductor; i.e., the raising it to a potential equal to that of the source. But the quantity which must be transferred to raise the conductor to this potential is a function of the capacity of this conductor. So that the time of the appearance of a charge at the remote end of a linear conductor, such as a wire, will depend not only upon the length but also upon the

capacity of this conductor. Moreover, a minimum of charge will appear at the remote end in a minimum of time; and this charge will gradually increase until the wire is fully charged. The time which has elapsed since the flow began, and which represents the speed of transference of the electrification, is therefore variable, since it is dependent upon the delicacy of the apparatus used to detect the electrification at the remote end of the wire. Evidently, therefore, the speed of electrical transference along a wire can have no constant value; since it varies, not only with the capacity of the conductor, but also with the delicacy of the devices employed for detecting the arrival of the potential difference at its end.

604. Current-strength.—Ohm's Law.—The quantity of electrification which is transferred depends obviously upon the time t during which the current flows; so that, for example, calling Q the quantity transferred measured in coulombs, and I the current measured in amperes, we have

$$Q = It. \qquad [70]$$

Moreover, since the current-strength is measured by the quantity transferred in unit time, and since this quantity is the greater, the greater the difference of potential between the ends of the conductor, I will evidently be directly proportional to $V - V'$, or E, the difference of potential. Again, experiment shows that the current is also dependent upon the dimensions and material of the conducting wire. Since no known material conducts perfectly, all conductors are said to offer **resistance** to the flow of electrification; this resistance being the reciprocal of the conductivity. Hence the current-strength, which varies directly with the conductivity of the wire, will vary inversely as its resistance. The current-strength in any conducting circuit, then, is a function of two variables, and of two only; the difference of potential directly and the resistance inversely; whence we have

$$I = E/R, \qquad [71]$$

in which R represents the resistance of the circuit. This relation was first enunciated by Ohm in 1827, and is known as **Ohm's law**. It may be stated as follows:

The current-strength in any circuit is directly proportional to the sum of the electromotive forces in the circuit and inversely proportional to the sum of the resistances.

605. Resistance. — Resistance appears to be the agency by which the electrical energy of a circuit is transformed into other forms of energy; but of its nature we know nothing. It can be defined only in terms of the difference of potential and the current-strength, and is the ratio between them, since $R = E/I$. In a conducting wire it varies directly with the length of this wire and inversely with its cross-section. It also varies with the material of which the conductor is made; so that if we call the absolute resistance of a wire of unit length and unit section, made of a particular material, the **specific resistance** of that material, we may say that the actual resistance varies directly as the specific resistance of the material of the conductor. Calling l the length of the conductor, s its section, and ρ the specific resistance of the material, we have

$$R = \rho l/s,$$

which, since $s = \pi r^2$ or $\frac{1}{4}\pi d^2$, becomes $R = \rho l/\pi r^2$ or $4\rho l/\pi d^2$. Resistance is measured practically in ohms, and specific resistance in microhms.

606. Electrokinetic Units.—Current-strength may be measured in units based upon the electrostatic units already considered (469, 482, 507). Thus the electrokinetic unit of current may be defined by the equation $I = Q/t$ as the number of absolute electrostatic units of quantity transferred in unit of time; and, from Ohm's law, $E = IR$, since $E = V - V'$, the electrostatic difference of potential, we may define the electrokinetic unit of resistance as the resistance of a conductor through which one electrokinetic unit of cur-

rent flows when one unit of electrostatic potential-difference is maintained between its ends. The dimensions of electrostatic difference of potential are $M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$ (507); and the dimensions of electrokinetic current-strength are $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$. Consequently the ratio of these quantities, which represents the dimensions of electrokinetic resistance, is T/L. The reciprocal of this is L/T; and this, which represents electrokinetic conductivity, is a speed.

In general, however, another absolute system of units is employed, based upon the magnetic effects of the current, and hence called an electromagnetic system of units. These will be considered in connection with electromagnetism (669).

607. Applications of Ohm's Law.—The law of Ohm may be applied to calculate the current-strength, not only in a simple circuit, but also in the conductors of a complex one. Two general modes of arranging electrical devices in a circuit are in use: in the first these devices are arranged consecutively one after the other (Fig. 296),

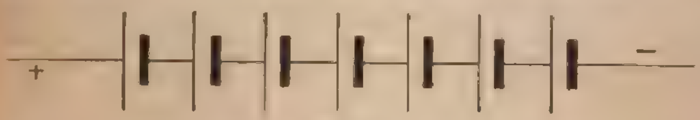


FIG. 296.

the current flowing successively through each of them, —*A*—*B*—*C*—*D*—; this is called an arrangement in **series**. In the second the devices are arranged side by

FIG. 297.

side (Fig. 297) or parallel to each other, the current flowing simultaneously through them; this is called an arrangement in **multiple**. Obviously these two methods

may be combined to constitute an arrangement in **multiple series**; as in Fig. 298 for example.

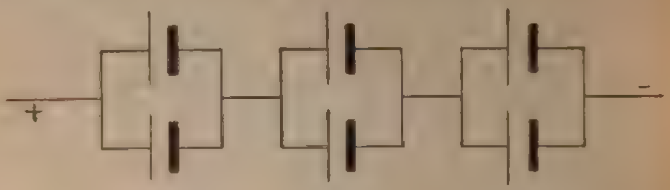


FIG. 298.

608. Series Arrangements. — Suppose several electrical generators—meaning by this term any device for maintaining a constant potential difference—to be connected in series; i.e., the negative terminal of one to the positive terminal in the next as in Figure 296.* Each generator develops a certain potential difference; and since these are added together in connecting the generators in series, the difference of potential of the whole, calling e that developed by one generator, is equal to Σe; or equal to ne if n equal generators be used. Moreover, each generator has a resistance r, depending upon the materials and mode of construction; whence the total resistance of the set will be Σr, or nr if n equal generators be employed. Hence calling R_i the total internal resistance, $R_i = \Sigma r$ or nr. If now we call R_e the resistance of the conducting wire used to connect the terminals of the first and last generators, i.e., the total external resistance, we have by Ohm's law:

$$I = \frac{\Sigma e}{\Sigma r + R_e} = \frac{ne}{nr + R_e} = \frac{E}{R_i + R_e}.$$

But in the circuit external to the generators we may have various devices; such as a galvanometer, a lamp, a motor, etc. So that the total resistance of the external

* The conventional mode of representing such generators is shown in the figure; the short thick lines indicating the negative side and the long fine ones the positive.

circuit R_e will be $w+g+l+m$; calling w, g, l, m the resistances of the wires, galvanometer, lamp, and motor. Whence

$$I=\frac{\Sigma e}{\Sigma r+w+g+l+m}=\frac{ne}{nr+w+g+l+m}=\frac{E}{R_i+R_e}. \qquad [72]$$

609. Multiple Arrangements.—Suppose a single generator to have its terminals connected by several wires in multiple. Evidently the current will divide into as many portions as there are wires. Moreover, the conductivity of the entire set of wires will be the sum of the separate conductivities; so that $C=c+c_{,}+c_{,,}+c_{,,,}+\ldots c_n$. But since $c=1/r$, this may be written $1/R=1/r+1/r_{,}+1/r_{,,}+1/r_{,,,}+\ldots 1/r_n$. Whence

$$\frac{1}{R}=\frac{r_{,}r_{,,}r_{,,,}+rr_{,,}r_{,,,}+rr_{,}r_{,,,}+rr_{,}r_{,,}}{rr_{,}r_{,,}r_{,,,}};$$

and $$R=\frac{rr_{,}r_{,,}r_{,,,}}{r_{,}r_{,,}r_{,,,}+rr_{,,}r_{,,,}+rr_{,}r_{,,,}+rr_{,}r_{,,}}.$$ Or the resistance of a number of conductors arranged in parallel or multiple, is the product of all the resistances divided by the sum of the products of the resistance of each conductor into all the others except one. Obviously if these resistances are all equal, $R=r^4/4r^3$ or $\frac{1}{4}r$; and hence the total resistance of four equal parallel conductors is one fourth of the resistance of any single conductor. If several equal generators be arranged in multiple as shown in Figure 297, the difference of potential E of the group will be that of a single generator only, e; but the resistance of the group R_i will be proportional to the reciprocal of the number of generators. In the same way the total resistance R_e of a number of equal devices, lamps for example, in the external circuit will be proportional to the reciprocal of the number of such devices. So that if n generators in multiple each of resistance r be connected to an external circuit containing m devices in multiple each of resistance $r_{,}$, we shall

have $R_i = r/n$ and $R_e = r_1/m$. Whence by the law of Ohm

$$I = \frac{E}{\frac{r}{n} + \frac{r_1}{m}} = \frac{mnE}{mr + nr_1}. \qquad [73]$$

610. Multiple-series Arrangements. — When both methods of arranging circuits are simultaneously employed, the formulas above given may be combined. Given N generators, arranged m in series and n in multiple, we have $mn = N$. So that if R_e be the external resistance, $R_i = mr/n$ and the law of Ohm gives us

$$I = \frac{me}{\frac{mr}{n} + R_e} \quad \text{or} \quad \frac{Ne}{mr + nR_e}.$$

If S devices each of resistance r_1 be arranged in multiple series in the external circuit, s devices being in series and t devices in multiple, $S = st$ and $R_e = sr_1/t$; and the current-strength will be

$$I = \frac{me}{R_i + R_e} = \frac{me}{\frac{mr}{n} + \frac{sr_1}{t}} = \frac{Nte}{mrt + nsr_1}. \qquad [74]$$

611. Branched Circuits.—Shunts. — The most frequent case in practice is where only two multiple circuits are employed but of different resistances. Either of these branched circuits is said to be a **shunt** circuit to the other. Thus, for example, if a galvanometer G and a coil S are placed on two parallel circuits (Fig. 299), the coil will act as a shunt to the galvanometer and will reduce the current flowing through it in proportion as its resistance is less in comparison. As in this case R, the total resistance between a and b, is equal to $GS/(G + S)$, we have, calling I' the total current flowing and supposing the potential difference E between a

and b to be constant, $I' = E/R = E/(GS/(G+S)) = E(G+S)/GS$. Since if the galvanometer alone were in the circuit, $I = E/G$, we have $I : I' :: 1 : (G+S)/S$; i.e., $I' = I(G+S)/S$; whence the fraction $(G+S)/S$ is called the multiplying power of the shunt, since it

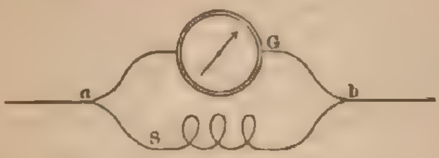


FIG. 299.

increases the total current in this ratio. Moreover, if the galvanometer alone were in circuit, the current $I = E/G$; and with the shunt alone $I'' = E/S$; whence $I'' : I :: G : S$. Now since the current I' when both are in circuit is equal to $I + I''$, i.e., to $1/G + 1/S$ or $(G+S)/GS$, it is evident that in this case this current must divide between the two circuits in the inverse ratio of their resistances. Of the total current in the circuit, then, the fraction $S/(G+S)$ will flow through the galvanometer and the fraction $G/(G+S)$ through the shunt. So that if the galvanometer have nine times the resistance of the shunt, one tenth of the current will flow through the galvanometer and nine tenths through the shunt.

612. Kirchhoff's Laws.—Two laws have been derived by Kirchhoff from Ohm's law which are frequently

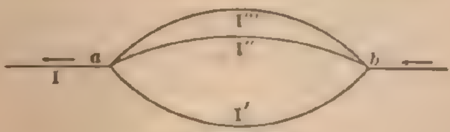


FIG. 300.

useful. Suppose a current flowing from b to a (Fig. 300) through the three branched conductors. Since at the

point a as we have just seen, $I = I' + I'' + I'''$, it follows that

$$I' + I'' + I''' - I = 0. \qquad [75]$$

Whence the first law:

I. The algebraic sum of the currents which meet at a point, in any arrangement of circuits, is always zero. This result is evident, since if the quantity brought to the point a by the branch conductors I', I'' and I''' be not equal to the quantity I carried away by the main conductor, there will be an accumulation of electrification at this point and consequently a rise of potential there; which is contrary to the hypothesis that the potential difference between a and b is maintained constant.

In the second case, suppose a closed circuit, a, b, c, d, e, f (Fig. 301), made up of six conductors. The current i_1 in ab will be $(V_a - V_b)/r_1$; whence $i_1r_1 = V_a - V_b$. In bc, $i_2r_2 = V_b - V_c$; and so on until in fa, $i_6r_6 = V_f - V_a$. On adding these equations together, the differences of potential disappear and we have

FIG. 301.

$$i_1r_1 + i_2r_2 + i_3r_3 + i_4r_4 + i_5r_5 + i_6r_6 = 0: \text{ i.e., } \Sigma ir = 0. \qquad [76]$$

Whence the second law of Kirchhoff:

II. In a closed circuit composed of several conductors arranged in series, the sum of the products obtained by multiplying the resistance of each conductor by the current flowing through it, is equal to zero.

In case the circuit contains electromotive forces the second law requires some modification. Here the differences of potential on summation do not vanish, but have a positive value. Hence the sum of ir is not zero, but has a value equal to the sum of E; or $\Sigma E = \Sigma ir$.

Under these conditions, the sum of the products of the resistance and the current-strength in each of the conductors, is equal to the sum of the electromotive forces in the circuit.

613. Fall of Potential in a Circuit.—If Ohm's law be plotted, making, in the formula $E = IR$, the ordinates differences of potential and the abscissas resistances, the result will be a straight line, inclined to the axis of abscissas by an angle whose tangent is represented by the current-strength.

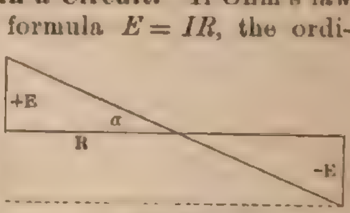


FIG. 302.

But by Ohm's law, I is an inverse function of R. Hence if E be constant, the angle will vary with R, decreasing as the resistance increases. Thus the fall of potential in an external circuit supposed homogeneous may be as represented in Figure 302, where $E/R = \tan \alpha = I$. While if we consider also the internal circuit (Fig. 303), of resistance R_i, we have the fall of potential in the generator $E' - E = IR_i$.

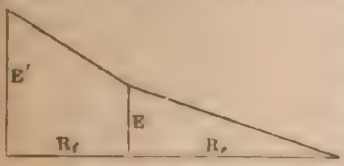


FIG. 303.

Since, however, the current is the same in all parts of the circuit, $E/R_e = (E' - E)/R_i$; whence $E : E' - E :: R_e : R_i$, and also $E' : E :: R_e + R_i : R_e$; as is evident from the figure. In general, if there be any closed circuit through which a current is flowing, the fall of potential in any portion of this circuit is proportional to the resistance of that part of the circuit. Or calling e the fall of potential $E' - E$, in the diagram above, $e : E :: R_i : R_e$. For the internal circuit we have $e = IR_i$, and for the external circuit $E = IR_e$. Whence $E + e$ or $E' = I(R_e + R_i)$; the result given by Ohm's law for the entire circuit.

614. Law of Maximum Current.—It is often desirable so to arrange a given number of generators that they will give the maximum current with a given external resistance. The laws of multiple-series circuits,

taken in connection with the law of Ohm, enable this to be done. Let N be the total number of generators, let them be arranged m in series and n in multiple, and let R_e be the given external resistance, r being the resistance of each generator. We have shown above (610) that the current obtained with the above arrangement of generators is

$$I = \frac{Ne}{mr + nR_e} = \frac{e}{\frac{r}{n} + \frac{R_e}{m}}.$$

Evidently when the quantity $r/n + R_e/m$ is a minimum, the current-strength I will be a maximum. When the product of two quantities is constant, their sum is least when they are equal. Now the product of r/n and R_e/m is rR_e/nm or rR_e/N. Here the numerator is constant since it is the product of the resistance of a single given cell by the given external resistance; and the denominator is also constant, since it is the given number of cells to be employed. Hence the sum $r/n + R_e/m$ is least when $r/n = R_e/m$. Multiplying both members by m, we have $mr/n = R_e$; but mr/n is the resistance of the N generators combined in n series of m each. Hence the rule that to obtain the maximum current with a given external resistance, the generators should be arranged so that their combined resistance equals that of the given external resistance.

615. Energy of the Current.—The potential energy of a quantity of electrification, Q coulombs for example, raised to a potential of V volts is QV volt-coulombs or joules. The energy contained in a condenser of capacity C farads charged to a potential V volts is $\frac{1}{2}CV^2$ joules, or since $C = Q/V$, is $\frac{1}{2}QV$ joules. When, therefore, a quantity Q flows through a wire from the potential V to V', its loss of energy is $QV - QV'$ or $Q(V - V')$; and this energy is expended in work. Whence $W = Q(V - V') = I(V - V')t$. If Q and V are expressed in electrostatic C. G. S. units, the work done will be obtained in ergs.

616. Law of Joule.—The experimental establishment of the energy of an electric current is due to Joule. The work W done by a current I, in the time t, according to the law of Joule, is equal to the product of the current-strength I, the time t, and the fall of potential E; or

$$W = (V - V')\,It = EIt. \qquad [77]$$

Since $E = IR$ by Ohm's law, we have $W = I^2Rt$; or if we substitute the value of $I = E/R$, we get $W = E^2t/R$; all of which are equivalent expressions. Since $I = Q/t$, the equation $W_s = EI = I^2R = E^2/R$ evidently expresses the work in ergs which is done by the current in each second; i.e., the rate of work or the activity. If E be measured in volts, R in ohms, and I in amperes, W will be expressed in joules per second or watts.

617. Law of Maximum Efficiency.—Since the work which is done in any circuit or part of a circuit is proportional to the fall of potential in that circuit or part of a circuit, and since the fall of potential is itself proportional to the resistance, it is evident that the work done in the different parts of a circuit is proportional to the resistances of those parts. Thus if in a given circuit the internal resistance is R_i and the external resistance R_e, the work per second done by the current in the internal circuit will be I^2R_i and in the external circuit I^2R_e; the whole work being of course $I^2(R_i + R_e)$. If E' is the potential difference of the generator on open circuit, $E' = I(R_i + R_e)$. And if the fall of potential in the generator be e and that on the external circuit be E, evidently $e = IR_i$ and also $E = IR_e$; and since $E' = E + e$, $E' = I(R_i + R_e)$ as before. In this case I^2R_i represents the energy expended per second in the generator and therefore uselessly; while I^2R_e represents the energy expended in the external circuit and therefore available for useful purposes. We may indicate by W_T the total work of the circuit, by W_U the useful work done by the generator, and by W_N the non-available work. In which case we have $W_T = I^2(R_i + R_e)$ or $E'I$, $W_U = I^2R_e$ or EI,

and $W_N = I^2R_i$ or eI. The efficiency of the arrangement is the ratio of the useful work to the total energy expended; or W_U/W_T. But this ratio is equal to $EI/E'I$ or E/E'; i.e., to $I^2R_e/I^2(R_i + R_e)$ or $R_e/(R_i + R_e)$. However, then, the total yield may itself vary, the efficiency may be increased by increasing still more rapidly the useful work. Now it is easily shown that if E' and R_i be constant, W_U is a maximum when $R_e = R_i$ (614). Under this condition the efficiency is equal to $R_e/2R_e$ or to 0·5; and the maximum useful work is one half the total work. As $W_U = \frac{1}{2} W_T = \frac{1}{2}E'I$, and $I = E'/2R_i$, we have for the maximum useful work the value $E'^2/4R_i$. It should be observed, however, that this condition of maximum work is not the condition of maximum efficiency. Evidently, the efficiency will be the greater as W_U approaches more nearly the value W_T. But as $W_U/W_T = E/E' = R_e/(R_i + R_e)$, it is clear that the efficiency will be the greater in proportion as the drop in potential in the generator is smaller, and therefore as E approaches E'; or in other words, in proportion as the resistance of the generator is smaller, and so the value $R_e/(R_i + R_e)$ approaches unity. Thus, for example, if $R_i = \frac{1}{9}R_e$, the ratio $R_e/(R_i + R_e)$ is 9 : 10 and the efficiency is 90 per cent. This ratio is frequently called the **economic coefficient** of the system and is denoted by n. In the above example the total work in the circuit is only one fifth of that done when the external and internal resistances are both equal to unity. But the useful work done is more than one third of the maximum; and hence the efficiency increases from fifty to ninety per cent. Evidently, therefore, in theory if the internal resistance were zero, the ratio $R_e/(R_i + R_e)$ would become unity and the efficiency would reach 100 per cent, its theoretical maximum.

618. Variable Period.—When one terminal of a generator is connected to a long insulated linear conductor, such as a telegraph wire, a current will flow into the conductor until its potential is raised to that of the terminal; the amount of charge being directly propor-

tional to the capacity of the conductor, and the time of charge inversely proportional to the current-strength, and therefore directly proportional to the resistance of the conductor. Conversely, if the end of the conductor be removed from the generator and connected to the ground, there will be a flow of current from the conductor to earth, until its potential is zero and a quantity of electrification equal to its charge has flowed to earth in the form of a current. If, however, the remote end of the wire, as well as the second terminal of the generator, be put to earth, then the potential will rise until it equals that of the first generator-terminal, only at the near end, and will fall toward the remote end, where it will be zero. When communication is first established, a wave of current traverses the wire, the increase of potential at any point being represented by the ordinates of a curve called the **arrival-curve**, the abscissas of which represent times. The time elapsing, after communication with the generator is established until the flow is constant, is called the **variable period**. It depends not only upon the potential difference at the generator-terminals, but also upon the resistance and electrostatic capacity of the conductor. And since both these quantities vary as the length, the time of charge varies as the square of the length. Thus in Wheatstone's experiment, the difference of potential necessary to produce a spark appeared at the end of a wire 365 meters long in somewhat less than a millionth of a second. But it does not follow that it would traverse a wire a million times as long in an entire second, as he supposed. In fact, since $l \propto \sqrt{t}$, it would travel only one thousand times as far in one second as in one millionth of a second, for the same potential difference and in a wire of the same material; which is less than 400 kilometers. The wave-front reaches the point a of the wire OR (Fig. 304) in the time t, and the points b and R in the times t' and t''. But when it has just reached b, it has a value $a1$ at a; and reaches the values $a2$ and $a3$ only after the wave-front has reached the end R or has attained the permanent state. If, however, the

contact with the generator be an instantaneous one, the condition of things is as shown in the second diagram

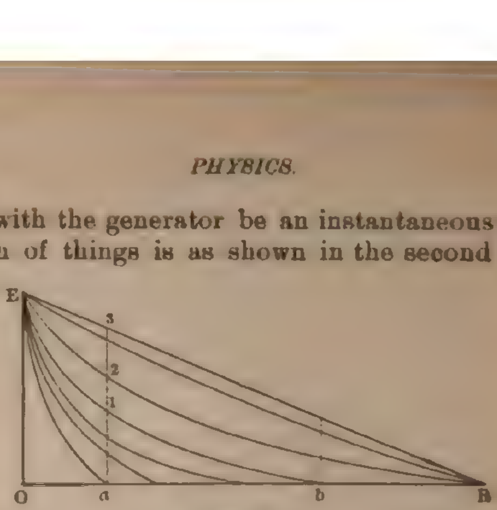


FIG. 304.

(Fig. 305). The potential rises from O to E at once; but as the supply of charge is cut off, it falls to 3 as the wave travels to b, and to 2 and 1 as it reaches c and B.

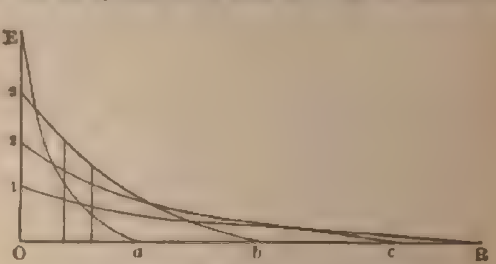


FIG. 305.

So that at every point on the conductor there is a rise of potential as the charge approaches it, and a fall as it recedes. A more abrupt wave results from a reversal of the electrification, such as is produced by connecting the positive and negative terminals of the generator to the conductor alternately. In application of these principles it is evident that the retardation of signals will be greater in a telegraph cable than in a similar air-conductor, since the cable being a condenser has a greater electrostatic capacity. The Atlantic cable, for example, shows a hundredth part of the maximum potential-value at the remote end, in about one fifth of a second; while it requires 108 seconds to attain 0·9 of its maximum, and would require an infinite time to attain this maximum itself. The signals in such a cable are alternately positive

and negative; and to increase their sharpness condensers of large capacity are employed at each of its ends.

SECTION II.—TRANSFORMATION OF ELECTROKINETIC ENERGY.

A.—THERMAL RELATIONS OF THE CURRENT.

(*a*) *Production of Heat from the Current.*

619. Conversion of Electric Current-energy into Heat-energy.—Whenever an electric current traverses a conductor, the entire energy of this current is transformed into some other kind of energy. This may be heat and light, the energy of chemical separation or the energy of mechanical motion. If none of the electric energy be expended in effecting chemical changes or in developing mechanical motion, the whole of it must evidently appear in the circuit as heat. It appears to be solely through the agency of what is termed resistance in the circuit that this transformation is effected; the amount of heat developed, other things being equal, being directly proportional to the resistance. From the law of Joule (616) the energy expended in one second in a circuit of resistance R through which a current of strength I is flowing, is the product of the resistance by the square of the current-strength, i.e., $W = I^2R$. But since the work in absolute units or ergs corresponding to one unit of heat (water-gram-degree) is $4{\cdot}2 \times 10^7$, or J (335), it follows that $H = W/J = I^2R/J$ in water-gram-degrees per second.

620. Distribution of Heat in the Circuit.—Moreover, the distribution of heat in the various parts of a circuit is proportional to the resistances of those parts. If R_i be the internal resistance and R_e the external resistance, I^2R_i/J and I^2R_e/J will be the heat-units developed in the two portions. And so if R_e consist of two parts r and r_1, in series, the heat produced in each will be I^2r/J and I^2r_1/J. If r_1 has 100 times the resistance of r, 100 times as much heat will be developed in it for a given value of I. Evidently in these cases the

fall of potential is proportional to the resistances; and so the heat-energy, which varies as EI and therefore is proportional, for an equal current-strength, to the fall of potential, is also proportional to the resistances.

EXPERIMENTS.—1. Join end to end a number of pieces of fine wire of different diameters but of the same length, and pass a current through them, gradually increasing its strength. The smallest wire, since its resistance is greatest, will be most heated.

2. Join a number of pieces of wire of the same diameter and length but of different materials, such as iron and copper. The iron may be heated to bright redness by a suitable current, while the copper, owing to its less specific resistance, remains invisible.

3. Pass through a long and fine iron wire a current sufficient to bring it to dull redness, and then shorten the wire progressively. The current increases as the resistance diminishes, until finally the wire is melted.

4. Bring the wire to dull redness as in the last experiment, and then plunge a loop of it into melting ice. By thus cooling one portion its resistance is lessened, the current-strength is increased, and the un-immersed portion of the wire becomes incandescent.

621. Effect of Temperature on Resistance.—As the last experiment shows, the specific resistance of conductors is a function of the temperature. In general, this resistance increases as the temperature rises. This is the case with the metals. But, on the other hand, certain substances, and notably carbon, diminish in resistance when heated. The same fact has been observed by Weston, in a special alloy of ferro-manganese and copper. Calling R_t the resistance at $t°$ and R_0 that at 0°, the expression

$$R_t = R_0(1 + \alpha t) \qquad [78]$$

gives the relation between these resistances, α being the coefficient of resistance-change with temperature, or in brief the temperature-coefficient; i.e., the change in resistance for unit length of a conductor when heated from 0° to 1°. The equation is similar to that given for heat-expansion and the coefficient α is obtained by its transformation thus:

$$\alpha = (R_t - R_0)/R_0 t.$$

he variation of resistance with temperature is not orm, a third term must be added to the equation:

$$R_t = R_o(1 + \alpha t + \beta t^2).$$

coefficient for metals and their alloys in general ositive, though it is much smaller for alloys than

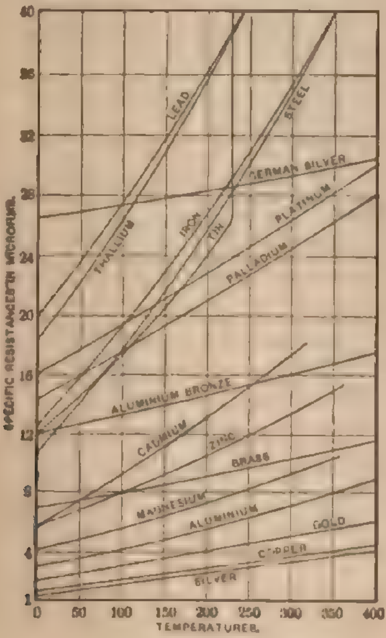


FIG. 306.

the simple metals; so that alloys are preferred for istance coils. Thus, for example, while silver has mperature-coefficient of 0·00380, copper of 0·00388, minum of 0·00390, platinum of 0·00247, iron of 0·00463, mercury of 0·00088, an alloy of 2 parts of gold and f silver has a coefficient of only 0·00065, an alloy platinum and 1 iridium a coefficient of 0·00133, of 2 platinum and 1 silver a coefficient of 0·00025, german-silver a coefficient of 0·00040. Platinoid, a

german-silver containing one or two per cent of tungsten, has a temperature-coefficient of only 0·00022. The table (Fig. 306) constructed by Benoit shows these results graphically. In it, the abscissas represent temperatures and the ordinates the specific resistances of the metals and alloys given, expressed in microhms. In general, the lines for alloys are straighter and their slope is less than in those for the pure metals. The coefficient of carbon, on the other hand, is negative and varies for different specimens from 0·0004 to 0·0005, so that the resistance of the filament of an incandescent lamp when lighted is only about half its resistance when cold. Nichols has shown that the resistance of the ferro-manganese-copper alloy above mentioned, when containing about 18 per cent of ferro-manganese, is practically independent of temperature; its coefficient being positive when the percentage is less than this and negative when it is greater.

As the temperature decreases, the variation in the specific resistance of the pure metals with temperature decreases also, the temperature-coefficient appearing to reach a point about $-123°$, where it has the same value for them all (Cailletet, Wroblewski). This result seems to confirm Clausius's statement that the specific resistance of a metallic conductor vanishes at the absolute zero.

622. Rise of Temperature in Conductors.—When a given amount of heat-energy is expended upon a conductor, the rise of temperature produced in it is a function of its specific heat, of its mass, and of its radiating surface. Were there no cooling by radiation, the temperature would continue to rise indefinitely. In fact it does continue to rise until the loss by radiation balances the heat received from the current. The heat produced by a current of I amperes in a wire of resistance R ohms is I^2R/J water-gram-degrees per second or is $I^2 \rho l/\pi r^2 J$, since $R = \rho l/\pi r^2$ (605). The heat lost by the conductor per second is proportional to $2\pi rl$, its surface, to ϵ, the heat radiated per second per unit of

surface per unit excess of temperature above the air, and to T, this excess of temperature; accepting Newton's law of cooling. Whence $I^2\rho l/\pi r^2 J = 2\pi r l \,.\, \epsilon \,.\, T$; and we have for the temperature of the conductor:

$$T = I^2\rho/2\pi^2 r^3 \epsilon J. \tag{79}$$

Hence the rise of temperature in a conductor is proportional directly to I^2, the square of the current-strength, and to ρ, the specific resistance; and inversely to the cube of the radius, r, and to ϵ, the emissive power. If I^2/r^3 be constant, i.e., if the radius of the conductor varies as the square root of the cube of the current-strength, the rise of temperature in the conductor will also be constant. In practice, a copper wire should not be made to carry more than 6 amperes per millimeter of cross-section if it is naked, or from 2 to 3 amperes if it is covered.

As to the value of ϵ, or the amount of radiation, experiment has shown that the loss from an unpolished copper surface per second is about 0·00025 of one water-gram-degree per square centimeter of surface per degree excess of temperature above the surrounding medium. Hence for every watt which it is required to dissipate, the conductor should have a surface of from 0·15 to 0·20 square centimeter.

EXAMPLE.—Required the limiting temperature of a copper wire 0·165 centimeter in diameter, through which a current of 10 amperes is flowing. Substituting in the above temperature-formula the corresponding values, ρ for copper being 1·65 microhms,

$$T = \frac{1^2 \times 1{\cdot}65 \times 10^3}{2 \times (3{\cdot}14)^2 \times ({\cdot}083)^3 \times {\cdot}00025 \times 4{\cdot}2 \times 10^7} = 13{\cdot}01^\circ.$$

Hence the temperature of the wire would rise until it reached 13·01° above that of the surrounding air, when it would remain constant.

623. Temperature of Fusion.—It is often desirable to calculate the current-strength required to bring a conductor of given dimensions and material to the fusing point. For this purpose it is necessary only to

substitute the value of this fusing point for T in the above equation.

EXAMPLE.—Required the diameter of a lead wire which will be just fused by a current of 7·2 amperes. Since for lead $T = 335°$ and $\rho = 19·85$ microhms, we have

$$r = \sqrt[3]{\frac{0·72^2 \times 19·85 \times 10^3}{2 \times (3·14)^2 \times ·00025 \times 4·2 \times 10^7 \times 335}} = 0·053 \text{ cm.}$$

And therefore the diameter is 0·053 × 2 or 0·106 centimeter.

If κ be the total radiation in watts per square centimeter, the total heat-loss from a wire of diameter d and length l will be $\pi dl\kappa$ watts. So that if the current-strength be I amperes and the resistance R ohms, the number of watts produced will be I^2R or $I^2(4\rho l/\pi d^2)$; since $R = \rho l/s$ and $s = \frac{1}{4}\pi d^2$. If the watts radiated equal the watts produced, the temperature is constant and $\pi dl\kappa = I^2(4\rho l/\pi d^2)$; whence $I^2 = d^3(\pi^2\kappa/4\rho)$ and $I = d^{\frac{3}{2}}\sqrt{\pi^2\kappa/4\rho}$. Calling a the constant quantity under the radical sign, $I = ad^{\frac{3}{2}}$ or $d = (I/a)^{\frac{2}{3}}$. Since κ is the radiation per square centimeter in joules per second or watts, and since ϵ is the water-gram-degrees or heat-units radiated per square centimeter per second for unit difference of temperature, the heat-units radiated for $T°$ difference of temperature per second will be $T\epsilon$. Moreover, since one water-gram-degree is equal to 4·2 joules or J, $T\epsilon$ water-gram-degrees per second will be equal to $T\epsilon J$ joules per second. Whence $T\epsilon J = \kappa$. If in the equation $I = ad^{\frac{3}{2}}$, d be made equal to unity, $I = a$; or a represents the current in amperes required to fuse a wire one centimeter in diameter. Preece has calculated the values of a from this formula for several metals and alloys with the results given in the following table:

FUSING CURRENT IN AMPERES.

Substance.	For d = 1 cm.	For d = 1 mm.
Copper	2530·0	80·0
Aluminum	1873·0	59·2
Platinum	1277·0	40·4

Substance.	For d = 1 cm.	For d = 1 mm.
German-silver........	1292·0	40·8
Platinoid............	1173·0	37·1
Iron.................	776·4	24·1
Tin..................	405·5	12·8
Lead.................	340·6	10·8
Alloy 2 lead and 1 tin.	325·5	10·3

hese values have great practical importance in electric ghting since it is found necessary to interpose a usible conductor termed a "cut-out" somewhere in the ircuit, of such dimensions that if subjected to a current trong enough to endanger the safety of the lamps or ther devices on that circuit, the conductor will melt nd thereby open the circuit. In using the above figres for this purpose, it should be remembered that if he fusible strip be a short one, the loss of heat by conluction to the clamps at its ends becomes material. Thus while a current of 11 amperes will fuse a lead wire f 1 millimeter diameter if it is ten centimeters long, it will require a current of nearly 18 amperes if it is only ne quarter as long.

624. Thermic Measuring-instruments.—Inasmuch s the total heat developed in a circuit is proportional o the square of the current-strength, it is evident that his total heat may be used to measure the currenttrength. This is called the **calorimetrical method** of neasuring currents. From Joule's equation $H = W/J = I^2Rt/J$, we have $I^2 = JH/Rt$. A coil of wire of nown resistance, say R ohms, is placed in a known mass, m grams, of a liquid of specific heat σ, at the temperaure $T°$. The current is allowed to flow through the wire for t seconds and the temperature $T_1°$ is again noted. The number of heat-units (water-gram-degrees) developed by the current, or H, is equal to $m(T_1 - T)\sigma$; and hence $I^2 = (Jm(T_1 - T)\sigma/Rt)$. Since this method determines I^2 it may be used either for direct or alternating currents. Corrections should be made for the materials of the calorimeter, for the change in the resist-

ance of the wire and the change in the specific heat of the liquid with temperature.

625. Expansion-meters.—The expansion of a conducting wire by the heat produced in it by the current has also been utilized for measuring currents; and since when R is constant, as it is when the meter is in shunt circuit with the main conductor, E is proportional to I, such meters are used for measuring differences of potential also. In the Cardew voltmeter, for example, the current traverses a long wire of platinum-silver, enclosed in a brass tube for protection; and the expansion of this wire by the heat developed, suitably amplified, is recorded by an index moving over a graduated dial, empirically graduated. Ayrton and Perry have modified this voltmeter and rendered it more sensitive; so that it indicates differences of 3 or 4 volts. Geyer has devised a registering amperemeter, consisting of a curved strip of german-silver and of an insulated wire which is fastened at one end to the strip and fixed at the other, like the string of a bow. When a current traverses the system, the wire is heated more than the strip, and by its greater expansion allows the strip to straighten and so, by means of a pencil at the junction, to trace a curve upon a moving paper. As in the former case, all these instruments indicate directly the value of I^2; and since this quantity is always positive whatever the sign of I, they may be used either for direct or for alternating currents.

626. Production of Light by the Current.—Whenever bodies are sufficiently heated they become luminous. The temperature at which substances emit light depends upon the character of the substance heated. Draper gives 525° and Weber 370° as the lowest temperature at which bodies become luminous. The increase of light with temperature is very rapid, a platinum wire heated by the current giving thirty-six times as much light at 1300° as at 1000° (Draper). Indeed the experiments of Violle show that if at 800° a platinum surface emits a light of 0·108, at 1000° the light is 1·82, at 1200° it is

17·8, at 1400°, 100, at 1600°, 327, and at 1775°, 587. So that the light emitted increases much more rapidly than the heat developed, being approximately proportional to the square of this heat. But inasmuch as the heat developed, in a lamp for example, is proportional to the square of the current-strength, it follows that the increase of light is proportional to the fourth power of the current-strength. In other words, by doubling the current supplied to a lamp it would emit sixteen times as much light. Since such a lamp supplied with current will rise in temperature until the heat produced and that radiated are equal, we have Stefan's law of radiation as follows:

The quantity of heat lost by radiation is proportional to the fourth power of the absolute temperature.

627. Electric Lighting.—Two general methods are in use for producing light by the action of an electric current. The older of these—first employed by Davy in 1809—is known as the **arc light**, and is produced by bringing the carbon terminals of a sufficiently powerful generator into contact and then separating them. The current continues to flow across the intervening gap, developing, in consequence of the great resistance there, a most intense heat which raises these terminals to vivid incandescence. If the carbons be horizontal, the upward current of air causes the luminous stream to assume an arched form; whence the name "electric arc."

In the second and more recent method, known as the **incandescence light**, the current is caused to traverse a continuous conductor of small section and of high resistance; the heat developed in this conductor being sufficient to raise it to a light-giving temperature.

628. The Electric Arc.—An examination of the ends of the carbons producing the arc light shows that the light is emitted from the solid carbons themselves, which are in a condition of vivid incandescence; the space between them being filled with ignited carbon vapor giving a bluish-purple light and having only a feeble radiating power. According to Rosetti's measurements, the

positive terminal is the hotter, its temperature being about 3900°; while that of the negative terminal is only 3150°. He gives for the value of the arc itself between the terminals, 4800°. With a direct current it is observed that the positive carbon, which is the brighter, is hollowed out in the form of a crater, while the negative carbon is pointed. If the experiment be made in vacuo, so that combustion is avoided, a transport of particles is observed to take place between the electrodes, passing from the positive to the negative carbon, and building the latter up at the expense of the former. Hence the positive carbon is consumed much more rapidly than the negative one and the position of the arc is continually changing.

To maintain an arc light requires a potential-difference between the carbons of about 50 volts; of which 30 volts measures the opposing or counter-electromotive force of the arc itself and 20 volts is the fall due to the resistance, which is variable with its length. For a light of 1000 candles the rate of work or activity EI is 50×15 or 750 watts; about three-fourths of a kilowatt.

629. Arc Lamps.—An electric arc lamp is a device for holding the carbons of an arc light and for maintaining the light constant by means of suitable mechanism. The latter function it is which has given the name **regulator** to such lamps. Two distinct operations must be performed by a regulator: first, it must separate the carbons to produce the light; and second, it must feed these carbons forward as fast as they are consumed, so as to preserve the light from extinction and to keep the length of the arc constant. Moreover, the lamp must be capable of operating independently, so as to be used in a circuit with other lamps without being affected by their variations. In case the lamp is to be used in the focus of a mirror or of a lens, as in lighthouse illumination,—in which case it is known as a focusing lamp,—it is required to perform still a third function; i.e., it must feed the carbons forward in the exact proportion

in which they are consumed, so as to preserve the arc constant in its position.

In commercial arc-lighting the lamps are ordinarily arranged in series; so that if 50 volts be the difference of potential required to maintain each one of them, 60 such lamps will require a potential-difference at the generator-terminals of 3000 volts. In this system, called **the series system** of lighting, the potential-difference is high and is variable with the number of lamps; while the current-strength is low—about 9 or 10 amperes—and is maintained constant. One of the most extensively used series lamps is the Brush lamp, in which a ring-clutch, surrounding the upper carbon-holder and controlled by an electromagnet in circuit with the carbons, lifts this holder as soon as the current passes, and thus separates the carbons to produce the light. Moreover, this magnet is antagonized by a second one in shunt circuit with it, for the purposes of regulation; since whenever the arc becomes too long and is in danger of extinction, the increased resistance thus developed throws more current into the antagonizing shunt magnet, enabling it to overcome the main magnet and so to allow the carbons to feed together. Since the result is due to the differential action of two magnets, one situated in the main and the other in a shunt circuit, such lamps are generally known as **differential lamps**.

One of the best focusing lamps in use is the lamp invented by Serrin (1859). The upper carbon is supported above the lower one by a holder bent twice at right angles; this holder by its weight operating a clockwork by means of a rack cut on its side. The lower carbon-holder is parallel with the first, and is raised by a chain operated by the clockwork, so that as the upper carbon descends the lower one rises to meet it, the gearing being so constructed that the lower or negative carbon rises one half as rapidly as the upper one falls; this being assumed as the ratio of consumption. An electromagnet in the main circuit acts on

an armature attached to an articulated parallelogram carrying the clockwork, so as to depress the lower carbon-holder which is attached to it, and at the same time to bring a detent into play to stop the motion of this clockwork. When not in use the carbons are in contact. On passing the current, the electromagnet draws down its armature with the attached parallelogram; thus establishing the arc at the carbon terminals and locking the clockwork. As the carbons burn away, the resistance of the arc increases; and the magnet losing its power, the antagonizing springs raise the parallelogram, releasing the clockwork and allowing the upper carbon to feed downward and the lower one upward in the ratio determined, of two to one. Lontin (1877) modified the lamp by putting the magnet in a shunt circuit and reversing its position. The carbons being separated when the lamp is not in action, the passage of the current affects the magnet only. Its armature is raised, the clockwork is released, and the carbons feed together. As soon as they touch, the shunt magnet loses its power and the parallelogram falls, carrying with it the lower carbon and establishing the arc; the shunt magnet regaining its power when the arc attains a length greater than that for which it is adjusted. The regulation is more perfect with this device than with that of Serrin.

630. Incandescence Lamps.—The first successful incandescence lamp was made by Edison in 1879. Previous experimenters had used continuous conductors of platinum and even carbon for this purpose. But Edison was the first to perceive clearly the conditions to be fulfilled in order to ensure success, not only as concerns the lamp itself, but also with regard to its economy when used upon a circuit. The incandescence lamp of Edison is shown in the figure (Fig. 307). The light-giving portion is a thread or filament of carbon, made by cutting a strip of bamboo to the proper dimensions and then carbonizing it at a very high temperature. This filament is then attached to conducting wires—which

are of platinum where they pass through the glass—and is enclosed in a pear-shaped globe of glass, which is then exhausted to a high degree. The resistance of the filament when hot is, for the 16-candle lamp designed to be used on a 110-volt circuit, about 240 ohms; so that the current required to maintain the light is about 0·46 of an ampere. Consequently the number of watts expended in the lamp is 50·6, and the efficiency of the lamp or the number of watts expended per candle is about 3·1. Inasmuch as the light emitted by an incandescence lamp increases so much more rapidly than the energy expended upon it, the economy is the greater the higher the temperature and the larger the number of watts expended. But this high temperature deteriorates the filament and shortens its time of service. So that efficiency must be sacrificed to some extent to durability. And the economy is a maximum when the cost of the increase in efficiency and that of the decrease in durability exactly balance each other.

Fig. [illegible]

In incandescence lighting the lamps are arranged generally in multiple upon the distributing circuits, the generator being of a kind suited to maintain a constant difference of potential between the two main conductors; this difference being that required by each single lamp, i.e., 110 volts in the Edison system. In comparison with the arc system in series, it is a very low pressure system, and therefore cannot give a dangerous shock when the wires are handled. The large volume of current required to feed so many lamps in multiple, necessitates the use of large mains; and this has led to the devising of extremely ingenious systems of distribution, one of the best of which is the three-wire system of Edison. In this system two generators are connected as in series, two mains lead from the extreme terminals, and a third main from the junction between the two generators. The lamps are placed between one of the two outer mains and the third

or middle one. When the same number of lamps is active on the two sides, the effect is the same as if the lamps of each pair are in series; and since this doubled resistance is met by the doubled potential difference of the two generators, the current through each lamp is the same, and they operate normally. Because the same current is now supplied at double the potential difference, it becomes possible to halve the cross-section of the mains, i.e., to make them of one fourth the diameter. In place of two conductors of unit diameter, therefore, there are in the three-wire system three mains, each of one fourth this diameter; or the ratio of mass in the conductors is $2 : \frac{3}{4}$ or $1 : \frac{3}{8}$; a saving of five eighths in the cost of the mains. If lamps are extinguished on one side, current flows along the third main to supply the excess required; and so each lamp is independent of all the rest.

(*b*) *Production of Current from Heat.*

631. Thermo-electrification.—While the conversion of electrical energy into heat is easy and complete, the conversion of heat into electrical energy is difficult and incomplete; the conditions being analogous to those controlling the relations of heat to mechanical energy. In 1821 Seebeck observed that, when one of the junctions of a circuit of two metals is heated, the potential-difference between these metals is increased at this heated junction; so that on closing the circuit a current may be obtained through it. This potential difference thus developed between two metals, is a function not only of the temperatures at the junctions, but also of the substances employed. It depends also upon the mean temperature of the junctions as well as upon the difference of temperature which exists between them. The potential-difference in a circuit of two metals at the mean temperature $t°$, when one junction is kept half a degree above $t°$ and the other half a degree below it, is called **the thermo-electric power** of these metals. The following

table gives approximately the thermo-electric powers for certain metals, the mean temperature being 20°. The potential-differences are given in microvolts per degree, and are referred to that of lead taken as zero (Jenkin):

THERMO ELECTRIC POTENTIAL-DIFFERENCE.

Bismuth	+ 97·0	Platinum	− 0·9
Cobalt	+ 22·0	Copper (pure)	− 3·8
German silver	+ 11·75	Antimony	− 6·0
Mercury	+ 0·418	Iron	− 17·5
Lead	0	Tellurium	− 502·0

EXAMPLES.—Thus, german-silver in this table is positive with regard to lead and iron is negative. Hence the current will flow through the heated junction from the german-silver to the iron; and if the difference of temperature between the junctions be 10°, the potential-difference between them, being the product of the thermo-electric power by this temperature-difference, will be 292·5 microvolts. The circuit being entirely metallic, its resistance is low; and therefore the current-strength in a thermo-electric circuit is quite considerable.

632. Thermo-electric Generators.—A thermo-electric generator is simply a transformer of heat-energy into electric energy; and consequently is subject to Carnot's law of efficiency. Even under the best conditions available in practice, however, the efficiency of this conversion is much less than the conversion into mechanical energy; since Rayleigh has shown that an iron-german-silver couple will transform into electric energy only one three-hundredth part of the heat that a perfect heat-engine, working between the same temperature-limits, will transform into mechanical energy. While this yield may and no doubt will be increased in the future, there seems no reason to believe that a thermo-couple will recover more heat in useful forms than does the steam-engine.

The earlier thermo-generators employed bismuth and antimony as the metals. The thermo-multiplier of Nobili and Melloni (1834), which was used, as we have already seen (364), for the detection of heat, was thus

constructed. Marcus (1865), and subsequently Farmer, used german-silver in combination with an alloy of antimony and zinc. Becquerel (1866) employed artificial copper sulphide as the positive element of the couple; finding that the potential difference between it and german-silver is ten times that between bismuth and copper. The thermo-generators in actual use at present are those of Noë and of Clamond. Both use a zinc-antimony alloy for the positive metal; but in the Noë generator the negative metal is german-silver, while in the Clamond it is iron. The alloy in the Noë generator is cast in the form of cylinders about 8 mm. in diameter and 22 mm. long, *a* (Fig. 308), the german-silver wire

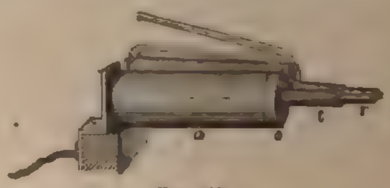


Fig. 308

being attached in the process of casting. A copper rod *r* cast in one end serves to conduct the heat to the junction. The couples are placed horizontally and arranged radially, their copper rods being in the center. By means of a gas or alcohol flame the inner ends of these copper rods are heated; and by means of an open cylinder of thin metal soldered to the outer end of the cylinder, the outer ends of the couples are kept cool by radiation. Each german-silver wire is soldered to the outer end of the next couple in order. The potential difference of each couple under working conditions is about 0·1 volt, and the internal resistance 0·025 ohm. In the Clamond generator, the alloy is cast in flat spindle-shaped plates *B* as shown in the figure (Fig. 309), the iron strips *L* being cast into the ends. These couples are then combined into a flat ring, the outer end of one couple being soldered to the inner terminal of the succeeding one. These rings are placed one above the other, separated by

rings of asbestos paper, and are connected together either in series or in multiple, as may be desired. In the cylindrical space enclosed by them is a gas-burner, *D*, made of an earthenware tube, pierced with holes. Air and gas enter at the bottom, and the flames in the annular space between the tube and the couples heat the inner ends of these couples, while their outer ends are cooled by radiation. A generator consisting of 120 couples arranged in series, and thus heated, gave a potential difference of 8 volts, and had an internal resistance of 3·2 ohms. Combined with an external circuit of equal resistance, electric energy was produced at the rate of 5 watts; the quantity of gas consumed being 180 cubic decimeters per hour. If we assume that one cubic decimeter of gas gives in burning 5200 water-gram-degrees, this expenditure of gas produces 260 water-gram-degrees per second, which is equivalent to 1084 watts. Hence the efficiency, or the ratio of the energy produced to the energy supplied, is as 5 : 1084 or less than $\frac{1}{200}$. To produce a kilowatt-hour would require, at the above rate, the consumption of 36 cubic meters of gas; costing, at six cents a cubic meter, the sum of about $2.16. Since the production of a kilowatt-hour by a steam-engine costs only about five or six cents at the most, the commercial use of existing thermo-generators as a source of electrical energy is practically prohibited.

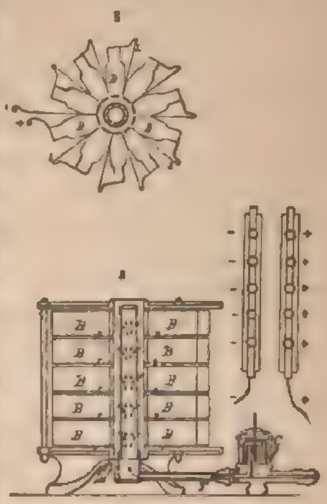


FIG. 309.

633. Pyromagnetic Generator.—Edison has made use of the fact that iron ceases to be magnetic at about 770° and nickel at 340°, for the purpose of effecting the conversion of heat-energy into electric energy. His

pyromagnetic generator consists in principle of a thin iron tube placed in a strong magnetic field and surrounded by a coil of wire. Under ordinary conditions this tube is of course magnetized. But on directing through it a hot blast and heating it to 770°, it loses its magnetism. This decrease of magnetism by heating the tube and the increase of magnetism by cooling it produces an electric current in the coil, the energy of which has the heat for its source. By arranging a series of such tubes in a circle, by providing a rotating screen to admit the hot blast into them in succession, and by suitably connecting the coils so as to assist each other, a continuous current may be obtained from the generator. A similar principle may obviously be utilized in the construction of a motor. This ingenious device, however, is evidently subject to the second law of thermodynamics, the experimental results thus far obtained with it not showing any gain in efficiency over a thermo-electric generator.

634. Phenomenon of Inversion.—Volta's law of contact (650) teaches us that no current flows through a circuit of dissimilar metals when their junctions are all at the same temperature. Whence it follows that the algebraic sum of the potential-differences developed at these junctions is zero. Seebeck's discovery that a current does flow in such a circuit when one of the junctions differs in temperature from the others, proves that the potential-difference of contact is a function of the temperature, so that now the algebraic sum of these differences is no longer equal to zero. Experiment shows that in general this potential-difference is directly proportional to the temperature; so that if for a given pair of metals e represents the potential difference at t_0, $e+\epsilon$ will represent this difference at t_0+1, $e+2\epsilon$ at t_0+2, etc., ϵ representing the increase of potential-difference for one degree.

EXAMPLE.—Thus at 18° the contact potential-difference between iron and copper is 146000 microvolts, the iron being positive to the copper. From the table above given, the thermo-electric power of

these metals at this temperature, i.e., the potential-difference for a temperature-difference of one degree, is — 13·7 microvolts. So that at 19° the potential-difference between iron and copper will be 146000 — 13·7 microvolts, at 20° 146000 — 27·4 microvolts, etc.

From this it follows that if a closed circuit be formed of an iron wire and a copper wire, and if one of the junctions be maintained at 20° and the other at 19°, the potential-difference at the former will be 13·7 microvolts greater than at the latter; and this potential-difference it is, which, due to the difference of temperature at the junctions, produces the thermo-electric current. This variation of the thermo-electric power with temperature, however, is not constant. Cumming (1823) observed that on increasing the mean temperature, even with a constant temperature-difference, the potential-difference between two junctions decreases, becomes zero, and finally increases in the other direction.

EXAMPLE.—Thus the thermo-electric power of copper and iron, which, as we have seen, is 13·7 microvolts at 19°, the copper being positive to the iron, disappears at 274·5°, their thermo-electric powers for this mean temperature being equal. At higher temperatures their relative position is reversed, the iron being now positive to the copper. If the temperature be 530° for one of the junctions, the other remaining at 19°, the higher temperature being as much above 274·5 as the lower one is below it, the potential-differences will be equal and opposite, and their algebraic sum will be zero. There will be no current.

The point of temperature at which a given pair of metals have the same thermo-electric power, and beyond which consequently their potential-differences are reversed in sign, is called the **neutral point**. The maximum potential-difference is obtained when one of the junctions is at this temperature and the other is as far away from it as possible. If the mean temperature be the temperature of the neutral point, no current whatever flows through the circuit. In order to calculate the potential-difference in any given case, therefore, it is necessary to know the temperature of the neutral point for the given metals.

635. Thermo-electric Diagram.—By means of a properly constructed thermo-electric diagram, therefore, all that is known about this subject may be represented. Such a diagram, first suggested by Lord Kelvin and afterward completed by Tait, is given in the figure (Fig. 310). In this diagram lead is taken as the standard

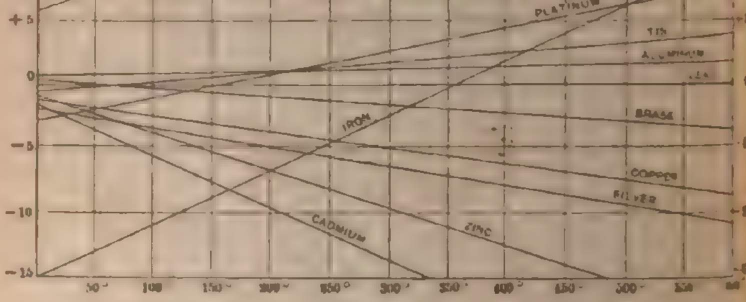


FIG 310.

metal, for a reason presently to be explained (637). The thermo-electric powers with respect to lead, given in microvolts, are represented as ordinates and the temperatures as abscissas. It will be observed that the variation of thermo-electric power with temperature, for each of the metals given, is represented by a straight line. And since the potential-difference is the product of the thermo-electric power for the mean temperature by the difference of temperatures, it is evident that this potential-difference for any two given metals is the area of the trapezoid enclosed between the given temperature ordinates and the variation-lines for the given metals. Where two metallic lines intersect, they have the same thermo-electric power; and the temperature corresponding is the neutral point. Evidently the areas of the triangles on the two sides of the neutral point will be equal if their bases are equal; i.e., if the temperatures

on opposite sides are equal. In other words, the potential-difference will be zero if the neutral point is the mean temperature.

Since the area which represents the potential-difference is proportional to κt, the product of the thermo-electric power κ and the temperature t, and since κ also varies as t, the potential-difference itself varies as t^2. So that the curve showing the variation of potential-difference with temperature is a parabola. The thermo-electric diagram of Gaugain is based upon this relation.

EXAMPLE.—It is required to calculate the potential-difference for a cadmium-palladium couple, the two junctions being maintained at 150° and 50°. The thermo-electric power of cadmium at 50° is − 4 microvolts as compared with lead. At 150° it is − 8 microvolts. That of palladium at 50° is + 7·5 microvolts and at 150° 10·7 microvolts. The thermo-electric power of the two metals at 50° is 7·5 − (− 4) or 11·5 microvolts; and at 150 it is 10·7 −(− 7 5) or 18·7 microvolts. The area of a trapezium is the product of the height into the mean value of the bases; i.e., is (18·7 + 11·5)/2 × 100 or 1510 microvolts. Again, in the diagram, the thermo-electric power for the mean temperature is evidently the mean value of these bases; which in the present case is 9·2 −(− 5·9) or 15·1. Whence the potential-difference for 100° is 15·1 × 100 or 1510 microvolts, as before.

More accurate results may be obtained, however, from a table of values. Inspection of the diagram shows that for a given metal the ratio of its thermo-electric power as compared with lead, to the distance measured from the neutral point to the mean given temperature, is the tangent of the angle which the line for this given metal makes with the axis of abscissas; i.e., the line for lead. Calling this angle α, we have $\tan \alpha =$ D. P. / T. D.; whence the potential-difference for 1°, or the thermo-electric power, is obtained by multiplying the temperature-difference between the neutral point with lead and the mean temperature by the tangent of the angle of inclination. Calling T_m the mean temperature and T_n the neutral point, $T_n - T_m$ will represent the temperature-difference and $(T_n - T_m) \tan \alpha$ will represent the potential-difference for 1° between the given metal and

lead. So if $(T_n' - T_m')\tan\alpha'$ be this difference for a second metal, the potential-difference between the first and second metal will be their algebraic sum; i.e., $(T_n - T_m)\tan\alpha - (T_n' - T_m')\tan\alpha'$. Again, these values may be tabulated by stating the thermo-electric power in microvolts at 0° with the mean temperature-correction added as a function of this temperature. The necessary constants are given in the following tables:

THERMO-ELECTRIC POWER IN MICROVOLTS. (TAIT.)

(At the Temperature t°.)

Iron	$+17{\cdot}34 - {\cdot}0487t$	Lead	0
Cadmium	$+2{\cdot}66 + {\cdot}0429t$	Tin	$-0{\cdot}43 + {\cdot}0055t$
Platinum	$+2{\cdot}60 - {\cdot}0075t$	Aluminum	$-0{\cdot}77 + {\cdot}0039t$
Zinc	$+2{\cdot}34 + {\cdot}0240t$	Palladium	$-6{\cdot}25 - {\cdot}0359t$
Silver	$+2{\cdot}14 + {\cdot}0150t$	German silver	$-12{\cdot}07 - {\cdot}0512t$
Copper	$+1{\cdot}36 + {\cdot}0095t$	Nickel	$-22{\cdot}04 - {\cdot}0512t$

THERMO-ELECTRIC TABLE. (JENKIN.)

	Neutral Point with Lead.	Tangent of Angle with Lead-line.
Iron	+ 357°	+ ·0420
Palladium	− 181°	+ ·0311
German-silver	− 314°	+ ·0251
Zinc	− 32°	− ·0289
Silver	− 115°	− ·0146
Copper	− 68°	− ·0124
Lead	——	——
Tin	+ 45°	+ ·0067
Aluminum	− 113°	+ ·0026
Cadmium	− 69°	− ·0364

EXAMPLE.—To compute the potential-difference for a cadmium-palladium couple whose junctions are at 50° and 150°, we have for $(T_n - T_m)\tan\alpha$ in the case of cadmium $(-69 - 100) \times -{\cdot}0364$ or $+6{\cdot}15$; and in the case of palladium $(-181 - 100) \times +{\cdot}0311$ or $-8{\cdot}74$. The difference of these values is 14·89, which, multiplied by the mean temperature-difference, gives 1489 microvolts. Or, to calculate the value by the first table, we have for the difference between cadmium and palladium $(2{\cdot}66 + {\cdot}0429t) - (-6{\cdot}25 - {\cdot}0359t) = 8{\cdot}91 + {\cdot}0788t$: which for $t = 100°$ becomes $8{\cdot}91 + 7{\cdot}88$ or

16·79 microvolts; and this multiplied by 100 gives 1679 microvolts for the total potential-difference.

636. Peltier Effect.—In 1834, Peltier noticed that when an electric current is sent through a junction of two metals it causes either an absorption or an evolution of heat according to its direction. And further, that absorption is produced whenever the current passes in the same direction as that developed by heating the junction, and evolution of heat is produced whenever the current passes in the opposite direction. This phenomenon is called the **Peltier effect.**

EXPERIMENT.—Peltier's cross consists of two bars, one of bismuth, *B*, the other of antimony, *A*, placed at right angles and halved together at the junction. If a current be sent from the generator *C* as indicated by the arrows (Fig. 311) through the junction from *A* to *B*, opposite to that produced by heating the junction, the current will develop heat at the junction by Peltier's law. And this heat will produce a current on the other side of the cross, flowing as the arrows indicate, from *B* to *A* through the junction, and from *A* to *B* through the galvanometer. If the generator current be reversed, cold will be produced at the junction; and in 1838 Lenz succeeded in freezing water at this junction, previously cooled by melting snow. If a current be sent through a compound bar, consisting of a bar of bismuth at the center and two of antimony at the ends, both phenomena will appear simultaneously; heat appearing where the current passes from antimony to bismuth and cold where it passes from bismuth to antimony. Two small air-thermometers placed at the junctions will indicate these opposite effects.

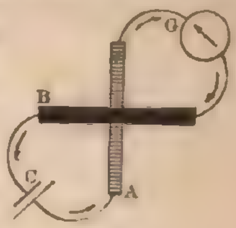


FIG. 311.

Since the Peltier effect is quite feeble, it is often masked by the Joule effect or the ordinary heating effect due to the current. But inasmuch as the Peltier effect is reversible, it may be detected and even measured by a suitable arrangement of the circuit. Edlund has suggested the form shown in the figure (Fig. 312). When the current passes as the arrow indicates, it goes from *A* to *B* through the left-hand junction and from *B* to *A* through the right-hand one; producing heat in the

former and cold in the latter. So that by enclosing the junctions in bulbs of glass connected by a small horizontal tube containing a drop of mercury, the motion of the mercury will be due solely to the Peltier effect. Evidently the Joule effect is the same at both junctions, and is represented by I^2r; while the Peltier effect is op-

FIG. 312

posite at the two and is proportional to IV. In the above apparatus, the total heat in ergs developed or absorbed at a junction is JH; and this is the sum of the Joule and the Peltier effects. Hence $JH = I^2r + IV$ at one junction and $JH' = I^2r - IV$ at the other. The difference $J(H - H') = 2IV$ measures the Peltier effect alone. If the heat be expressed in water-gram-degrees, the fall of potential at the junction expressed in volts is $V = 4{\cdot}2H/I$. In general it is exceedingly small; amounting in the case of the junction of copper with an alloy of bismuth and ten per cent of antimony to only 0·0219 volt at 25° and to 0·0274 volt at 100°. This appears as a counter-electromotive force developed by the generator current. Evidently the Peltier effect vanishes at the neutral point.

637. Thomson Effect.—Since the energy of a thermo-electric current can have no other source than the heat supplied to the circuit, the inversion-effect discovered by Cumming led Lord Kelvin to the conclusion that in a thermo-electric circuit there must be some other counter-electromotive force than that observed by Peltier. He

supposes a couple the hot junction of which is at the neutral point, the other being at a lower temperature. Since at the hot junction the Peltier effect is *nil*, the heat cannot be absorbed there. Moreover, at the cold junction heat is evolved, not absorbed. Hence there must be an absorption of energy in the wires themselves in virtue of the difference of temperature at their ends. Experiment confirmed this conclusion, and showed that an electric current in an unequally heated conductor tends to reduce differences of temperature in certain metals such as copper, and to increase these differences in other metals such as iron. When therefore the current traverses a copper wire from its cold to its hot end, there is an absorption of heat, and *vice versa*. This is called the **Thomson effect**, or sometimes the electric convection of heat. In a copper-iron thermo-electric couple with the hotter junction at the neutral point (Fig. 313), the current passes from cold to hot in the copper, and from hot to cold in the iron, and there is absorption of heat in both metals, their resulting potential-differences being added. Since Le Roux in 1867 showed that the Thomson effect is null or exceedingly small in lead, this metal is taken as zero in the thermo-electric diagram.

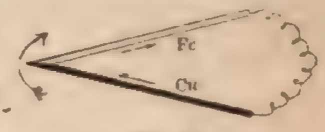


FIG. 313.

638. Energy of the Thermo-current.—Since the energy expended in the circuit of a thermo-current, like that of any other electric current, follows Joule's law, this energy must be derived from the heat absorbed by the circuit itself. We have seen that this absorption of heat may take place at a junction, producing a difference of potential there, the Peltier effect, or it may take place in one of the metals of the couple producing a difference of potential between the ends of this metal, the Thomson effect. The algebraic sum of these two differences of potential is that of the thermo-electric couple.

EXAMPLES.—1. The total potential-difference for a copper-iron couple, whose junctions are at 0° and 100°, is obtained by multiply-

ing the thermo-electric power by the temperature-difference. The thermo-electric power is, by the table, (17·34 — ·0487t) — (1·36 + ·0095t) = 15·98 — 5·82t × 50 = 13·07. And this multiplied by 100 is 1307 microvolts as the potential-difference.

2. The Peltier potential-difference, being the product of the thermo-electric power by the absolute temperature of the junction, is for the hot junction (12·47 — 2·31) × 373, or 10·16 × 373 = 3790 microvolts. For the cold junction it is (17·34 — 1·36) × 273, or 15·98 × 273 = 4363 microvolts. Since these two values are in opposite directions, their algebraic sum is 573 microvolts, the efficient potential-difference.

3. The Thomson potential-difference is the product of the difference of the thermo-electric power at the two ends by the absolute temperature of its middle point. Hence for the iron it is (·0487 × 100) × (273 + ½(0 + 100)), or 1573 microvolts. For the copper it is (·0095 × 100) × 323, or 307 microvolts. Since these two potential differences are in the same direction the total potential difference is their sum or 1880 microvolts.

4. The potential-difference of the Peltier effect being in one direction and that of the Thomson effect being in the other, their difference or 1307 microvolts is the thermo-electric potential difference of the couple as above.

639. Method of computing Energy by Diagram.— The same general result may be reached by construction, using the principle of the thermo-electric diagram of

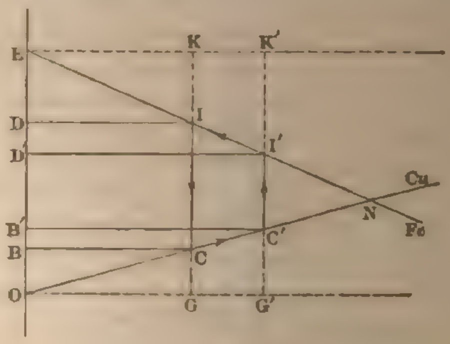


FIG. 314.

Tait (Fig. 314). Let the two inclined lines represent the rate of variation of the thermo-electric power with temperature for copper and iron respectively, as compared

with lead, this rate being positive for copper and negative for iron; the current therefore flowing through the heated junction from the copper to the iron as the arrows show. The thermo-electric power is represented by the ordinates and the absolute temperature by the abscissas. Hence O is the absolute zero, and OG and OG' the temperatures of the two junctions respectively. Since the area $CC'I'I$ is the product of the mean thermo-electric power by the temperature-difference, it represents the total potential difference in the circuit. And therefore if unit current flows through it, this area will also represent the energy of this current. The area $D'B'C'I'$ which is the product of the thermo-electric power at the hotter junction by the absolute temperature of this junction, represents the Peltier effect at this junction, as the area $DICB$ represents this effect at the colder one; heat being absorbed at the former and evolved at the latter. The area $BB'C'C$, being the product of the difference of the thermo-electric power at the two ends of the copper wire by the mean temperature-difference, represents the Thomson effect in this metal; and the area $DD'I'I$, this effect in the iron. Since the current passes from cold to hot in the copper and from hot to cold in the iron, there is absorption in both metals. There is absorption of heat then from C to C', from C' to I', and from I' to I; and therefore the area $BCC'I'ID$ represents the energy taken in, while the area $DICB$ represents that given out. The difference $CC'I'I$ represents the energy utilized in the current in the circuit; or if the current be unity, this area represents the potential-difference. Since the line for lead is parallel to the temperature-axis, the inclination of the copper line to both these lines is the same. Whence the tangent of the angle, or the ratio of the thermo-electric power to the absolute temperature, is GC/OG. Consequently, GC for copper (positive) and KI for iron (negative) represents what Lord Kelvin calls the specific heat of electricity.

B.—CHEMICAL RELATIONS OF THE CURRENT.

(*a*) *Conversion of Electrokinetic Energy into Chemical Energy.*

640. Convection-transference.—Besides the transference of electrification which takes place in metallic conductors, there is a transference which takes place in liquids, producing chemical changes in them. This transference appears to be of the nature of an atomic convection, each atom or atomic group which is concerned in it being electrically charged and carrying its charge with it in the direction of transference. The electrical charge upon a free atom is thus calculated by Lodge. Experiment shows that about $1·5 \times 10^{13}$ positive electrostatic units must be expended upon the hydrogen, and an equal number of negative units upon the oxygen, of one gram of water in order to set these constituent gases free. Hence if we assume that this gram of water contains 10^{24} molecules, there will be expended upon each molecule 3×10^{-11} electrostatic units; one half this charge going to the two atoms of univalent hydrogen, the other half to the single atom of bivalent oxygen. Each atom of hydrogen therefore will have a charge of nearly 10^{-11} electrostatic unit. And since the charge on all univalent atoms or atomic groups is the same, it follows that every such atom or atomic group which is capable of taking part in this transference must be charged with 10^{-11} electrostatic unit. This quantity therefore appears to be a natural unit of electrification; the smallest quantity which actually takes part in any chemical change. It is less than the hundred-trillionth of a coulomb. Moreover, the electrical charge of the atom increases with its valence, a bivalent atom or atomic group having twice this charge, a trivalent atom three times, and so on. Furthermore, it is the view of von Helmholtz that each kind of matter has a specific attraction for electricity, some kinds for positive, other kinds for negative; and hence that chemism itself is

due essentially to the electrical attraction of oppositely charged atoms.

641. Electrolysis.—In some liquids, such as mercury, the transference of electrification is effected by conduction precisely as in solid metals. In others, such as petroleum and turpentine, there is no transference through the liquid, but only displacement within it; the liquid acting like a solid dielectric. In still other liquids, such as acids, alkalies, and salts for example, either fused or dissolved in water, the transference takes place solely by atomic convection. This process of transference is called **electrolysis** or electrolytic conduction; and the substances in solution whose constituent atoms or atomic groups act to carry the electric charges are called **electrolytes.** These constituents themselves are called **ions**; and the conductors by which the current passes into and out of the solution are called **electrodes**; the one by which the current enters being called the positive electrode or **anode**, and the other the negative electrode or **kathode.** Those ions which go to the anode on electrolysis are of course negative, and are called **anions**; while those which go to the kathode are positive, and are called **kathions.** These names were proposed originally by Faraday.

EXAMPLE.—If an aqueous solution of hydrogen chloride be subjected to the action of the current it will be electrolyzed. The electrolyte hydrogen chloride will yield the kathion hydrogen and the anion chlorine. The hydrogen ions will travel in one direction in the liquid, carrying their positive electrical charges to the kathode. The chlorine ions will travel in the opposite direction, transferring their negative charges to the anode.

642. Laws of Electrolysis.—We owe to Faraday (1833) the establishment of the general laws governing electrolysis. These laws are two in number:

I. The amount of chemical change which takes place in any electrolytic circuit is directly proportional to the quantity of electrification transferred through that circuit.

II. For the same quantity of electrification

transferred through any circuit, the amounts o the different electrolytes decomposed in th circuit—and therefore the amounts of the ion set free—are proportional to the chemical equi valents of these different electrolytes or ions.

The first law asserts simply a proportionality betwe the mass of the electrolyte decomposed (or of the io set free) and the current; i.e., the quantity of electrif cation transferred per second. Experiment shows th one absolute electrostatic unit of quantity sets fr $3·46 \times 10^{-15}$ gram of hydrogen; i.e., one coulomb se free 10·38 micrograms. And conversely, to set free on gram of hydrogen 96340 coulombs must be transferre through the circuit. If therefore 50 coulombs be thu transferred, 519 micrograms of hydrogen will b liberated; and this whatever be the time occupied i this transference. So that a current of five amperes fo ten seconds effects the same amount of decompositio for example, as a current of one ampere for fifty second If we call the quantity of any electrolyte which is de composed by a unit current in unit time the **electro chemical equivalent** of that electrolyte, then the first la states that the quantity decomposed by a given curre in a given time is simply the product of the electr chemical equivalent by the current-strength and by th time of the experiment. This law is a necessary cons quence of convection-transference; since if the charg for a given ionic atom be constant, the amount of ele trification transferred must be proportional to th number of such atoms concerned in the transference.

The second law asserts that if several electrolyte be included in the same circuit, so that the same curre traverses them all, the relative amounts of these ele trolytes which are decomposed will be proportional t their chemical equivalents. The chemical equivalent o an ion is defined as the smallest quantity of it whic enters into combination with or replaces an atom o hydrogen; i.e., as the quantity of that ion which i chemically equivalent to an atom of hydrogen. Sinc

the valence of any atom is the number of hydrogen atoms to which this atom is equivalent, it is evident that if we divide the atomic mass by the valence we shall obtain the mass which is equivalent to one atom of hydrogen; i.e., the chemical equivalent. Thus the chemical equivalents of the univalent ions hydrogen, chlorine, and sodium, for example, are 1, 35·37, and 23, being numerically the same as their atomic masses; while the chemical equivalents of the bivalent ions oxygen, calcium, and zinc are 7·98, 19·95, and 32·44, respectively; these numbers being the quotients of their atomic masses divided by two. So the chemical equivalent of a trivalent ion is one third, that of a quadrivalent ion is one fourth, of the atomic mass. In the case of an electrolyte, its chemical equivalent may be defined as that quantity of it which contains only a single monad atom of the same kind; i.e., either positive or negative. Thus the molecule of hydrogen chloride, for example, contains but a single positive atom and a single negative one. Hence its chemical equivalent is 36·37, the same numerically as its molecular mass. A water-molecule, however, contains two monad hydrogen atoms, and hence its chemical equivalent is 17·96; or one half its molecular mass.

According to the second law, therefore, the relative quantities of hydrogen, of sodium, of oxygen, and of zinc set free in the same circuit are proportional to the numbers 1, 23, 7·98, and 32·44, respectively. So that if we call the chemical equivalent of a substance expressed in grams its **gram-equivalent**, it is evident that for a given current the same number of gram-equivalents of the electrolyte will be decomposed and the same number of ionic gram-equivalents set free whatever be the nature of this electrolyte. Now, as above stated, one coulomb, i.e., a current of one ampere for one second, sets free 10·38 micrograms of hydrogen, and hence the mass of oxygen or of sodium thus set free will be $10·36 \times 7·98$ or $10·36 \times 23$, i.e., 82·8 micrograms of oxygen or 238·7 micrograms of sodium. So that the statement of the second law is that the electro-chemical equivalent of any elec-

trolyte or of any ion is simply the product of the electro-chemical equivalent of hydrogen by the chemical equivalent of the acting substance. In general, therefore, if ϵ represent the chemical equivalent of an ion and m the mass of hydrogen set free, expressed in micrograms per coulomb, the absolute mass of the ion set free, also expressed in micrograms per coulomb, i.e., its electrochemical equivalent, will be $m\epsilon$. Moreover, the mass set free by the transfer of n coulombs, will evidently be $mn\epsilon$. Since, however, $Q = It$, the n coulombs is equal to a current of a amperes for t seconds, or $n = at$; so that the mass of the ion in micrograms set free in t seconds by a current of a amperes, will be $mat\epsilon$.

This law, also, flows necessarily from the fact of convection-transference. Since the same electrical charge is transferred by each ionic atom whatever its nature, and since the relative masses of these ionic atoms are proportional to their chemical equivalents, it follows that equal quantities of electrification, on passing through different electrolytes, require for their transport equivalent quantities of these ions.

643. Voltameter.—Upon the first law above given, Faraday based a method of measuring the strength of a current. Since the amount of an ion set free is proportional to the quantity of electrification transferred, it is obvious that by determining the mass of the ion evolved in a given time, the amount of the electrification transferred may be ascertained. The apparatus required for the purpose is called a **voltameter**, and the method the **voltametric** method. The ions ordinarily employed in this method are hydrogen, copper, and silver.

The hydrogen voltameter consists of a glass vessel provided with platinum electrodes and filled with dilute sulphuric acid. On passing the current, both hydrogen and oxygen gases are evolved, but the former only is collected, the latter being allowed to escape. From the known volume of the hydrogen set free, corrected for pressure and temperature, its mass is easily calculated. And since one coulomb sets free 10·38 micrograms, the

number of coulombs transferred through the electrolyte is obtained. If the duration of the experiment be t seconds, the quotient of the coulombs divided by t gives the average strength of the current in amperes.

In the copper voltameter a saturated solution of copper sulphate is made use of, slightly acidified with sulphuric acid. Two copper plates are suspended in the solution, serving as electrodes. On passing the current the anode is attacked and dissolved, and copper is deposited on the kathode. So that the increase in the mass of this kathode represents the amount of copper deposited. If this be expressed in micrograms, then since one coulomb sets free 328 micrograms of copper, the quotient of the copper deposited divided by 328 gives the number of coulombs which has traversed the solution. And this number, divided by the time of the experiment in seconds, gives the mean value of the current-strength in amperes.

It is evident that the results will be the more accurate in proportion as the chemical equivalent of the ion employed is higher. In the most precise voltametric work, therefore, the silver voltameter is preferred, since the chemical equivalent of this metal is nearly 108. The solution employed is generally a five per cent solution of the nitrate, slightly acidified with nitric acid. Two silver plates are made use of as the kathode, and one, placed between them, as the anode. The strength of the current should not exceed from two to five milliamperes per square centimeter of kathode surface. The electro-chemical equivalent of silver as determined by Rayleigh is 1117·94 micrograms per coulomb. Kohlrausch obtained the value 1118·3 micrograms and Mascart the value 1115·6 micrograms per coulomb.

644. Table of Electro-chemical Equivalents.—In the following table the electro-chemical equivalents of some of the most important elemental ions are given, all of these ions being positive except oxygen, chlorine, and nitrogen, which are negative. By dividing the atomic mass in the second column by the valence in the third,

the chemical equivalent given in the fourth column is obtained. And this number multiplied by the electro-chemical equivalent of hydrogen in micrograms per coulomb gives the electro-chemical equivalent of the ion in the fifth column, also in micrograms per coulomb. The numbers in the sixth column are the reciprocals of those in the fifth, given in grams. The electro-chemical equivalents of compound ions such, for example, as the univalent radical hydroxyl OH, and the bivalent radical oxysulphuryl SO_4, are similarly obtained; the chemical equivalent of such a radical being the sum of its component atomic masses divided by its valence.

ELECTRO-CHEMICAL EQUIVALENTS.

Element.	Atomic Mass.	Valence.	Chem Equiv.	Electro-chemical Equivalents. Micrograms per coulomb.	Coulombs per gram.
Hydrogen......	1	1	1	10·38	96340
Oxygen......	15·96	2	7·98	82·83	12070
Chlorine........	35·37	1	35·37	367·10	2721
Nitrogen........	14·01	3	4·67	48·47	20600
Aluminum	27·04	3	9·01	93·50	10700
Lead............	206·40	2	103·20	1071·00	933·7
Zinc	64·88	2	32·44	336·70	2970
Nickel	58·60	2	29·30	304·20	3287
Mercury........	199·80	2	99·90	1037·00	964·3
"	"	1	199·80	2074·00	482·2
Copper..........	63·18	2	31·59	327·90	3050
"	"	1	63·18	655·80	1525
Silver...........	107·7	1	107·7	1118·00	893·5
Gold	196·2	3	65·4	678·90	1473·9

645. Secondary Actions.—The results of simple electrolysis are frequently complicated by supplementary chemical actions taking place at the electrodes. If a solution of hydrogen chloride be electrolyzed, the two ions hydrogen and chlorine appear as such at the electrodes and become free. But if a solution of sodium chloride be so treated, the kathion sodium on being set free immediately attacks the water surrounding the kathode, becoming converted into sodium hydroxide and setting free hydrogen. So when hydrogen sulphate is electrolyzed, while hydrogen appears at the kathode, the anion SO_4 decomposes the water at the anode, uniting

with its hydrogen to form hydrogen sulphate again and setting the oxygen free (Fig. 315). The oxygen and hydrogen gases, therefore, obtained in the ratio of one to two by volume when dilute sulphuric acid is acted upon by the current are due to a secondary action, water itself not being an electrolyte. For a third example, sodium sulphate may be mentioned. Secondary decomposition takes place at both electrodes, sodium hydroxide being formed and hydrogen being set free at the kathode, and sulphuric acid being formed and oxygen set free at the anode. A similar secondary action occurs when the anode is acted upon by the anion. If a solution of copper sulphate be electrolyzed by means of electrodes of copper, the anion SO_4 will attack the anode and form copper sulphate with it; the quantity of copper thus dissolved at the anode being exactly the same as the amount deposited at the kathode. If a solution of lead nitrate be subjected to electrolysis, metallic lead will be deposited on the kathode and lead peroxide upon the anode. So if lead electrodes be used in dilute sulphuric acid, the oxygen set free at the anode will attack the lead and convert it into peroxide.

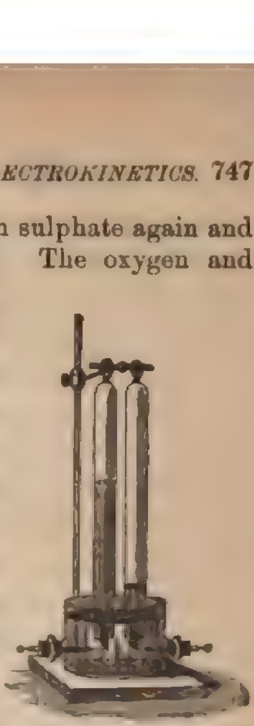


FIG. 315

EXPERIMENTS.—I. Fill a U-tube with a strong solution of sodium chloride, add to it sufficient red litmus solution to color it distinctly, introduce into it a pair of platinum strips to act as electrodes, and pass a current through it. Hydrogen will be set free at the kathode, and the solution will be turned blue by the sodium hydroxide formed by secondary action at this point; while the solution in contact with the anode will be bleached by the evolved chlorine.

II. Repeat the experiment with solution of sodium sulphate, making the litmus purple in color by an exact neutralization. On passing the current, the litmus will be reddened at the anode by the sulphuric acid formed there, and blued at the kathode by the sodium hydroxide there produced. At the same time oxygen and hydrogen gases will be evolved.

646. Migration of the Ions.—It has been observed that when a solution of copper sulphate is electrolyzed between copper electrodes the solution surrounding the kathode becomes continually weaker, while that near the anode continually increases in strength. So that with horizontal electrodes blue streaks can be seen descending when the anode is made the upper plate; while if the kathode is above, the upper layers of the liquid become colorless. When platinum electrodes are used, the liquid becomes gradually weaker at both electrodes, but it weakens two or three times as fast at the kathode as at the anode. This result, due clearly to the migration of the ions, was explained by Hittorf (1853) upon the hypothesis that different ions travel through the liquid at different rates; this rate for the SO_4, for example, being greater than that for the Cu. Kohlrausch has experimentally confirmed this view, and has shown that each ion has its own rate of motion in a given liquid, independent of the particular ion with which it may have been combined. Moreover, he has proved that hydrogen travels faster than any other ion, its speed in nearly pure water, under a fall of potential of one volt per linear centimeter, being about 1·08 centimeters per hour; that of potassium under similar circumstances being 0·205 centimeter, of sodium 0·126 cm., of lithium 0·094, of silver 0·166, of chlorine 0·213, of iodine 0·216, and of NO_3 0·174 centimeter. It is upon the sum of the speeds of the two opposite ions that the conductivity of a liquid depends; and therefore acids which are hydrogen compounds have a higher conductivity than their salts.

647. Theory of Electrolysis.—One of the fundamental facts of electrolysis is that the products of the decomposition appear only at the electrodes, no such products being set free within the electrolyte itself. The first attempt to account for this fact was made by Grotthus (1805), who supposed the molecules of the electrolyte, KCl for instance, to be arranged in rows between the electrodes, their positive faces being turned toward the kathode and their negative faces toward the anode (Fig. 316).

The end molecules in immediate contact with the electrodes are first decomposed, the anion Cl escaping at the anode and the kathion K at the kathode. Then the kathion of the end molecule on the positive side combines with the anion of the next contiguous molecule; and the anion of the end negative molecule combines with the kathion of the molecule next to it; a series of successive decompositions and recompositions taking place along the entire line, and resulting in the liberation of free ions only at

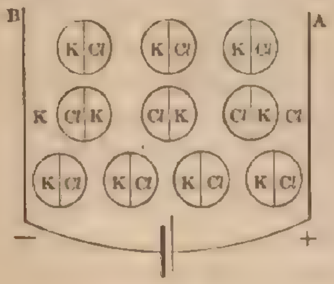


FIG. 316.

the electrodes. This theory is insufficient, since it requires a finite force to effect the decomposition within the electrolyte; and no such force has ever been observed, the experiments of von Helmholtz proving that the interior of an electrolyte can stand not the slightest electrostatic stress. Hence Clausius (1857) modified the theory by introducing into it certain considerations based on the kinetic theory of matter. He supposes that by collision of the moving molecules of the electrolyte some of them are dissociated or separated into their ions, and that it is these dissociated and charged ions alone which are influenced by the electrodes. Arrhenius (1887) extended this view of the subject, and showed that solutions of salts and of strong acids and bases, i.e., those substances which constitute the chief electrolytes, contain these substances as such only to a small extent, by far the greater part being dissociated into ions. The physical properties of a solution of potassium chloride for

example, such as its osmotic pressure and the depression of its freezing point, show that almost all of this salt splits up into its ions potassium and chlorine on being dissolved. It is these dissociated and free ions, one set of which is positive and the other negative, and each of which carries the same numerical electrical charge, that the theory of Kohlrausch (1886) supposes to act in effecting electrolytic transference. The motion of these ions is proportional to the slope of potential through the liquid; but for a given slope of potential and a given solvent each ion is supposed to have a speed of travel which is specific to itself. The actual value of this speed for certain ions has just been given. Electrolytic conduction then appears to consist of an actual motion of free and charged ions; a continuous procession of positively charged kathions moving toward the kathode, and a similar procession of negatively charged anions towards the anode. The cause of this motion is evidently the difference of potential at the electrodes. And when these dissociated ions arrive at these electrodes, the difference of potential which has brought them there may or may not be sufficient to separate their electric charges from them. If it is, the charge neutralizes that of the electrode, the ionic atoms combine with others of the same kind, and the substance is evolved in the free state. If it is not, the charged ions, accumulating at the oppositely charged electrodes, set up there an opposing or counter-electromotive force which is called **polarization.**

648. Work done in Electrolysis. — According to Faraday's second law, equivalent quantities of different electrolytes are decomposed, and equivalent quantities of the ions set free, by the same quantity of electrification. But since the amount of energy absorbed in the decomposition of these different electrolytes is not the same for them all, it follows that the work done in effecting this decomposition cannot be the same; and, therefore, that the amount of work done in any circuit by one coulomb, for example, is not constant, but depends upon the nature of the electrolyte which it traverses. Now

the heat-energy absorbed in decomposing M grams of an electrolyte is evidently MH, in which H is the heat absorbed in decomposing one C. G. S. unit of mass; and the work done in this decomposition is MHJ joules. On the other hand, the electrical work done in any circuit is always equal to the product of the quantity of electrification transferred and the fall of potential in the circuit; i.e., to EQ. Since to decompose M grams of an electrolyte would require $Q\gamma$ coulombs (γ being the electrochemical equivalent in grams per coulomb) we have the equation $EQ = HJ\gamma Q$; or $E = HJ\gamma$. In other words, the work in joules which is required to effect the decomposition of one electrochemical equivalent of an electrolyte represents numerically the difference of potential in volts which exists between the electrodes. Consequently, to obtain the potential-difference in volts necessary to decompose a given electrolyte, it is necessary only to calculate the energy absorbed in its decomposition—or, what is the same thing, the energy set free in forming it—expressed in joules.

EXAMPLES.—Thus, for example, the heat of combination of hydrogen with oxygen is 34180 water-gram-degrees per gram. Since its electrochemical equivalent is 10.38 micrograms or 0·00001038 gram per coulomb, the heat of decomposition of one electrochemical equivalent of water is 0·355 w. g. d. And since one w. g. d. is equal to 4·2 joules, 0·355 w. g. d. = 0·355 × 4·2 or 1·49 joules. Hence the difference of potential required to decompose water is 1·49 volts. Less than this fails to do so.

Again, when zinc dissolves in sulphuric acid, the value of H is 1670 w. g. d.; and since γ is ·0003367, the value of $H\gamma J$ is 1670 × ·0003367 × 4·2 or 2·36 joules.

When copper dissolves in sulphuric acid, H has the value 909·5 and γ the value ·0003279. Whence $H\gamma J$ has a value of 909·5 × ·0003279 × 4·2 or 1·252 joules.

If now in any process zinc is dissolving in sulphuric acid and copper is being deposited from sulphuric acid at the same time, the work done is positive in the first case and negative in the second, and the difference, or 2·360 − 1·252 joules, is 1·108 joules. Consequently the potential-difference between the copper and zinc will be 1·108 volts.

(b) *Conversion of Chemical Energy into Electrokinetic Energy.*

649. Production of Electrical Energy from Chemical Energy.—In electrolysis, work is done upon the electrolyte by the current, and the electrical energy expended appears as chemical energy in the separated ions. Conversely, the chemical energy of separated ions may be transformed into electrical energy. Thus for every electrochemical equivalent of an element which goes into combination, one coulomb of electrification becomes free. The potential under which it is set free, since the quantity of electrification is unity, is represented, as above stated, by the heat-energy evolved by the union, expressed in joules. Thus, for example, when one electrochemical equivalent of zinc, 336·7 micrograms, dissolves in sulphuric acid, the heat-energy set free is 2·36 joules. If, however, the energy evolved in this chemical action be electrical, it is represented by EQ, and its value is also 2·36 joules. Hence, since Q is unity, the potential-difference developed is 2·36 volts, as a maximum. This production of electrical energy by means of chemical action has given origin to various devices for the purpose, called **hydro-electric batteries.**

650. Volta's Theory of Contact.—By means of his condensing electroscope, Volta proved that when two different metals are brought into contact and then separated, they are both equally charged, but with opposite electrifications; zinc, for example, under these circumstances becoming positive, and copper negative. Since this difference of potential produced by contact is developed whenever any two dissimilar substances whatever are made to touch, the law of Volta may be stated thus:

Whenever two substances, at the same temperature, are brought in contact, a finite difference of potential is established between them, this difference of potential being independent of their dimensions, of their form, of the extent

of the surfaces in contact, and of the absolute value of the potential upon either of them, and dependent only upon the chemical or physical characters of the substances themselves, and the medium in which they are immersed.

The mechanism of this electrification, according to Lodge, may be thus stated: Assuming with von Helmholtz that all substances have a specific attraction for electricity, but in different degrees, a piece of zinc and a piece of copper in air will attract the negatively electrified oxygen atoms differently, the zinc more than the copper. A flow of oxygen atoms to both plates will take place, and the union between them will set free the negative charges; this process going on until the zinc as well as the copper becomes so highly charged negatively that the action ceases. Because therefore the attraction of the zinc is the greater, its charge will be the greater, its negative potential continually rising. As a matter of fact, a plate of zinc, insulated in air, has a potential 1·8 volt below that of the surrounding atmosphere; while a plate of copper has a potential only about 0·8 volt below. In consequence the apparent difference of potential between zinc and copper in air is about one volt. When now the two metals are made to touch, a flow of electrification takes place across the junction, until the potential-difference between them is equalized. But evidently, since the attractions for electricity are not the same, the charges will not be simultaneously equalized. The zinc will remain positively, the copper negatively, charged, although both are at the same potential. The apparent difference of potential between zinc and copper in air, as observed in Volta's experiment and in the still more refined one of Lord Kelvin, is due to the fall of potential in the air near the metals. Since the zinc is 1·8 volt below the air and the copper 0·8 volt below, the metals when in contact are evidently 1·3 volt below, this being the mean difference; the original potential-difference between each metal and the air in contact with it remaining unchanged. In

consequence there is a fall of potential of one volt from the air in contact with the zinc to the air in contact with the copper. This is what the electrometer indicates, and which is ordinarily called the potential-difference between zinc and copper. In air no further effect takes place; since the air, being a dielectric, is simply thrown into a slight electrostatic strain by the opposite charges on the two metals.

If, however, the two metals be immersed in an electrolyte, the result is quite different, since there are in the liquid the potentially free or dissociated atoms necessary to electrolytic conduction. A procession of negatively charged atoms continually moves toward the positive zinc, and one of positively charged atoms toward the negatively charged copper, thus preserving the zinc continually at a negative potential and the copper at a positive one; so that on connecting the metals by means of a conducting wire outside of the liquid, a current flows from the copper to the zinc plate through this wire.

By assuming then an attraction, either between the atoms of matter themselves, or between these atoms and electricity, it follows, since every free atom has its own charge, that an isolated substance must be charged and have a potential different from that of the surrounding medium; the energy of this charge being derived from the potential energy of the attracting system. And further, that when two such conducting substances are immersed in an electrolyte and placed in contact, the chemical potential energy of the system is continually converted into the potential electrical energy of the terminals, and this again into the kinetic energy of current flow. In the external circuit this energy appears as heat, light, mechanical motion, etc.

651. Volta's Contact Series.—Metals, like other electrics, may be classed in a series in the order of their electrochemical characters. Moreover, the potential-differences in air between them may be measured and the series may be made quantitative. The following

table gives these contact potential-differences for various substances as observed by Ayrton and Perry at a temperature of 18°. The values with an asterisk were obtained by calculation; those without, by direct experiment.

CONTACT POTENTIAL-DIFFERENCES IN AIR IN VOLTS

	Carbon.	Platinum.	Copper.	Iron.	Lead.	Zinc.
Carbon........	0	0·113*	0·370	0·485*	0·858	1·096
Platinum.......	− 0·113*	0	0·238	0·369	0·771	0·981
Copper.........	− 0·370	− 0·238	0	0·146	0·542	0·750
Iron............	− 0·485*	− 0·369	− 0·146	0	0·401*	0·600*
Lead...........	− 0·858	− 0·771	− 0·542	− 0·401*	0	0·210
Zinc...........	− 1·096	− 0·981	− 0·750	− 0·600*	− 0·210	0

In this table the positive character of the substances named increases from above downward and also from left to right; carbon being the most strongly negative (or least strongly positive), while zinc is most strongly positive. A minus sign indicates that the substance named above it is negative to the substance named to the left of it. Thus platinum is negative to iron. Moreover, the potential-difference between any two substances in the table is very nearly the sum of the intermediate differences of potential. Thus the potential-difference between carbon and lead is that between carbon and platinum − 0·113, plus that between platinum and copper − 0·238, plus that between copper and iron − 0·146, plus that between iron and lead − 0·401, which is − 0·898; which is as near − 0·858 as could be expected, the metals measured being those of commerce.

652. Law of Successive Contacts.—We may represent conventionally the potential-difference between two substances A and B by the symbol $A \mid B$; in which the positive substance is placed on the right, and the numerical value of the expression is the excess of this potential-difference; as for example Pt | Fe = 0·369 volt. Evidently the dissimilar substances thus placed in contact may be of any kind whatever; not only may they be two metals, but also two liquids, a metal and a liquid,

and even two pieces of the same metal at different temperatures. If now we write in this way a number of metals connected in series, thus:

$$\mathrm{Pt} \mid \mathrm{Cu} + \mathrm{Cu} \mid \mathrm{Fe} + \mathrm{Fe} \mid \mathrm{Pb} + \mathrm{Pb} \mid \mathrm{Zn} = \mathrm{Pt} \mid \mathrm{Zn},$$

we express the fact above stated that the extreme potential-difference Pt | Zn is simply the sum of the mean potential-differences in the series. Since these are respectively $-0{\cdot}238$, $-0{\cdot}146$, $-0{\cdot}401$, and $-0{\cdot}210$, their sum $-0{\cdot}995$ is very closely $-0{\cdot}981$, the value given for Pt | Zn. So that if the same metal terminate both ends of the series, it follows, inasmuch as $E \mid A = -A \mid E$, that the difference of potential between these ends is zero; thus:

$$A \mid B + B \mid C + C \mid D + D \mid E + \ldots . + N \mid A = 0.$$

Hence it results: 1st, that when several metals form a contact-series the potential-difference between the extreme members of the series is the same as if these metals were in direct contact; and 2d, that when a series of metals all at the same temperature forms a closed circuit, the algebraic sum of the potential-differences in that circuit is zero; and hence there can be no current through the circuit.

The conditions are changed, however, when the temperature of one of these contacts is altered, or when the dissimilar metals constituting the ends of the series are immersed in an electrolyte. In the former case a new difference of potential is developed in the circuit in virtue of the Peltier effect; and this produces a thermo-electric current. In the latter case a chemical source of energy intervenes, as we have already shown; a new potential-difference being thus produced on these terminal plates, thereby causing a hydro-electric current in the closed external circuit.

653. Hydro-electric Generators.—Voltaic Cells.—If a number of pairs of zinc and copper disks be arranged

alternately in contact, the potential-difference of the combination will evidently be that of a single pair only; or if an additional disk be added to make the terminal metals the same, this potential-difference will be zero. But now if an electrolyte be placed between each pair of disks, the difference of potential will increase with the number of pairs. In 1800, Volta himself constructed a generator upon this plan, using for the purpose disks of silver and of zinc, each pair being separated by a disk of cloth moistened with salt water, the silver disks all facing in one direction and the zinc disks in the other. Owing to its form, the disks being piled one on the other, the apparatus became known as **Volta's pile**; the term **voltaic cell** being applied in later years to similar arrangements using liquids, and **voltaic battery** to a group of such cells connected together. As the pressure of the upper portions of the column in Volta's pile forced the liquid out of the cloth disks in the lower portions and so impaired the effect, Volta constructed another battery by placing the disks in a series of glass cups, the silver disk in each cup being joined by a wire to the zinc disk in the next. If dilute sulphuric acid be used as the electrolyte, the difference of potential for each cell will be about one volt.

654. Polarization.—Suppose a plate of zinc and one of copper to be immersed in dilute sulphuric acid. The negative ion SO_4 will be set free on the zinc surface, combining with it to form $ZnSO_4$ and charging it negatively. The positive ion H will be set free on the copper surface, and will escape as free hydrogen, charging the plate positively. If a wire be made to connect the two plates, a current will flow through it, the energy expended coming from the conversion of the zinc into sulphate. Inasmuch, however, as the solution of zinc in sulphuric acid is capable of giving a potential-difference of 2·36 volts (649), while in fact the potential-difference between the zinc and copper plates in the above cell is only about one volt, it is evident that this evolution of hydrogen upon the copper plate has developed a

counter-electromotive force of about 1·36 volts. The production in a voltaic cell of an electromotive force contrary to that normal to the action going on within it, and therefore weakening the current, is called **polarization** (647). Moreover, the hydrogen exerts also a mechanical action in virtue of its gaseous state. It covers the copper surface with a non-conducting gaseous layer, which prevents contact with the liquid to a greater or less degree, and thus increases the resistance of the cell. Since by Ohm's law the current in any circuit varies inversely as the resistance in it, the current yielded by such a battery rapidly falls in strength after the circuit is closed. Raising the plates out of the liquid frees their surfaces from the gas, and thus restores their action on subsequent immersion.

655. Constant Voltaic Cells.— Such cells as those just described fail to furnish a constant current. They may be made to do so, however, by surrounding the plate on which the hydrogen is set free by some solution capable of chemically absorbing this gas. Daniell in 1836 proposed the use of copper-sulphate solution for this purpose. The **Daniell cell** consists of a glass jar containing dilute sulphuric acid in which the zinc is placed; and of a smaller jar within this, made of porous earthenware, and containing a saturated solution of copper sulphate, within which the copper plate is placed. When the cell is in action, copper is deposited continually on the copper plate and the polarization is practically zero. In order to avoid the resistance of the porous cylinder, C. Varley (1855) proposed to keep the two solutions separate by the action of gravity, in virtue of their differing densities. The **gravity cell** consists of a glass jar having copper-sulphate crystals at the bottom, surrounding the copper plate. Upon this saturated solution rests the lighter dilute sulphuric acid in which the zinc is suspended. On closed circuit, the zinc dissolves to form zinc sulphate, and the copper of the copper sulphate is deposited on the copper plate; so that there is a sharp line of demarkation between the solutions, and the copper solution

does not come in direct contact with the zinc plate. It is essential to the proper working of this cell that it be kept on closed circuit.

Grove in 1839 suggested nitric acid, and Poggendorff in 1842 suggested chromic acid, as the depolarizing substance. In the **Grove cell**, the kathode is of platinum, and it is immersed in nitric acid contained in a porous cup; while the zinc plate or anode is plunged in dilute sulphuric acid contained in an outer jar. Bunsen in 1842 substituted carbon for the platinum; and subsequently employed chromic in place of nitric acid. In these cells the hydrogen combines with the loosely held oxygen of the nitric or chromic acid, which surrounds the kathode; and thus preserves the contact with the liquid.

In the **Leclanche cell**, the kathode is of carbon, and is surrounded with a mass of solid manganese dioxide. This is reduced by the hydrogen evolved during the action of the cell, and therefore acts as the depolarizing agent. The zinc is immersed in a solution of ammonium chloride; and as this is neutral, it exerts no action until the circuit is closed. Hence this form of cell is largely used for open circuit work, such as call-bells, telephones, and the like.

In the **Lalande-Chaperon cell**, an improved form of which has been introduced in this country by Edison, the kathode consists of a compacted plate of copper oxide, the exciting liquid being a moderately strong solution of potassium hydroxide. The zinc dissolves in the alkaline solution and the copper oxide is reduced to metallic copper by the evolved hydrogen. This cell is remarkably free from local action, and has a very low internal resistance.

656. Local Action.—Theoretically, there should be no consumption of zinc in a voltaic cell when it is yielding no current. In fact, however, whenever commercial zinc is placed in dilute acid, the evolution of hydrogen at once begins. This is due to inequalities in the zinc, either physical or chemical, one portion of the zinc being

more positive than another; and so a voltaic action is set up locally between adjacent portions of the zinc plate. This **local action** may be prevented by covering the zinc surface with mercury, or amalgamating it, as it is termed; by which all parts of it are reduced to electrical equality and no solution of the zinc takes place on open circuit. In all cells using acid or alkaline liquids in contact with the zinc, amalgamation is an advantage. In general the term "local action" is applied to any changes going on in a cell on open circuit which reduces the potential energy of the cell.

657. Standard Cells.—It is convenient to have a concrete standard of electromotive force, for determining relative values of this unit by direct comparison. Such voltaic cells as are used for this purpose are called **standard cells**. Two types of standard cells have been employed—the Daniell cell and the Clark cell. The best form of standard Daniell cell is that devised by Kelvin (1871). A short and rather wide glass jar contains at bottom the zinc plate immersed in saturated zinc-sulphate solution. Above this the copper plate is suspended; the half-saturated solution of copper sulphate which is to cover it being introduced from an exterior funnel, connecting with a siphon-tube terminating horizontally within the jar at the surface of the zinc-sulphate solution. By raising the funnel the liquid it contains flows into the jar and rests upon the heavier zinc-sulphate solution. At the conclusion of the experiment the funnel is lowered and the liquid flows back into it. The electromotive force of this cell is 1·072 volts at 15°; and its temperature-coefficient is practically negligible. Modifications of this battery have been proposed by Lodge, by Fleming, and by the author. The Latimer-Clark cell (1873), as modified subsequently by Rayleigh (1885) and by Carhart (1890), consists of a glass tube having a platinum wire sealed into its lower end, into which is poured pure mercury, then a paste made of mercurous sulphate and zinc-sulphate solution (saturated at 0°) containing a

little zinc carbonate. Upon this paste rests a saturated zinc-sulphate solution, and the zinc cylinder is supported by a cork partly in the solution and partly in the paste. The tube is then filled with marine glue. The electromotive force of the Rayleigh cell is 1·437 volts, and that of the Carhart cell 1·442 volts, both at 15°. The temperature-coefficient of the Carhart-Clark cell is 0·0004 at 0°. The Clark form of cell on account of its material advantages has been generally adopted in practice.

658. Energy of the Voltaic Cell. — The potential-difference between the terminals of a voltaic cell may in certain cases be calculated. In the Daniell cell, for example, this potential-difference is the algebraic sum of the separate contact potential-differences between the constituents. Arranging these thus we have:

$$\begin{array}{ccccc} CuSO_4 \mid Cu & + CuSO_4 \mid ZnSO_4 & + Zn \mid ZnSO_4 & + Cu \mid Zn & \\ +0·070 & -0·095 & +0·430 & +0·750 & =1·155 \end{array}$$

The sum of the separate contact-differences of potential, therefore, according to Ayrton and Perry's tables, is 1·155 volts, while the actual potential-difference of the cell, measured directly, varies from 1·068 to 1·081.

Moreover, the potential-difference may also be calculated from the energy of the zinc consumed. The quantity of electrification set free by the solution of one electrochemical equivalent of zinc in acid is one coulomb; but the energy set free in this operation is 2·36 joules; i.e., 2·36 volt-coulombs. That is to say, the one coulomb to generate this energy must fall through a potential-difference of 2·36 volts. In the same way one electrochemical equivalent of copper sets free one coulomb of electrification, but 1·252 joules of energy; whence the one coulomb must fall through 1·252 volts. In the Daniell cell, the zinc is dissolved, but the copper is deposited. The work in the former case is positive, and in the latter negative. The difference, or 1·108 volts, is therefore the effective potential-difference of the cell.

A similar calculation may be made for the Grove cell. Arranging the potential-differences as follows:

$$\begin{array}{llll} HNO_3 \mid Pt + & H_2SO_4 \mid HNO_3 + & Zn \mid H_2SO_4 + & Pt \mid Zn \\ +0\cdot672 & +0\cdot078 & +0\cdot241 & +0\cdot981 \quad = 1\cdot972 \end{array}$$

we see that their sum gives 1·972 volts as the potential-difference of the entire cell; while the value as obtained experimentally is about 1·94 volts.

659. Thermal Considerations.—In the case of a voltaic cell, we have assumed that $EQ = MHJ$; i.e., that the electrical energy produced is equal to the chemical energy expended. Von Helmholtz has pointed out, however, that this is true only when the electromotive force of the cell is independent of temperature; i.e., when its temperature-coefficient is zero. In the Daniell cell, this is practically the fact; and hence the electromotive force calculated by the above equation is closely identical with the observed value. Since in this equation $M = Q\gamma$, we have $E = JH\gamma$; or $4\cdot2H\gamma$, expressed in joules per water-gram-degree. But $\gamma = 10\cdot38\epsilon$, where ϵ is the chemical equivalent of the ion; and $H\gamma = 10\cdot38H\epsilon$; or calling $H\epsilon$ the heat of combination h of one chemical equivalent of the ion, we have $E = 4\cdot2 \times 10\cdot38h = 44h$ microvolts. If in this equation E be made equal to unity, $h = 1/44$ or ·02272. Hence ·02272 water-gram-degree corresponds to one microvolt or 22720 water-gram-degrees to a volt. From thermodynamic considerations von Helmholtz writes the above equation $E = \cdot000044h \pm T(dE/dT)$; in which the added term represents an electromotive force existing as a function of the absolute temperature T, and which may act with or in opposition to the normal one. Recent researches have made it probable that this is a Peltier effect produced when the current traverses the junctions in the battery, either of metals with liquids or of liquids with each other. Thus, for example, the observed electromotive force of a cell $Pb \mid PbSO_4 \mid ZnSO_4 \mid CuSO_4 \mid Cu$ was 0·61 volt at 20°; while that calculated from the heats of formation of lead and copper sulphates was only 0·383. Now by

direct experiment the Peltier effect Cu | $CuSO_4$ | Cu was found to be 0·00066, and the effect Pb | $PbSO_4$ | $ZnSO_4$ | $PbSO_4$ | Pb was found to be − ·00011 volt. The absolute temperature-coefficient dE/dT for both is ·00077 and that of $T(dE/dT)$ is 293 × ·00077 = 0·225 volt. Now 0·383 + 0·225 = 0·608 volt, as above.

660. Energy of the Voltaic Circuit.—Having the potential-difference E of a voltaic cell when on open circuit expressed in volts, the total energy expended by such a single cell in the time t is evidently EIt joules; where I is the current strength in amperes. Or as $Q = It$, we have $W = EQ$ joules. For a battery of n such cells in series, the potential-difference is nE or E' and the work nEQ or $E'Q$, the current and time remaining constant. If, however, these be made to vary, then by Ohm's law the current I' produced by n cells in series, each of resistance r, the external resistance R_e remaining constant, is $nE/(nr + R_e)$ (608). The current I' produced by n such cells in parallel is $nE/(r + nR_e)$ (609). While if m cells are arranged in series and n in multiple, the current I' produced is $mnE/(mr + nR_e)$ (610). Calculating from these formulas the values of E' and I', the total energy expended in the circuit will be the continued product $E'I't'$ joules.

Since the work done in the several portions of a circuit is proportional to the resistances of those portions, the total energy above obtained may be divided into two parts: one the useful work expended in the external circuit; the other the unavailable energy expended uselessly as heat in the battery itself. Calling R_i and R_e these two resistances, the fall of potential through the external circuit will be $ER_e/(R_i + R_e)$, while that in the battery will be $ER_i/(R_i + R_e)$. Hence the external work done will be $WR_e/(R_i + R_e)$, the internal work being $WR_i/(R_i + R_e)$.

That arrangement of cells in a battery which will produce the maximum useful effect depends upon the resistance of the external circuit, and has been already

discussed under the law of maximum efficiency (61
The rule for maximum current has also been consider

661. Secondary Batteries.—Gautherot (1801) o served that a pair of platinum wires after use in el trolysis was capable of giving an inverse current; a Ritter (1803) constructed a pile of plates differing fr the pile of Volta only in the fact that all the plates w of copper, alternating with disks of moistened cardboa On connecting this Ritter's pile with the Volta pile fo short time, he found that it became charged and w capable of producing an inverse current. Volta (180 explained correctly the result as depending upon t decomposition of the electrolyte between the plates; due in fact to the polarization of the electrodes. Pla (1860) proved the exceptional efficiency of lead for t purpose, and constructed the first batteries of this s which were commercially useful. Such batteries a termed **secondary** or **polarization batteries.**

EXPERIMENTS.—1. Place in dilute sulphuric acid two plati plates, and connect them with two or more Daniell cells arrange series. After a few minutes disconnect the cells and put the pl in circuit with a galvanometer. A current will traverse galvanometer in the inverse direction to that which charged plates; but this current will be very brief in duration.

2. Charge the plates a second time, and by means of an e trometer measure the difference of potential between them. It w be found to be about 1·5 volts; which is the electromotive fo between oxygen and hydrogen.

3. Deposit upon one of these plates a layer of peroxide of le by making it the anode in a solution of lead nitrate. Immerse t plate in dilute sulphuric acid, together with the hydrogen pla the last experiment. The difference of potential between them w be found to be the same as before.

4. Place two platinum plates in solution of lead nitrate, and the action of a current cover one with metallic lead and the o with lead peroxide. On immersing them in dilute sulphuric a the potential-difference will be found to be about two volts.

5. Finally, let a platinum plate covered with lead peroxide immersed in dilute sulphuric acid with a plate of amalgamated The potential-difference will reach a value as high as 2·5 volts.

6. Place two lead plates in dilute sulphuric acid and connect th with several Daniell cells until gas is evolved from them. T

connect them with a galvanometer of moderately high resistance, and note the time required to discharge the plates. Reverse the connection with the battery and charge the lead plates a second time. It will now be observed that the time of discharge is considerably greater than before. By a repetition of this process, the yield of current (i.e., the storage capacity) is continually increased.

In all these cases the electrolyte is decomposed, its ions being carried to the respective electrodes and either deposited upon or united with them. The energy expended by the current is stored up as the potential energy of the separated ions; this potential energy being re-converted into the energy of the current when the circuit is closed through the galvanometer; whence the name **storage batteries**, or **accumulators**, applied to these cells. Moreover, the work done in separating the ions being QE joules, the current-energy given by them on reuniting will also be QE joules; i.e., will be Q coulombs under a potential-difference of E volts. Thus the energy which must be expended upon nine grams of water to decompose it is about 142500 joules; and this is also the energy which the one gram of hydrogen it contains would evolve on uniting with the eight grams of oxygen. But since the atomic charges in the nine grams of water are only 96340 coulombs, it follows that in order to develop this energy the fall of potential should be about 1·49 volts. The potential-difference of a secondary cell, therefore, depends solely upon the contacts within it; while the amount of energy stored up in it is a function only of the mass of the ions deposited upon the electrodes. The energy of such a cell, unlike that of a condenser, is, in joules, the entire product of its potential-difference in volts by the quantity of its charge in coulombs; or in watt-hours, is the continued product of the volts by the amperes by the hours.

662. Actual Storage Cells or Accumulators.—The secondary or storage cell of Planté consists simply of two sheets of lead, rolled together in a spiral and immersed in dilute sulphuric acid; contact being prevented by strips of rubber placed between the plates.

On passing a current through such a cell, oxygen is set free on the anode and unites with it to form peroxide; while the hydrogen evolved on the kathode reduces any oxide there to metallic lead. On discharging the cell and reversing the current, the oxide formed from the plate by the corrosion due to the electrolytic oxygen is reduced by the hydrogen now set free upon this plate; while the oxygen corrodes in its turn the other plate. So that finally a sufficiently thick layer of peroxide is obtained on the anode and of metallic lead on the kathode, to yield a current of several amperes for many hours, with the difference of potential existing under these conditions between metallic lead and lead peroxide; i.e., two volts. The capacity of the cell is given either in ampere-hours, or in watt-hours; i.e., a battery which will give a current of ten amperes for twenty hours has a current-capacity of 200 ampere-hours; or if the potential-difference be 100 volts (50 cells each of 2 volts) it has an energy-capacity of 20000 watt-hours. If discharged at the rate of ten amperes, the rate of work of the current would be 1000 volt-coulombs or joules per second; i.e., one kilowatt; and as 746 volt-amperes or watts is a horse-power, the activity of the battery would be nearly 1·5 horse-powers. This rate in theory it would maintain for 20 hours.

One objection to the Planté cell thus produced is the length of time needed to "form" the layer of active matter upon the plates by electrolytic corrosion. Brush in 1879 and Faure in 1880 suggested, in order to shorten the time of formation, to apply a layer of lead oxide directly to the lead plates before immersing them in the acid. By the action of the current, hydrogen is set free on the kathode and reduces this oxide to metallic lead, and oxygen is evolved on the anode and raises the oxide to peroxide. In its present form this type of cell consists of a support-plate or grating preferably of an alloy of lead with a few per cent of antimony, the perforations in this plate being filled with lead oxide under pressure. After the cell is formed the oxygen or anode plate con-

tains lead peroxide, the hydrogen or kathode plate metallic lead. The storage capacity of these cells is about 12 ampere-hours per kilogram of lead.

The Main storage battery uses peroxide of lead and zinc. The positive plate consists of thin laminæ of sheet lead separated by powdered graphite, riveted together and thickly perforated. It is practically a Planté plate, and is formed in the same way by corrosion under the action of the current, the layer of peroxide being about a millimeter thick. The negative plate is of zinc with a core of copper, the whole kept well amalgamated. The solution is dilute sulphuric acid containing zinc sulphate. On charging, zinc is electrolytically deposited on the kathode plate while the anode plate is peroxidized, the storage capacity depending on the quantity of the ions deposited on the plates; being ordinarily from 16 to 18 ampere-hours per kilogram of metal in the cell. The difference of potential between zinc and lead peroxide, however, is 2·5 volts; and this gives this cell a great advantage, since with the same current-capacity in ampere-hours, one fourth more energy-capacity in watt-hours is obtained. The chief defect of the battery appears to be its tendency to local action; but this, it is claimed, may be completely overcome by keeping the zinc well amalgamated.

The theoretical advantages of secondary batteries are very considerable. They have in general a high electromotive force and a low internal resistance, and hence are suited to deliver a large volume of current. Moreover, there is no waste in their operation, no consumption of material; the only wear in cells of the Faure or active-matter type being due to corrosion of the positive plates, which must be renewed at intervals. The electric energy which is stored in them on charging, however, is recovered again on discharge only to the amount of 70 or 80 per cent. The defects of this form of cell are found in practice to be numerous. The electromotive force on discharge must not be allowed to fall below 1·8 volts, since then the lead oxide is converted

into sulphate and the cell is injured. Moreover, if a certain discharge rate is exceeded the plates buckle and are brought in contact. Indeed, the present forms of active-matter cell seem to require constantly the attention of experienced persons.

A secondary cell differs from a primary one only in the fact that in the former the energy has an electrical origin and in the latter a chemical one. In a zinc-peroxide cell, for example, made up with the zinc of commerce, the first discharge takes place as a primary battery, the energy being stored in the zinc from the carbon which has reduced it chemically from its ore. If after discharge it is re-charged by a current, the re-deposition of zinc restores the cell to its primitive condition. But now the energy stored up in the zinc has an electrical origin, and the cell acts as a secondary battery.

C.—MAGNETIC RELATIONS OF THE CURRENT.

(a) *Electromagnetism.*

663. Oersted's Discovery.—Although a relation between electric and magnetic phenomena had long been suspected, it was not until 1819 that experimental proof of this relation was obtained. In that year, Oersted of Copenhagen showed that a wire carrying an electrical current caused the deflection of a magnetic needle.

EXPERIMENT.—1. Place a wire conveying a current near to and parallel with a magnetic needle at rest in the meridian (Fig. 317).

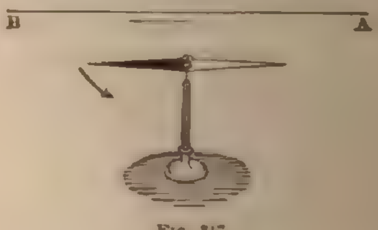


FIG. 317.

The needle will turn about its point of suspension and come to rest in a position more or less inclined to the meridian.

2. Place the wire above the needle and pass the current through it in a direction from south to north. The north-seeking end of the needle will turn toward the west. If the current pass from north to south, the north-seeking end of the needle will turn to the east. Place the wire below the needle and the phenomena will be reversed; the north-seeking end turning to the east when the current flows from south to north and to the west when it flows from north to south.

3. Bend the wire into a loop and place it so that the current flows over the needle along one side of the loop and under the needle along the other. If it flows from north to south over the needle it will flow from south to north beneath it. Both these actions conspire to turn the north-seeking end toward the east, and the effect is multiplied.

4. Place the conducting wire vertical and opposite to the marked end of a magnetic needle (Fig. 318). If the current-flow be upward, as shown in the figure, the marked or north-seeking end of the needle will move toward the east.

Fig. 318.

664. Ampère's Rule.—To keep these facts in mind, Ampère proposed the following simple rule: Suppose a person lying in the conductor and facing the needle; if the current pass from his feet toward his head, the north-seeking end of the magnet will be deviated toward his left hand. To eliminate the earth's magnetic action, Ampère used a dipping needle (590), with its axis of rotation parallel to the earth's lines of force. Under these conditions the needle places itself rigorously perpendicular to the conducting wire. Moreover, he inverted Oersted's experiment, making the conductor movable and the magnet fixed. If the wire conveying the current be vertical and be placed in the plane of the meridian opposite the end of the magnet, then since action and reaction are equal a person lying in the wire and looking in the direction of the lines of force of the magnet (i.e., toward the south-seeking or from the north-seeking end), the current flowing from his feet toward his head, will experience a force tending to move the conducting wire toward his left hand.

665. Current-field.—Inasmuch as a freely suspended magnetic needle always tends to place itself parallel to the lines of force of a magnetic field, it is evident that this action of the current must be due to the production of a magnetic field surrounding the conductor, and extending to a greater or less distance from it.

EXPERIMENTS.—1. Place a card on which iron-filings have been sprinkled, over a wire conveying a current, and tap the card gently with the finger. The filings will be seen to arrange themselves in lines perpendicular to the direction of the wire; showing that the lines of force of a current-field are normal to the current-flow.

2. Pass the wire through the card perpendicular to its plane (Fig. 319) and gently tap it as before. The filings will be seen to arrange themselves in concentric circles about the wire as an axis, which circles represent the lines of force.

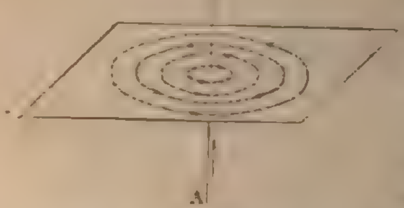


FIG. 319.

3. Place a small magnetic needle near the card. If the current flow from below upward, the needle will take a position tangent to these circles, with its north seeking end pointing in the direction of the flow of force indicated by the arrows. So that the positive direction of the lines of force in a current-field is opposite to the motion of the hands of a watch when the current-flow is from below upward, and *vice versa*.

(If a glass plate be used in these experiments in place of the card, the phenomena may be projected on a screen.)

Ampère conclusively demonstrated that the field produced by a current is identical in all respects with the field produced by a magnet. The above experiments show that the lines of force of a current-field are circles having the wire for their center, and having their plane perpendicular to its length. Whence it follows that the equipotential surfaces are radial planes parallel to the axis of the conductor and equidistant. Since unit work must be expended on unit pole to move it from one of these surfaces to the next, to carry it completely round the wire at unit distance would require Fl ergs. But the force F exerted by a current of strength I on unit pole at

unit distance is $2I$; and the distance moved, l, is 2π. Whence the work done is $4\pi I$ ergs, and there are $4\pi I$ equipotential surfaces. Conversely, energy corresponding to $4\pi I$ ergs is lost for every revolution of unit pole under the action of the current. In this case I is to be expressed in electromagnetic measure (669).

666. Current-field Intensity.—The intensity of a magnetic field at any point has been defined (530) as the force with which it acts on a unit pole placed at the point; i.e., $H = f/m$. Since Faraday showed with the voltameter that the magnetic effects of a current are directly proportional to its strength, and Biot and Savart that these effects in the case of an indefinitely extended current vary in the inverse ratio of the distance between the conductor and the pole, we have evidently $H_i = ki/d$; in which k represents the force of unit current at unit distance. Biot and Savart sent the current through a long vertical wire and hung a small magnetic needle near it in a plane perpendicular to the meridian (Fig. 320). Noting the number of oscillations n under the action of the earth's field alone H', then passing the current and again noting the oscillations, first at a distance d where the intensity of the current-field is h and then at d' where it is h', three equations are obtained, $n^2 = KH'$ (in which K is the needle-constant), $N^2 = K(H' + h)$, and $N'^2 = K(H' + h')$. Combining these, $(N^2 - n^2)/(N'^2 - n^2) = h/h'$; and this ratio of intensities was found in all cases to equal the inverse ratio of distances; or $h/h' = d'/d$. Consequently, if the magnetic mass of the pole be m, we have $f = kim/d$; or, the force with which a straight current indefinite in length acts upon a magnetic pole is perpendicular to the plane

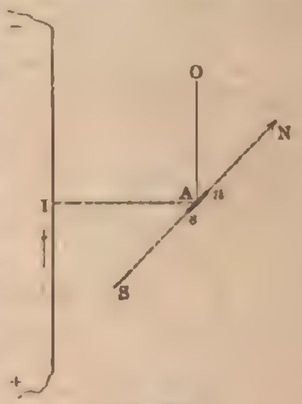


FIG. 320

passing through the current and the pole, is directed toward the left of the current (i.e., toward the left hand of a person swimming with the current and looking in the negative direction of the lines of force), and in magnitude is inversely as the distance between them. Evidently $fd = kim$; and fd is a couple whose magnitude is kim, tending to make the pole rotate about the current as an axis. A complete magnet has no tendency to move, however, since $f_n = -f_s$.

667. Law of Laplace.—That reciprocal action of a current-element and a pole which satisfies the experiment of Biot and Savart has been shown by Laplace to be a simple consequence of the law of inverse squares as given by the equation $\phi = (kim\int ds \sin\alpha)/d^2$; in which ϕ is the mutual force exerted, m the magnetic mass of the pole, i the strength of the current, ds the length of the element, d the distance between them, and α the angle between the element and a line drawn to the pole from its middle point. Since $m/d^2 = F$, which is the strength of the field at O due to the mass m at P, and since $ds \sin\alpha$ is the height of the parallelogram (Fig. 321) constructed with the element ds and the

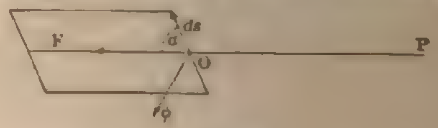


FIG. 321.

force F as sides, $Fds \sin\alpha = dA$; and Laplace's law may be written $\phi = \int kidA$; or $\phi = \int IdA$ if $k = 1$. In other words, the force experienced by a current-element placed in a magnetic field is equal to the product of the current-strength expressed in electromagnetic measure by the area of a parallelogram having the element and the field-intensity as sides. Evidently this force ϕ will be normal to the plane of the parallelogram, and its direction will be toward the left of an observer placed in the current and looking in the direction of the force.

668. Circular Current.—To apply this law, the force with which a circular current acts on a unit pole placed on its axis at some point P (Fig. 322) may be calculated. Since the angle which any element of the circle makes with a line drawn to P is 90°, $\sin \alpha = 1$; and if the current be unity, Laplace's formula gives us $\phi = \int ds/d^2$. Resolving this force ϕ, which is perpendicular to AP, into two components,

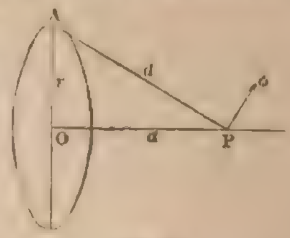


FIG. 322.

one along the axis and the other normal to it, we have for the former component $\phi \cos OAP = \phi r/d = F$; whence $F = \int rds/d^3$. The element of current in this case is the length of a circle of radius r; i.e., $2\pi r$, and hence $F = 2\pi r^2/d^3 = 2S/d^3$; which is the force in dynes exerted along the axis by an electromagnetic unit of current upon a unit pole placed at the point P. At the center of the circle, $r = d$; and hence $F = 2\pi/r$. With a current of I units and a pole of strength m, $F = 2\pi mI/r$. If the coil has n turns, $F = 2\pi nmI/r$. Moreover, since $H_I = F/m$, we have $H_I = 2\pi I/r$.

669. Electromagnetic System of Units.—A system of units has been constructed based on the electromagnetic actions of the current, and hence called the electromagnetic system. From the above expression for F, we have $I = Fr/2\pi m = Fr^2/lm$, since $2\pi r = l$; and hence we may define the electromagnetic unit of current as *that current which flowing through unit length of a circular arc of unit radius, exerts unit force upon a unit pole placed at its center.* The electromagnetic system of units is obtained, therefore, by making the numerical coefficient unity in its fundamental law, the law of Laplace, $F = kim \int ds/d^2$; just as the electrostatic system of units is produced by making this coefficient unity in the fundamental electrostatic law of Coulomb, $F = kQ^2/d^2$. A C. G. S. electrostatic unit of quantity is that quantity

which attracts a similar quantity at a centimeter dist with the force of a dyne. A C. G. S. electromagnetic of quantity is that quantity which flowing per se through a circular arc a centimeter in length an centimeter in radius will exert the force of a dyne C. G. S. unit pole placed at its center. Unfortuna these two units of quantity have not the same v The electromagnetic unit of quantity is about 3 × times greater than the electrostatic unit; and of co the same ratio is preserved throughout the syst From the expression above $I = Fr^2/lm$, we get $= [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]$ as the dimensional electromagnetic eq tion of unit current; and since $Q = It$, $[Q] = [L^{\frac{1}{2}}M^{\frac{1}{2}}]$ from the expression $Q = d\sqrt{F}$ we get for the dim sional electrostatic equation of unit quantity [Q $[L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}]$. Dividing, we find for the ratio LT^{-1}, wh is the dimensions of a speed. And we observe 3×10^{10}, which is the ratio of the electromagnetic to electrostatic unit of quantity, represents the spee light. Calling this ratio v, we may represent the rela of the electrostatic units to the electromagnetic unit the C. G. S. system as follows:

1	Electromagnetic	unit of	quantity $= v$	Electrostatic	unit	
1	"	"	current $= v$	"	"	
1	"	"	capacity $= v^2$	"	"	
v	"	units of	potential $= 1$	"	unit	
v^2	"	"	resistance $= 1$	"	"	

From this relation the following table of value obtained:

VALUES OF PRACTICAL UNITS.

Units.	Name.	E. S Value.	E. M V
Quantity...	Coulomb	3×10^{9}	10^{-1}
Current	Ampere	3×10^{9}	10^{-1}
Potential-difference.	Volt	$\frac{1}{3} \times 10^{-2}$	10^{8}
Resistance.........	Ohm	$\frac{1}{9} \times 10^{-11}$	10^{9}
Capacity...........	Farad	9×10^{11}	10^{-9}

670. Galvanometer Measurements.—A galvanometer is an instrument for measuring the strength of a current by means of the deflection which this current produces upon a magnetic needle placed in its field. For detecting the presence or the direction of a current, a rectangular coil, consisting of a number of turns of wire placed in the meridian and enclosing the needle, is quite sufficient. But for accurate measurement of the strength of a current, the instrument must be so constructed that the deviation of the needle is proportional either to the current-strength itself, or to some function of it. Two forms of galvanometers are in common use, in one of which the current is proportional to the tangent of the deflection-angle, and in the other to the sine of this angle. These instruments are called **tangent** and **sine galvanometers** therefore, respectively.

671. Tangent Galvanometer.—The needle of a galvanometer is acted on by two forces: one the deflecting force of the current, the other the restoring force of the earth's field. When the plane of the galvanometer coil is in the meridian, the lines of force of its field are perpendicular to those of the earth's field, and the needle, solicited by two perpendicular forces, takes up a resultant position of equilibrium between them. Calling ml (Fig. 323) the moment of the needle, H the horizontal intensity of the earth's field and I that of the current, ϕ being the deflection-angle, then the length of the arm is $l \cos \phi$ for the current and $l \sin \phi$ for the earth's magnetism. Whence the earth's couple on the needle is $MH \sin \phi$ and that of the current $MI \cos \phi$. In the position of equilibrium these two couples are equal; and hence $I = H \tan \phi$. If a second current produce a deflection ϕ', we have $I' = H \tan \phi'$; whence $I : I' :: \tan \phi : \tan \phi'$; or

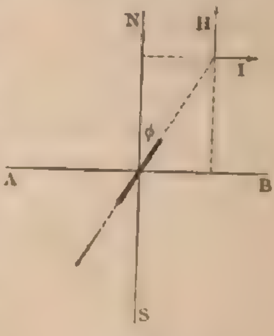


FIG. 323.

in this instrument the deflecting powers of two currents are directly proportional to the **tangents** of the deflections.

672. Sine Galvanometer.—In this instrument, the coil is always maintained parallel to the needle, by rotating its plane as the needle is deflected. Consequently the component of the earth's couple is balanced by the entire force of the coil, and we have $IM = HM \sin \phi$. Whence $I = H \sin \phi$, and the deflecting force of the current is proportional to the **sine** of the deviation-angle. Since $\sin 90°$ is 1, while $\tan 90°$ is ∞, the sine compass can be used to measure only very weak currents compared with those which are measured by means of the tangent galvanometer.

673. Absolute Galvanometers.—To determine the value of the current-strength in absolute measure, the constant of the galvanometer must be determined. This constant is the intensity of the field in C. G. S. units which is produced by unit current. If the galvanometer be so constructed that the constant can be calculated from its dimensions, then it is called an **absolute galvanometer.** Suppose the coil to be constructed of a single circle of wire of radius r. Then the deflecting force of this coil. i.e., the intensity of its field at the center, is $2\pi I/r$ (668). Whence we have $2\pi I/r = H \tan \phi$; and $I = (Hr \tan \phi)/2\pi$. The value $2\pi/r$ is the deflecting power of the coil for unit current-strength. It is called the **galvanometer-constant.** Representing it by G, the above expression becomes $I = (H/G) \tan \phi$. The ratio H/G is the galvanometer **reduction-factor,** and is frequently represented by k. In the equation $I = k \tan \phi$, it represents the current strength required to give a deflection-angle whose tangent is unity; i.e., an angle of 45°. Hence by multiplying the tangent of any other deflection-angle by the reduction-factor, the absolute current-strength in C. G. S. units is obtained.

674. Experimental Determination of the Reduction Factor.—If, however, the galvanometer-constant cannot be measured directly, then it is necessary to determine the reduction-factor experimentally. This may be done

by means of a calorimeter or a voltameter. A current is sent through the galvanometer, placed in series with the calorimeter, the same current going through both; this current being adjusted to give a deflection of about 45°. The deflection is read at intervals, and the temperature of the liquid in the calorimeter noted. If, for example, m grams of water be raised $T°$, then mT water-gram-degrees of heat have been evolved. But a current of strength I' has been flowing for the same time t through a resistance R; and this current has developed I'^2Rt/J water-gram-degrees. Whence $mT = I'^2Rt/J$ and $I' = \sqrt{mTJ/Rt}$ C. G. S. units of current. If the copper voltameter be used, the current-strength in absolute electromagnetic units is obtained by dividing the mass of copper deposited in the time t, expressed in micrograms, by 3279 and by t. Having the mean current-strength I' during the experiment, determined by either of the above methods, and also the mean angle of deflection ϕ', we have $k = I'/\tan\phi'$.

675. Uniform Galvanometer-field.—The theory of the tangent galvanometer now given assumes the ratio of H to G to be constant, and supposes, therefore, the current-field to be as uniform as the earth's field. In practice this result is secured in two ways: first, by making the needle very short, i.e., one tenth to one fifteenth of the radius of the coil, so that it does not move out of the region where the lines of force are parallel; and second, by providing the galvanometer with two equal and parallel coils having the needle placed midway on their common axis.

676. Reflecting Galvanometers.—For use in accurate measurement, Lord Kelvin has devised reflecting galvanometers of great delicacy (Fig. 324). The needles form an astatic system, the upper combination being cemented on the back of a silvered mirror, and the whole suspended by a quartz fiber. The coils are circular, are small in diameter, and are quite thick; and they surround closely one, or sometimes both, of the needle systems; thus producing a very intense field.

If the mirror M be concave, then by means of a lamp and a narrow opening in front of the flame, placed at O (Fig. 325), a line of light is thrown on the mirror and reflected back to B upon a graduated millimeter scale just above the opening. If the mirror be plane, the deflections upon the scale are read off directly by means of the telescope T; the ratio of the scale-number to the distance between the scale and the mirror expressed in millimeters being the tangent of twice the angle α moved by the mirror. Since the scale is so short, and the distance at which it is placed is about a meter, no appreciable error is committed in taking the arcs for the tangents and in considering the current-strengths as directly proportional to the scale-readings. By the use of shunts, reducing the current through the galvanometer to one tenth, one hundredth, or one thousandth, the range of these serviceable instruments may be very considerably extended. The magnetic system is controlled by means of the curved directing-magnet shown in the figure.

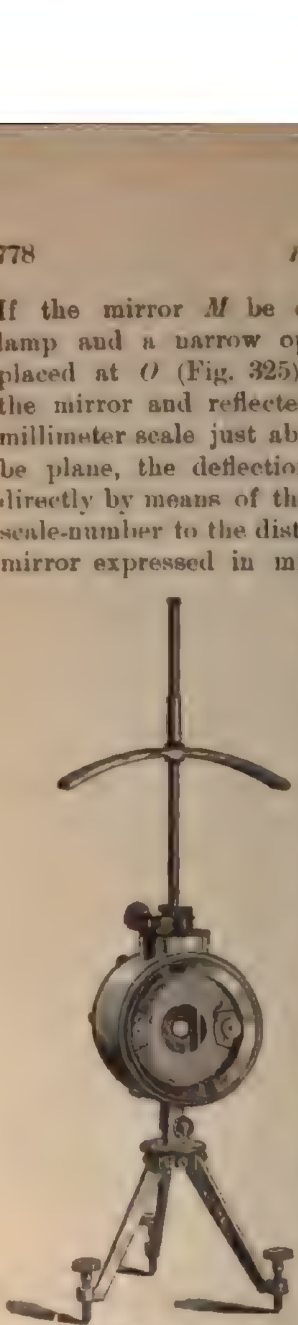


FIG. 324

In the Deprez-d'Arsonval galvanometer (1882) the magnet is fixed and the coil is movable (Fig. 326). A compound U-magnet of steel is supported vertically on a stand, its poles being upward. Between these poles is suspended a rectangle consisting of a large number of turns of fine wire, the ends of which terminate above and below in hard-drawn silver or copper wires serving

as the supporting axis. The upper wire is fastened to an adjustable screw, held by a lateral pillar. The

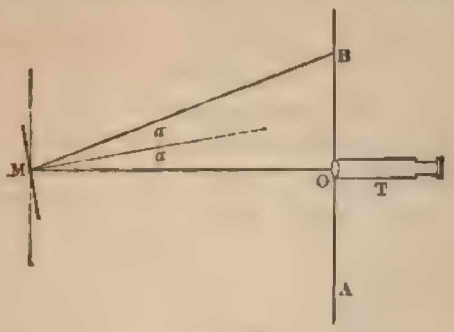


FIG. 825.

lower wire is attached to a flat elastic strip of metal, the tension of which is regulated by a similar screw. Through these wires the current passes to the coil. Within the rectangle an iron tube is supported, serving to concentrate the magnetic field. The upper end of the rectangle carries a mirror, by means of which the angles of deflection are read with a telescope and scale. When a current passes through the coil, it tends to rotate so as to place its plane, which normally coincides with that of the poles of the magnet, perpendicular to this direction, and thus to enclose the maximum number of lines of force. The couple due to the current is antagonized, and finally balanced, by the couple of torsion of the wire. To avoid the set sometimes experienced in

FIG. 826.

the wire, as well as to increase the sensitiveness of t[he] instrument, Gerard replaced the straight wire by a hel[ix] of phosphor-bronze wire, the rectangle being suspend[ed] on a cocoon fiber or a quartz fiber in its axis. On sho[rt-] circuiting the instrument and oscillating the rectangl[e] the currents induced in the coil bring it speedily to re[st.] Whence the name **aperiodic** given to this galvanometer.

677. Galvanometric Mode of Measuring Potenti[als.] —Evidently, if the galvanometer terminals be connect[ed] to points at different potentials, a current will flo[w] through it. And hence the amount of current th[us] passing may be used to measure this difference [of] potentials. In order that the instrument may not its[elf] produce too great a fall in the potential to be measur[ed,] however, a galvanometer for measuring potential-diffe[r-] ences must be made of high resistance, say from five [to] fifty thousand ohms or more. And then the curre[nt] through it, and therefore the deflection it gives, is close[ly] proportional to the difference of potential at its term[i-] nals. If the use of no current is permissible, then clea[rly] the measurements must be made with an electrometer.

678. Ballistic Galvanometer.—It is often desirab[le] to measure the discharge of a condenser galvanomet[ri-] cally. For this purpose, since the time is so short, th[e] ballistic principle is made use of. From dynamics [we] have $Ft = Ms$ (75); i.e., the impulse of a force is mea[s-] ured by the momentum it produc[es.]

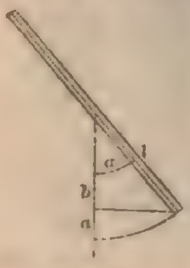


FIG. 327.

Moreover, the speed s acquired by [a] body in falling through a distance [a] under the action of the force of gravit[y] g is $s = \sqrt{2ga}$. Hence $Ms = \sqrt{2M^2g}$[...] or in the case of the earth's magnet[ic] field, where the force acting is not [Mg] but mH, $Ms = \sqrt{2mHMa}$. Now a is t[he] direct distance through which the bo[dy] has moved in the direction of the fo[rce] (Fig. 327); hence $a = l - b = l - l \cos \alpha = l(1 - \cos \alpha)$ [=] $l(2 \sin^2 \frac{1}{2}\alpha)$. Moreover, the force F exerted by the c[ur-] rent on a galvanometer needle, $= GmI$; and hence Ft [...]

$GmIt = GmQ$. Whence $GmQ = \sqrt{4mHMl \sin^2 \frac{1}{2}\alpha} = 2\sqrt{mHMl} \sin \frac{1}{2}\alpha$; and

$$Q = 2\sqrt{\frac{HMl}{mG^2}} \sin \tfrac{1}{2}\alpha. \qquad [79]$$

Consequently, when the galvanometer-needle is moved by an impulse like the discharge of a condenser, the quantity of electrification traversing the galvanometer is directly proportional to the sine of half the angle of deflection. The needle of a ballistic galvanometer should have a slow period of oscillation, so that during the time of the discharge it may not be sensibly displaced. It should be short and of small mass, should be strongly magnetized, and should have a moderately strong directing field. The coil should have a large number of turns.

679. Ammeters and Voltmeters.—For commercial uses, measuring instruments called **ammeters** and **voltmeters** are employed; these being empirically graduated so that with the former instrument the current-strength in amperes, or, with the latter, the potential-difference in volts between its terminals, may be read off directly. The voltmeter is generally only a high-resistance ammeter. It is placed in shunt circuit.

680. Electromagnets.—Since an electric current produces a magnetic field in its vicinity, and since a bar of iron is magnetized when placed in a magnetic field, it is evident that a bar of iron may be magnetized by a current. Such a bar thus magnetized is called an **electromagnet**; the first electromagnet having been made by Arago in 1820. Moreover, since the lines of force are perpendicular to the axis of the current, the bar is to be placed at right angles to the wire; and the effect is multiplied by winding the wire many times around the bar, after the manner of a helix. The direction of polarity in the bar depends only upon the direction of the current in the helix. Maxwell represents the direction of the lines of force in a helix as the same as that

in which the axis of a corkscrew advances when it is rotated in the same direction as that in which the current flows in the helix. Ampère's rule shows that when swimming with the current and facing the interior of the helix the marked end is on the left hand. Or looking down upon one end of a helix, it will be a marked end if the current circulates in it counter-clockwise, Fig. 328; and *vice versa*. The intensity of the field produced by an element of the current is given by the law of Laplace, this intensity being the ratio of the whole force F produced by the current to the strength of the

FIG. 328.

pole m, on which it acts; i.e., $H_l = F/m$ (668). For an actual current, the field intensity depends upon its form as well as on the current-strength. A system of equal circular currents small and parallel to each other was called by Ampère a **solenoid**. If a helix be wound with an even number of layers, it becomes equivalent to a solenoid, and consists of a series of circular currents normal to its axis, the components of the spires of the uneven layers along the axis being balanced by those of the spires of the even layers, since the current in these components flows oppositely; i.e., in the one outwardly, and in the other inwardly. In such a helix, of very considerable length as compared with its diameter, the magnetic field produced by the current may be considered uniform, and the lines of force parallel to its axis. Calling l its length, N the number of turns of wire in it, and I the current-strength, the intensity of field $H_l = 2\pi I\,r$ $= 4\pi NI/l$, since $l = 2\pi rN$; or if $n = N/l$, the number of turns in unit of length, $H_l = 4\pi nI$.

EXPERIMENT.—Coil a wire into a helix, and attach its ends to plates of copper and zinc. Immerse these plates in acidulated water contained in a weighted glass vessel (Fig. 329), and float the whole in water. As shown by the arrows, a current will flow through the helix, and will develop magnetic polarity in it; so that the helix will place itself with its axis in the meridian, and its ends will re-

spond to magnetic attractions and repulsions. This apparatus is known as De la Rive's ring. The solenoid *So* (Fig. 333) acts similarly.

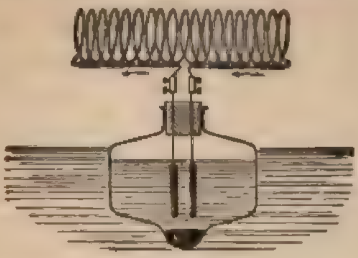


FIG. 329.

681. Equivalence of Currents and Magnets.—Since a magnetic field of the same character can be produced either by a current or by a magnet, it is pertinent to inquire when these are equivalent. The intensity of field produced by a circular current of strength I and radius r (and, therefore, of surface πr^2,) at a point P on its axis distant d from its plane (Fig. 330), is $H' =$

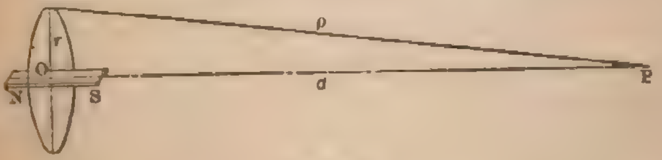


FIG. 330.

$2\pi r^2 I/d^3 = 2SI/d^3$ (668). If a magnetic needle whose moment is ml be placed at P, it will be acted on by a couple of the value of the product of these quantities, $2SI.ml/d^3$. Substituting now for the circular current a bar magnet NS whose moment is M, it will produce a couple acting on the needle whose value is $2M.ml/d^3$ (553). Evidently these couples will be equal when $M = SI$; in other words, a magnet is the equivalent of a current when its moment is equal to the product of the current-strength by the area of the circular current. In the case of a magnetic shell, the action of a closed current is identical

with that of a magnetic shell of the same contour, when the magnetic power of the shell is equal to the current-strength in electromagnetic units.

682. Closed Magnetic Circuits.—The analogy between magnetic and electric closed circuits, originally suggested by Faraday, has led to the application of the law of Ohm, in form at least, to the former; as appears to have been done first by Rowland in 1873. The magnetomotive force $\mathfrak{M}$ of the one corresponds to the electromotive force E of the other; the magnetic resistance $\mathfrak{R}$ to the electric resistance R, and the flow of magnetic force Φ to the flow of current I. Consequently, we may write the law of Ohm for the magnetic circuit in the form $\Phi = \mathfrak{M}/\mathfrak{R}$ analogous to $I = E/R$, its form for the electric circuit. The magnetomotive force $\mathfrak{M}$ which generates the flow of induction is represented by $4\pi NI$, in which N is the whole number of turns in the solenoid, and I the current flowing through it. The specific magnetic resistance, i.e., the resistance of unit length of a magnetic conductor of unit cross-section, is the reciprocal of the magnetic permeability, μ. If ρ be the specific magnetic resistance for a rod of given material, l its length, and s its section, we have the total magnetic resistance or $\mathfrak{R} = \rho\frac{l}{s} = \frac{1}{\mu}.\frac{l}{s}$; in entire analogy with electric resistance. Hence the complete expression for the flow of magnetic force in a circuit is:

$$\Phi = 4\pi NI \Big/ \left(\frac{1}{\mu}.\frac{l}{s}\right) = 4\pi NI\mu s/l. \qquad [80]$$

As in the case of dielectrics, the specific magnetic resistance of air is taken as unity; so that para-magnetic substances whose permeability is greater than that of air have a less specific resistance, and diamagnetic substances a greater one. Specific magnetic resistance is very variable, since it depends not only on the nature of the substance itself, but also on the magnetomotive force applied to it and on the previous magnetic history of

the body magnetized. On applying an armature to a U-magnet, its magnetic potential falls, just as the electric potential falls at the terminals of a generator when it is short-circuited. But the flow of magnetic induction increases in the one case exactly as the flow of current increases in the other. A piece of iron placed near a magnet, constitutes a part of the magnetic circuit of greater permeability than the air, and therefore of less resistance. The flow of force is increased, and the iron is attracted by the magnet.

683. Magnetization by Currents.—The magnetism developed in a magnetic substance when placed in a magnetic field depends not only upon the intensity of the field, but also upon the nature of the substance, upon its physical condition, upon the temperature, etc. While a current-field induces magnetism precisely in the same way as a magnet field, yet since a current-field may be made much stronger, a higher induction may be obtained by means of it. Electromagnets are hence preferred for all purposes where intense magnetic action is desired. The intrinsic energy of magnetization as already shown is half the potential energy of a given magnet in a given field (560); i.e., if MH represent the potential energy of a magnet of moment M in a field of intensity H, one half this value will represent the work which must be expended on it in order to magnetize it. Hence $W = \frac{1}{2}MH = \frac{1}{2}mlH$. To separate the armature of such a magnet to a distance l, the force exerted is W/l or $\frac{1}{2}mH$. But $m = JS$, and for a closed circuit $H = 4\pi J = B$. Whence $F = 2\pi J^2S = B^2S/8\pi$ dynes, or $4 \times 10^{-5}B^2S$ grams. So that for a magnet of good steel, having a permanent magnetic induction B of 10000 units, the lifting force will be about 4000 grams per square centimeter of cross-section. In the case of an electromagnet, the same result is reached thus: The flow of induction for a bar of iron of section S is BS. But this flow has just been shown to be $4\pi NI\mu S/l$. So that $B = 4\pi NI\mu/l$; or calling $4\pi NI$ the magnetomotive force $\mathfrak{M}$, $B = \mathfrak{M}\mu/l$. Whence $F = (S/8\pi)(\mathfrak{M}\mu/l)^2$ dynes.

For a given magnetomotive force, the magnetic induction B is directly proportional to the permeability and inversely proportional to the length of the magnetic circuit. For an electromagnet to have a high lifting power, the cores must be short and large, in order that the permeability may remain as great as possible. The value of B in iron has been pushed as high as 20000 C. G. S. units, or twice the value obtained in steel. An electromagnet of this iron, under these conditions, would have a lifting power of 16000 grams nearly, per square centimeter of cross-section (Joubert).

684. Ampere-turns.—The above value of F expressed in grams is $4 \times 10^{-5}(\mathfrak{M}\mu/l)^2S$. Since $\mathfrak{M} = 4\pi NI$, the value of $NI = 125l/\mu \,.\, \sqrt{F/S}$; in which I is given in amperes. The expression NI is the product of the number of turns of wire in the coil by the current-strength, and represents the magnetizing power of the coil. From the above formula, therefore, the number of **ampere-turns** required to produce an electromagnet of a given lifting power may be calculated, the dimensions of the core and the permeability of the iron being given. For example, if $\mu = 300$, $F = 500$ kilograms, the two cores being each 25 cm. in section and 15 cm. long, the centers being separated 10 cm., l will be 50 cm. Substituting in the equation $NI = 125l/\mu \,.\, \sqrt{F/S}$, we have

$$125 \times \frac{50}{300} \times \sqrt{\frac{500000}{50}}$$ or 2083 ampere-turns required.

(*b*) *Electrodynamics.*

685. Principles of Electrodynamics.—The mutual action of currents upon each other, which constitutes the subject of **electrodynamics**, was developed by Ampère in 1821. The principles enunciated by him are as follows:

I. Currents which are parallel and which flow in the same direction, attract each other; while parallel currents which flow in opposite directions repel each other.

II. Currents which are not parallel attract

each other if they both flow toward or both flow from the point of intersection; and they repel each other if one flows toward and the other away from this point.

Evidently this action is due in all cases simply to the magnetic current-fields; and since equilibrium is unstable unless the lines of force are parallel and in the same direction, the above principles may be generalized by saying that whenever two currents are brought near each other a mechanical stress is produced between them in such a direction as to make the flow of force, and hence their potential-energy, a maximum.

EXPERIMENTS.—1. Support a rectangle of wire *H* (Fig. 331) upon a suitable stand *S*, so that it can rotate freely around a vertical

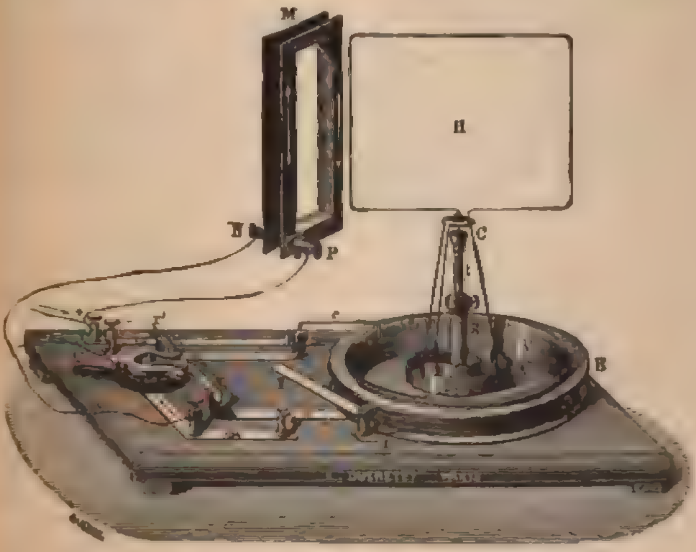


FIG. 331.

axis. Send a current through it and bring near to and in front of one of its sides a bundle of parallel wires *M* through which a current flows in the same direction. The rectangle will turn so as to bring its side into contact with the parallel wire.

2. By means of the commutator *C*, reverse the current in the rectangle so that it now flows in the opposite direction to that in the

side of the [illegible] of parallel wires nearest it. That side of the rectangle will [illegible] from the parallel wire.

3. Place the [illegible] of wires horizontally above the rectangle intersecting the [illegible] of it at its middle point, and making an angle with this [illegible] current flow from A to B (Fig 332) through the bundle of wires, and from C to D through the side of the rectangle contiguous to it; i.e., if on the lower side it flow toward the intersection O and on the upper side from it, the rectangle will rotate clockwise so as to make the currents parallel. If, however, the current in the bundle of wires be made to flow in the opposite direction from B to A; i.e., if on the upper side it be made to flow toward the [illegible], while the current in the rectangle flows from it, and [illegible], then the rectangle will rotate in the other direction or [illegible]kwise.

FIG. 332.

4. Suspend a helix of [illegible] so that its lower end just dips into mercury, and send a current through it. Owing to the attraction of the parallel currents in the adjoining spires the helix will shorten and lift the end out of the mercury, thus breaking the contact. The weight of the coil will lower the end again and re-establish it; and so on, the coil continuing to vibrate vertically. This apparatus is known as Roget's oscillating spiral.

The above apparatus may be employed to show the converse of Oersted's experiment, the magnet being fixed and the conducting wire movable.

EXPERIMENTS.—1. Bring the marked pole of a magnet near one side of the rectangle, the magnet being horizontal. If the current in the rectangle is flowing from below upward, the rectangle will move so as to leave the marked pole on the left of an observer lying in the wire and facing the pole. If the current be downward, the direction of rotation will be reversed.

2. Place the magnet horizontally above the rectangle, its axis forming a small angle with the upper side. The rectangle will rotate so as to place its plane perpendicular to the axis of the magnet. Holding the magnet so as to pass through the rectangle, all the actions conspire to make the two perpendicular.

3. Insert a magnet into the axis of the oscillating spiral above mentioned. If its poles be opposed to those of the coil the coil will be brought to rest. If the poles of the magnet and those of the coil agree in name, the oscillations will be increased in amplitude.

Since the lines of force of the magnet are parallel to its length, while those of the rectangle are perpendicular to its plane, stable equilibrium requires that the plane of the rectangle shall be at right angles to the axis of the magnet. Indeed, if the suspended wire be in the form of a circle, and a sufficient current be sent through it, the circle will turn so as to place its plane east and west, and its axis north and south. Thus the axis will be parallel to the magnetic meridian, and the circuit will enclose within its contour the maximum number of lines of force, according to Maxwell's rule. This sensitiveness to the earth's field is sometimes undesirable; and it may be obviated by winding a double rectangle (Fig. 333, H'), so that the direction of the

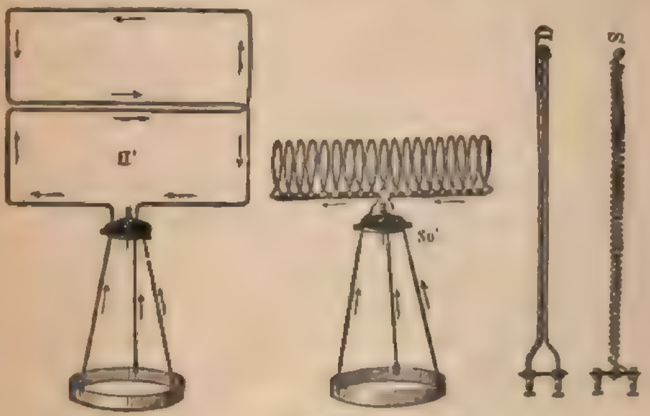


FIG. 333

current in the upper half of each portion is opposite to that in the lower, the whole thus constituting an astatic system.

686. Quantitative Relations.—The following relations were established by Ampère: (1) The force exerted between two current-elements is directed along the line joining their centers, is proportional to the product of their lengths and to the product of their current-strengths; it varies inversely as the square of their distance and is a function of their relative directions;

(2) the direction of the force changes when that of either of the currents is reversed; if both currents are reversed, it remains unaltered; (3) the action of two current-elements, one of which is perpendicular to the other at its center, is zero; (4) a sinuous current (Fig. 333, *S*) has the same action as a straight current (Fig. 333, *D*) between the same points; and hence two such currents, if opposite in direction, neutralize each other and produce an **adynamic** system. Representing by a and b (Fig. 334) two current-elements of lengths l and

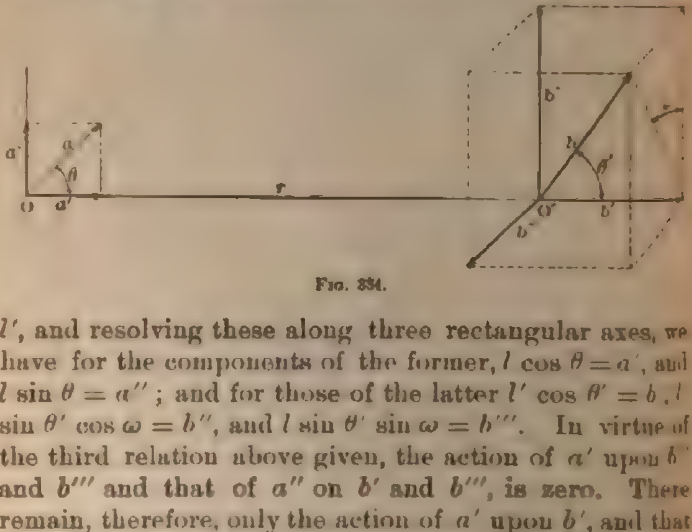


FIG. 334.

l', and resolving these along three rectangular axes, we have for the components of the former, $l \cos\theta = a'$, and $l \sin\theta = a''$; and for those of the latter $l' \cos\theta' = b'$, l' $\sin\theta' \cos\omega = b''$, and $l' \sin\theta' \sin\omega = b'''$. In virtue of the third relation above given, the action of a' upon b'' and b''' and that of a'' on b' and b''', is zero. There remain, therefore, only the action of a' upon b', and that of a'' upon b''. The action of a' upon b' is that of two elements in the same straight line; and the action of a'' upon b'' is that of two parallel elements, perpendicular to the line joining them. The force exerted between the components of each of these pairs is proportional to the product of their lengths, to the product of their current-strengths and to the square of the distance r between them. For the action of a'' on b'', therefore, we have

$$f = \frac{II'll'}{r^2}(\sin\theta \sin\theta' \cos\omega),$$

and for the force acting between a' and b', we have

$$f' = -\frac{\mathfrak{i}\mathfrak{i}'ll'}{2r^2}(\cos\theta\cos\theta'),$$

since the effect of the two elements acting in the same line is half that of the two parallel elements at the same distance, and is in the opposite direction. The total action F which takes place along the line OO' in virtue of both these actions is their algebraic sum evidently:

$$F = \frac{\mathfrak{i}\mathfrak{i}'ll'}{r^2}(\sin\theta\sin\theta'\cos\omega - \tfrac{1}{2}\cos\theta\cos\theta'). \qquad [81]$$

But if ϕ be the angle which the two elements make with each other, we have $\cos\phi = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos\omega$; and substituting this value in the equation for F, we have

$$F = \frac{\mathfrak{i}\mathfrak{i}'ll'}{r^2}(\cos\phi - \tfrac{3}{2}\cos\theta\cos\theta'), \qquad [82]$$

which is the form of the equation given by Ampère. If the two elements are parallel to each other and perpendicular to the line joining them, $\phi = 0$, and $\theta = \theta' = 90°$; whence for the force between these elements we have $F = \mathfrak{i}\mathfrak{i}'ll'/r^2$. So if the elements are in the same straight line, ϕ, θ, and θ' are all zero, and $F = -\mathfrak{i}\mathfrak{i}'ll'/2r^2$. Since two parallel currents in the same direction attract each other, the value of F in the first case above,

FIG. 335.

being positive, indicates an attraction; whence in the second case, being negative, it indicates a repulsion. That the consecutive elements of the same current actually do repel each other Ampère proved by floating a wire

frame F on mercury, as shown in the figure (Fig. 335). On establishing contact, the frame recedes from the conducting wires.

687. Electrodynamic System of Units.—Strictly speaking, the current-elements above given should be written dl and dl'; and then if, by means of the formula, the force exerted upon a current of length l by a current of indefinite length be calculated, it will be found to equal $\mathfrak{I}\mathfrak{I}'l/r$. If now, in the expression $F = \mathfrak{I}\mathfrak{I}'l/r$, the acting force, the length of the current, and the distance of the indefinite current each be made unity, and if the currents themselves be made equal, these currents will each be equal to unity. That is to say, if $\mathfrak{I} = \mathfrak{I}'$, $\mathfrak{I} = \sqrt{Fr/l}$. And we may define the electrodynamic unit of current as that rectilinear current which is attracted by an indefinite rectilinear current placed at a distance equal to its length, with a unit of force. From the dimensions of the other units, their values in the electrodynamic system can be readily computed.

688. Electrodynamic Action of two Closed Circuits.—Suppose two closed circuits in the form of squares (Fig. 336), having sides a and b, and carrying currents

FIG. 336.

$\mathfrak{I}$ and $\mathfrak{I}'$, in the direction of the arrows, the plane of b being perpendicular to that of a and passing through its center, d being the distance between their centers. Let the circuit a be fixed, and b capable of rotation about YY' as an axis. This latter circuit is evidently acted on

by a couple due to the mutual action of the currents. The side a' repels b'' with a force $ab\mathfrak{I}\mathfrak{I}'/d''^2$; and the component of this force normal to the side b'' is $ab\mathfrak{I}\mathfrak{I}'$ sin $Ob''a'/d''^2 = a^2b\mathfrak{I}\mathfrak{I}'/2d''^3$. In the same way a'' produces upon b'' an attraction whose component along the normal has the same value; and hence the total force is $a^2b\mathfrak{I}\mathfrak{I}'/d''^3$. Since the arm of the couple is $\frac{1}{2}b$, the moment of this couple is $a^2b^2\mathfrak{I}\mathfrak{I}'/2d''^3$. On the side b', there is produced in a similar way a similar and like couple. So that the total moment acting to rotate the circuit b is $\frac{1}{2}a^2b^2\,\mathfrak{I}\mathfrak{I}'(1/d'^3 + 1/d''^3)$. If d be so great with reference to the dimensions of the squares that we may write $d = d' = d''$, then the total rotating force $R = a^2b^2$ $\mathfrak{I}\mathfrak{I}'/d^3$; or calling a^2 and b^2 the surfaces of these squares, $R = SS'\mathfrak{I}\mathfrak{I}'/d^3$. If now we make $S = S' = 1$, $\mathfrak{I} = \mathfrak{I}'$ and $R = \dfrac{1}{d^3}$, we have $\mathfrak{I} = 1$; and since these results are independent of the form of the circuit, we may say that the electrodynamic unit of current is that current which traversing two circular conductors each of surface equal to unity, placed with their planes normal to each other and at a considerable distance, so that the center of the first is on the normal erected to the center of the second, produces in the first circuit a couple the moment of which is the reciprocal of the cube of the distance between the centers (Weber). If the square b be fixed and a be movable about the axis XX', the moment will be one half that above calculated.

689. Ratio of the Electrodynamic to the Electromagnetic Unit of Current.—It has been shown (668) that a current of surface S produces upon a magnetic needle of moment M at a distance d a couple whose value is represented by $R' = 2SIM/d^3$; where I is the current-strength in electromagnetic measure. But magnets are equivalent to currents when $M = SI$ (681); i.e., when the moment of the former is numerically equal to the product of the surface into the current-

strength of the latter. Hence we may replace M in the above formula by $S'I'$; and we have $R' = 2SS'II'/d^2$. Since these couples are equal, $R' = R$, and $2SS'II'/d^2 = SS'\mathfrak{I}\mathfrak{I}'/d^2$. If the surfaces be equal in both cases, and $I = I'$ and $\mathfrak{I} = \mathfrak{I}'$, we have $\mathfrak{I}^2 = 2I^2$; or $\mathfrak{I} = I\sqrt{2}$. That is to say, the current-strength in electrodynamic units is equal to that in electromagnetic units multiplied by the square root of two; and the electromagnetic unit is equal to the electrodynamic unit multiplied by $\sqrt{2}$. The electromagnetic unit of current, then, is 1·4 times as large as the electrodynamic unit. Hence, to obtain the value of the rotation-couple above given, when the current flowing through both circuits is measured in electromagnetic measure, it is necessary to multiply by two the value obtained from the formula.

690. Electrodynamometers.—These principles are applied in practice to the construction of current-measuring instruments, first proposed by Weber in 1846 and called **electrodynamometers.** They consist substantially of two coils of wire placed with their axes either perpendicular to each other or in the same straight line; one of these coils being fixed, the other capable of motion. They may be divided into two classes corresponding to this construction, the one class being called torsion electrodynamometers and the other balance electrodynamometers. In Weber's instrument, which belongs to the first class, the smaller and movable coil is suspended bifilarly within the larger and fixed coil, with its axis at right angles to that of the latter. The two suspending wires carry the current to the inner coil; and when this coil is rotated by the current, this bifilar suspension develops a restoring couple which is proportional to the sine of the angle of deflection. As the electrodynamic couple is proportional (1) to the product of the two current-strengths in the coils, (2) to the cosine of the angle of rotation, and (3) to a constant depending on the construction of the instrument, we have $\kappa \sin \alpha = II' \cos \alpha$. Whence $II' = \kappa \tan \alpha$; or if the same current is sent through both coils, $I^2 = \kappa$

tan α; i.e., the square of the current-strength is proportional to the tangent of the angle of deviation. Since the couple acting in the electrodynamometer is a function of the square of the current-strength, it is entirely independent of the direction of the current, the square of both positive and negative currents being positive. Hence this instrument is used to measure alternating as well as direct currents. The actual current-strength, however, is measured only in the case of the direct current; the quantity measured in the case of alternating currents being $\sqrt{(I^2)_m}$, or the square root of the mean square of the current-strength.

The electrodynamometer of Siemens & Halske (Fig. 337) is intended for industrial measurements. The

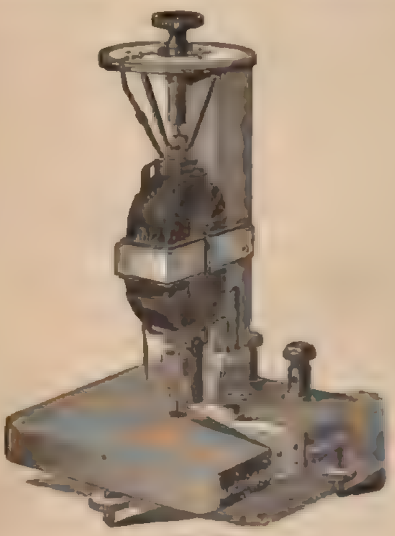


FIG. 337

interior coil is rectangular in shape and is fixed. The outer coil is also rectangular, but consists of only a single turn or a few turns of wire. It is supported on a cocoon or a quartz fiber, with its plane perpendicular to that of the inner coil. Its two ends dip into mercury cups at different levels beneath it, by which it receives current.

Surrounding the suspending fiber is a wire helix, the lower end of which is attached to the movable rectangle, while the upper end is fastened to a milled head carrying an index movable over a graduated circle. A second index fastened to the movable rectangle points to the zero of the circle when this rectangle is perpendicular to the other one. This index should face the other one when no current is passing and no torsion exists on the helix. On passing a current through both coils in succession a couple is developed which causes the rectangle to rotate. By turning the milled head in the opposite direction, the torsion of the helix is made to balance this couple, and the index on the rectangle returns to zero. The angle of torsion is then read off on the circle and the current-strength obtained by comparison with a table of values given by an empirical calibration of the instrument. Generally, the fixed coil is double, and consists of two wires, a coarser and a finer one; thus rendering the instrument available for a wider range of current-strengths.

Since the couple producing the rotation in these instruments is kII', or kI^2 when the coils are in series, it is evident that if one of the coils, the movable one for example, be made of very fine wire and be connected as a shunt to the other coil, the current traversing it will be the measure of the potential-difference at the terminals of the instrument; and hence the couple will be k_1eI; or in other words, the couple will be proportional to the energy of the current, and the electrodynamometer will act as a watt-meter.

691. Electrodynamic Balances. — Recently Lord Kelvin has contrived improved electrodynamometers of the second class, in which the force or couple due to the electric action is balanced by weights. The main difficulty in this class of instruments is in conveying the current to the movable coil, especially when this current is of considerable strength. This is accomplished in these instruments by means of a metallic hinge or joint, on which the movable part is suspended. This movable

part, as shown in the diagram (Fig. 338), consists of two coils, *A* and *B*, supported at the ends of a horizontal beam like that of a balance. Two fixed coils, *a* and *b*,

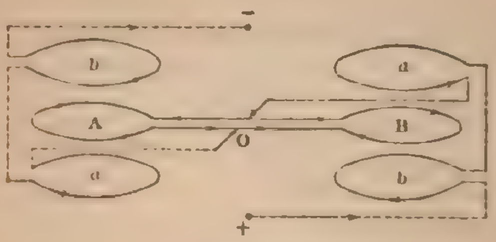


FIG. 338.

are arranged opposite each of the movable ones, one being above it and the other below it; so that when the current passes through the system the movable portion turns about its center *O*. Weights are employed to bring it back to zero, and by knowing the weight required, the current may be indicated. Several types of instrument have been constructed, giving a range from 0·1 ampere to 2500 amperes; or by using fine coils, from 10 to 200 volts. The form of these balances is shown in the annexed figure (Fig. 339). Evidently, by the use of both

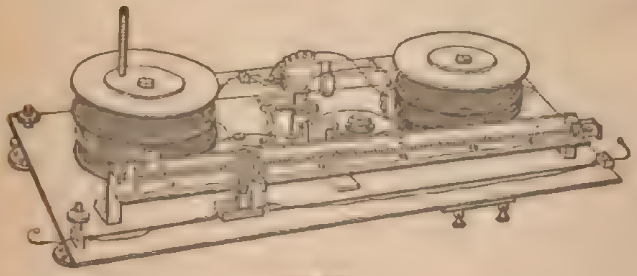


FIG. 339

high and low resistance coils, these instruments may be used as watt-meters; and by certain peculiarities of

construction they have been converted into direct reading instruments.

692. Electrodynamic Rotations. — Under suitable conditions, the mutual action between two currents may be made to produce continuous motion. Thus, for example, suppose that a vertical current *ab* (Fig. 340) be

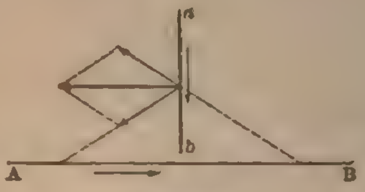


FIG. 340.

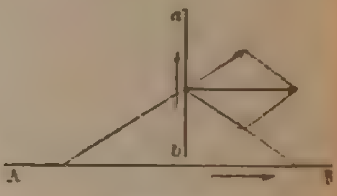


FIG. 341.

made to approach an indefinitely long horizontal one *AB*. Since on the left side of the point of intersection both currents approach this point, and on the right side one current recedes from it, while the other approaches it, the electrodynamic action will be attractive in the left-hand quadrant and repulsive in the right-hand one, as is shown by the arrows; the resultant of which will be a force parallel to and in the opposite direction to the indefinite current. If the finite current *ba* flow from the intersection (Fig. 341), it will experience a force tending to displace it parallel to the indefinite current and in the same direction. Precisely the same result follows in the case of radial currents with reference to circular ones. A radial current *Oa* or *Ob* (Fig. 342) flowing toward a circular one experiences a force tending to move it in the opposite direction to that in which the circular current is flowing. These actions were simultaneously demonstrated by Ampère by means of the simple

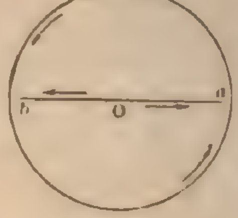


FIG. 342.

apparatus shown in the figure (Fig. 343), which is used on the stand shown in Fig. 331. A circular coil of wire B on this stand encloses a flat copper dish containing diluted sulphuric acid. In the center is a vertical rod S having a mercury cup at its upper end, in which rests the pivot of the rectangular frame F, the lower ends of which are fastened to a flat metal ring dipping into the acid.

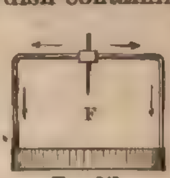


FIG. 343.

When the current in the rectangular frame flows as the arrows indicate, that in the coil being clockwise, a continuous counter-clockwise rotation takes place, due to the conjoint effect of the vertical currents and of the radial currents, both of which produce rotation in the same direction.

693. Electromagnetic Rotations.—Since every current is surrounded by $4\pi I$ equipotential surfaces, which are radial planes parallel to its axis (665), an amount of energy equal to $4\pi I$ ergs expended upon unit pole by the current will cause it to travel once round the current-axis. If the mass of the pole be m, $4\pi mI$ ergs must be expended. So that the expenditure continuously of this energy will cause continuous rotation, the speed depending on the rate of expenditure. Since action and reaction are equal, the current may also be made to revolve about the pole; this result being dependent upon the construction of the apparatus. Take, for example, a marked pole, N (Fig. 344), and suppose a semicircular conductor, AB, placed near it, movable about a diameter as an axis, this axis coinciding with that of the magnet.

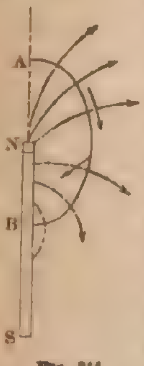


FIG. 344.

If the current flow from above downward as shown, the conductor will tend to revolve about its axis in the direction viewed from above, of the hands of a watch. These conditions are very easily realized experimentally as follows:

FIG. 345.

EXPERIMENT.—Close a tube of glass of suitable size wi corks (Fig. 345). Through the upper cork pass a wire ending below in a hook and supporting a wire hung on the hook. The lower cork carries net. If now mercury be poured in so as to co end of the movable wire, and a current be sent t the apparatus, the wire will rotate continuousl the pole; in one direction if the pole is the mark in the other if it is the unmarked one. Reversal current reverses the direction of the rotation.

Faraday in one of his experiments att the magnet by a flexible cord to the cond entering the bottom of a vessel filled mercury. On touching the mercury s at its center with the other conductor upper pole of the magnet rotated abou conductor as an axis. Ampère modified experiment by weighting a cylindrical m with platinum at its lower end and then ing it vertically in mercury (Fig. 346, *A*). If o the conducting wires, *a*, be placed in a globu

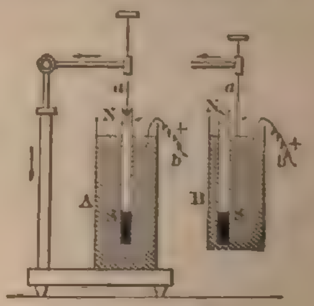


FIG. 346.

mercury in a cavity formed on its upper end, an other wire, *b*, in the mercury in the vessel, the net rotates on its own axis in a direction deter by the direction of the polarity of the magnet and current. If the vertical wire, *a* (Fig. 346, *B*), be to touch the surface of the mercury, the weighted m

being placed excentrically, this magnet will revolve about the conducting wire. With the marked pole uppermost, and the flow of the current as shown, the rotation in both cases is counter-clockwise. Indeed, if a cylindrical magnet be supported vertically between two pivots (Fig. 347), and one of the conducting wires be touched to one of these pivots, while the other wire be pressed on the circumference of the magnet at its middle point, the magnet will rotate.

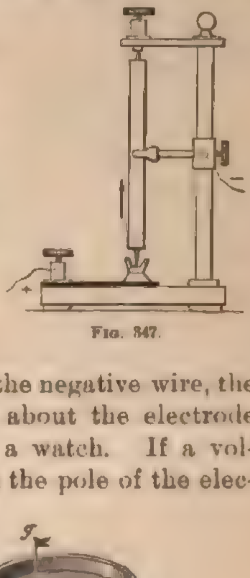


FIG. 347.

Liquids and even gases may be made to rotate in the same way as solids. Davy showed the rotation of mercury by placing two platinum electrodes just below the surface of this liquid, and then bringing down from above a magnetic pole. If this be a marked pole and it be brought over the negative wire, the mercury is depressed, and rotates about the electrode in the direction of the hands of a watch. If a voltameter C (Fig. 348, A) be placed on the pole of the elec-

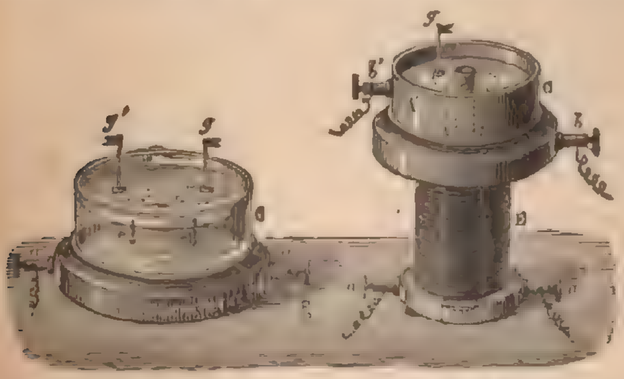


FIG. 348, A. FIG. 348, B.

tromagnet E, the liquid about the two electrodes e, e' will rotate in opposite directions, as is shown by the motion of the two cork floats g, g' (Jamin). By using concentric

annular electrodes A, A' in a circular trough C containing dilute sulphuric acid and having the magnet pole in the center (Fig. 348, B) the acid liquid may be made to rotate, the motion being made apparent by the cork float g on the surface (Bertin). If in a vacuum tube the discharge be made to take place parallel to the axis of a magnet placed in its center, this discharge will rotate about the magnet as an axis (De la Rive). In a similar way the electric arc may be made to rotate about the pole of a magnet suitably placed.

694. Ampère's Theory of Magnetism.—The practical identity in character between the action of a solenoidal current and a solenoidal magnet leads to the conclusion that magnetism itself is simply a vortical electric current, as Ampère long ago suggested. Numerous experiments show the rotary nature of the magnetic action with reference to the current. A current and a pole neither attract nor repel each other; they tend to rotate about each other, the action being at right angles to the line joining them. If a meter or two of a flexible conductor, such as gold thread, be made to carry as strong a current as it will bear, and a vertical bar magnet be brought near it, the thread will coil itself into a spiral, half of it twisting round the marked end and the other half twisting as a part of the same spiral round the south end (Lodge). If the conductor were made rigid and the magnet flexible, the magnet would coil itself round the current in like manner. If both are rigid, the motion is limited by this fact; though it is seen clearly to be due to the same action, although modified by the constrained condition. So if a vertical wire, hanging freely between the poles of a U-magnet (Fig. 349), receive a current passing through it to the mercury cup in which its lower end dips, it will be thrown out of the mercury by the conjoint action of the two poles, to the right or left according to the conditions of the experiment. This will break the current, and gravity will bring the wire again into the mercury; and so it will oscillate. If the vertical conductor form one radius of a star, a second radius will

be brought in contact with the mercury as the first one leaves it; thus producing a continuous rotation, as in Barlow's spur-wheel. The same effect is obtained with a disk. If a current be sent through a stream of mercury

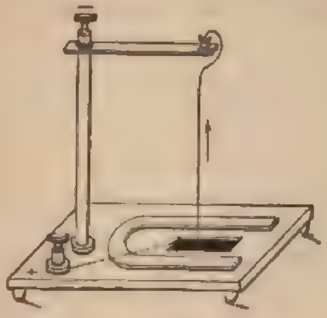


FIG. 349.

falling from one pole to the other of a strong U-magnet with its plane vertical, the liquid will at once twist itself into a flat spiral (Thompson). It would appear, therefore, that precisely as a current is surrounded by a whirl of lines of magnetic force, so a magnet is equally surrounded by an electrical current-whirl (Fig. 350); the same figures illustrating both conditions. In the former case the vertical axis represents a current surrounded by its magnetic whirl, and in the latter it represents a line of force surrounded by its current-whirl. In the second figure both are shown (Fig. 351); the axis being a portion of the electric circuit, this being surrounded by a single magnetic line of force, and this again surrounded with electric whirls; each magnetic line of force round a current being an electric vortex ring (Lodge). If therefore lines of force are to be regarded as the axes of current-whirls, the rotation will have the effect of a force tending to increase the

FIG. 350.

diameter of the whirls and to shorten the length of the axis; just as a flexible tube filled with liquid and rotated on its axis would increase in diameter and shorten in length. The effect will be that a line of magnetic force will tend to shorten itself lengthwise and tend to increase its cross-section; thus accounting for the observation of Faraday, that magnetic attraction is in the direction of the lines of force and is due to this tendency to contraction, while magnetic repulsion is perpendicular to these lines of force and is due to the tendency of the lines of force to repel one another.

FIG. 351

The phenomena of magnetism, however, belong to the molecule, and not to the mass. So that these electrical whirls must be rotations perpendicular to the magnetic axis of the molecule. Ampère's theory, therefore, supposes simply that electric currents circulate round the molecules of a magnetic substance, thereby polarizing them. The permanence of these molecular currents, like that of the motion of the heavenly bodies, depends solely upon the existence of non-resistance; for in this way only can their energy be dissipated. By no known process can these Amperian currents be produced or destroyed. The act of magnetization therefore consists simply in rotating the molecules so that their magnetic axes all face in the same direction; and then the substance is magnetically saturated. If, however, molecules exist round which no currents or only very weak currents circulate, the effect of placing such substances in a magnetic field will be to develop such currents, and this in such a direction that repulsion will exist between it and the field; thus producing the phenomena characteristic of diamagnetism (Lodge). The direction of the Amperian currents is determined by Ampère's law.

D.—MAGNETO-ELECTRIC INDUCTION.

(*a*) *Classification of Inductions.*

695. Induction Currents.—In 1831, Faraday succeeded in obtaining electric currents by means of magnetic actions. This induction of currents he produced in three different ways: (1) by the relative motion of a magnet and of a conducting wire; (2) by the relative motion of a current and of a conducting wire; and (3) by varying the strength of the current in a second wire placed near the conducting wire; the conducting wire itself, in which the induced current was developed, forming in all cases a closed circuit. Evidently, from the laws of electromagnetism, these three methods resolve themselves into a single one, viz., the variation of the strength of a magnetic field in presence of a conducting wire.

EXPERIMENTS.—1. Connect a coil of wire with a galvanometer, and introduce a bar magnet into its center. The galvanometer needle will be deflected, but only so long as the motion continues; the needle returning to zero when the magnet comes to rest. Withdraw the magnet from the coil; the needle will be again deflected, but now in the opposite direction.

2. In place of the magnet, introduce into the coil which is connected with the galvanometer a second coil of wire carrying a current. The galvanometer needle will be deflected in one direction as the second coil enters the first, and in the opposite direction as it is withdrawn from it; remaining at rest when the coil is still.

3. While the second coil is within the first, connect a fine wire across its terminals. The needle will be deflected in the same direction as when the coil is withdrawn; and on removing the wire an opposite deflection will take place. Here evidently the fine wire acts as a shunt and carries a portion of the current; and so diminishes that flowing in the inducing coil. The effect will be the greater therefore in proportion as this shunt wire is larger and shorter.

4. Repeat Experiment 2, and when the galvanometer needle is

at rest, drop several iron wires successively into the interior of the second or inducing coil. The current-induction in the outer coil will be greatly increased by the magnetism thus induced.

5. Wind an iron ring with two wire coils placed at opposite extremities of a diameter. On passing a current through one of these coils strong currents are induced in the other, which are in opposite directions on making and on breaking the inducing circuit. This is the experiment of Faraday.

The general law of current-induction may therefore be stated as follows: Whenever any modification whatever takes place in the flow of magnetic force traversing a closed circuit, a temporary electrical current is produced in this circuit, the duration of which corresponds to that of the variation of the flow. For convenience, induced currents produced by the mechanical variation of permanent or electromagnet fields are called **dynamo** or **magneto-electric currents**, while those produced by the variation of current-fields are called **self-induced** or **mutually-induced electric currents**; the induction being called **self** or **mutual electric induction, respectively.**

696. Direction of Induced Currents.—Law of Lenz.—If the direction of the currents thus induced be examined, it will be observed that the current generated in the outer coil above mentioned on introducing the inner coil into it, is in the opposite direction to that flowing in the inner coil; and on removing it is in the same direction. The former, therefore, is called an **inverse**, the latter a **direct**, current. In general, whenever a magnetic field diminishes in strength in the vicinity of a closed circuit, a current is induced in this circuit flowing in the same direction as that which would be required to produce such a magnetic field; i.e., a direct current. While whenever the field increases in strength the current induced is such as would by itself produce a field opposite in direction to that acting; i.e., an inverse current. Since two like parallel currents attract each other, the production of an induced direct current by

removing the inducing current will develop an attraction between the circuits; a repulsion being produced whenever an inverse current is induced by the approach of the inducing circuit. Hence the law of Lenz: Whenever by the relative motion of two circuits a variation in the flow of force is made to take place in one of them, a current will be induced in this circuit whose direction is such as to oppose the motion by its electrodynamic action.

Since the motion of an electric circuit in a magnetic field tends to take place always in such a direction as to make the flow of force through its contour a maximum, it follows that if by mechanical means the circuit be displaced so as to increase this flow of force through it (1) a current will be induced in the circuit in the inverse direction to that which would produce the flow, and (2) this current will oppose the movement. Conversely, if the motion diminish the flow, the induced current will be direct, and again the motion will be opposed. Hence for a given variation of the flow of force the direction of the induced current is such as to oppose the variation. If the flow increases, the induced current opposes this increase, and therefore must be inverse to the current producing the flow; if it diminishes, the induced current tends to increase it, and therefore must be direct; i.e., in the same direction as that producing the flow. The current changes sign, therefore, whenever the variation changes its direction. And this takes place whenever the flow of force passes through a maximum or minimum.

697. Magnitude of Induced Currents.—The strength of an induced current, like that of a voltaic current, is expressed by Ohm's law; i.e., is the ratio of the sum of the electromotive forces in the circuit to the sum of the resistances. The value of this electromotive force may be found thus (Kelvin, von Helmholtz): Suppose a voltaic cell of electromotive force E to be placed in a circuit of total resistance R. The current I will be E/R; and the energy expended in the element of time dt will

be $EIdt = I^2Rdt$. Let this circuit be moved in a magnetic field so that the flow of force Φ which traverses it in the positive direction is made to vary by an amount $d\Phi$ in the time dt; and call the work thus done dW. If the work is done by the cell itself, the energy, and therefore the current in the circuit, will be diminished thereby, say to I'; and we shall have in place of the above equation, $EI'dt = I'^2Rdt + dW$. But since $dW = I'd\Phi$, we have by substituting and dividing by I', the equation $Edt = I'Rdt + d\Phi$; whence solving for I' we have $I' = \dfrac{E - d\Phi/dt}{R}$. The current-strength now traversing the circuit is evidently that due to the electromotive force $E - d\Phi/dt$; i.e., it is as if the electromotive force of the cell were diminished by an antagonistic or counter-electromotive force $d\Phi/dt$ or e, due to the induction caused by the variation in the flow of force traversing the circuit. If, however, the circuit be moved by an external force so that the flow of force varies in the opposite direction, the counter-electromotive force $d\Phi/dt$ or e will be negative, will add itself to that of the cell, and the current-strength will be increased. Obviously, the development of this counter-electromotive force depends only on the rate of variation of the field, and not at all upon the previous existence of a current in the wire. Consequently the law of the development of electromotive force by induction may be thus stated: The total electromotive force induced in any circuit at a given instant is equal to the time-ratio of the variation of the flow of magnetic force across this circuit. It follows that the total quantity of electricity put in motion by any displacement of a magnetic system is equal to the quotient of the variation of the flow of force corresponding to this displacement by the resistance of the circuit. Thus if the flow vary from Φ_2 to Φ_1 in a circuit whose resistance is R, the quantity of electricity Q put in motion is $(\Phi_2 - \Phi_1)/R$. It is independent of the time and of the mode of the variation. The current I, however, is a function of the time,

since $Q = Idt$; whence if the variation of the flow $\Phi_2 - \Phi_1$ take place in the time t, the current $I = \frac{e}{R} = \frac{1}{R} \cdot \frac{\Phi_2 - \Phi_1}{t}$ $= (\Phi_2 - \Phi_1)/Rt$. Or, in other words, if the strength of the field vary from Φ_2 dynes to Φ_1 dynes in the time t (i.e., if the number of lines of force vary from Φ_2 to Φ_1) the current-strength in electromagnetic units will be obtained by dividing this difference by the resistance of the circuit and by the time. Evidently if Φ_2 is the greater, the current will be a direct or positive one; while if Φ_1 is the larger, the current will be negative or inverse. Again, if a rectilinear conductor of length l move with a speed s through a field of force of strength H parallel to itself and perpendicular to the lines of force, the electromotive force developed in it will be Hls; the current will be Hls/R; and the work done by or upon it in the time t will be $H^2l^2s^2t/R$. Thus in Fig. 352 let CC' be the rectilinear conductor of length l, and let it move parallel to itself with a speed s along the rails AA', BB', whose plane is perpendicular to the flow of force H. Then the flow of force per unit length per unit displacement will be H, the flow per length l will be Hl, and per s units of displacement per second, i.e., the electromotive force, will be Hls. Whence by Ohm's law $I = Hls/R$; and the energy expended in the time t will be $W = I^2Rt = H^2l^2s^2t/R$. If, for example, a meter bar move horizontally in the earth's field with a uniform speed of 20 meters per second, the difference of potential developed at its ends, assuming the vertical component of the earth's magnetism to be 0·58 dyne, will be $0·58 \times 100 \times 2000 = 116000$ C. G. S. or ·00116 volt.

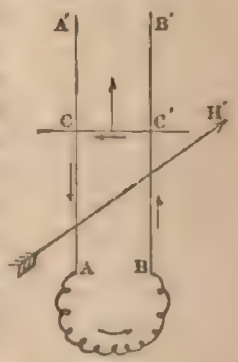


FIG. 352.

698. Recapitulation.—An induced current in a closed conducting wire, then, is always the result of a variation in the strength of a magnetic field in which

this wire is immersed. Hence it is to be observ[ed] (1) that two equal and opposite variations produ[ce] equal and opposite currents; (2) that the duration [of] the induced current is the same as that of the variati[on] of the field; (3) that the total quantity of electrificati[on] transferred is independent both of the time and of th[e] mode of the variation; and (4) that in a uniform fie[ld] no induced current is produced unless the conducti[ng] wire is so displaced or deformed as to vary the flow [of] magnetic force through its circuit.

EXPERIMENTS.—Produce a uniform magnetic field by placing t[wo] iron cubes on the poles of an electromagnet. The lines of for[ce] will be straight lines passing from one cube to the other perpe[n]dicular to its surfaces; as may be shown by a sheet of paper an[d] some iron filings.

1. Connect the two ends of a straight wire with a galvanomet[er] and place the wire horizontally in the field, its length being perpe[n]dicular to the lines of force. On moving it in the direction of [its] length, or parallel to the lines of force from one side of the field [to] the other, no effect is produced on the galvanometer; while if t[he] wire be moved vertically so as to move across the lines of force, t[he] galvanometer needle is strongly deflected. Since the wire and t[he] galvanometer form a closed circuit, it is evident that the flow [of] force through this circuit is varied only by this latter motion, t[he] number of lines of force passing through the circuit being increa[sed] or diminished according to the direction of the motion.

2. Make a ring of several turns of covered wire, of a less dia[m]eter than the distance between the pole pieces, and connect it to [the] galvanometer. Place it in the field (a) with its plane vertical a[nd] with its axis parallel to the lines of force. Move it either vertica[lly] or horizontally in its own plane, or parallel to itself in the directi[on] of its axis; no deflection is obtained, since evidently in the la[tter] case the number of lines of force traversing the ring (i.e., the fl[ow] of force through it) is unaltered: and in the former case as ma[ny] leave it on one side as enter it on the other, and the total flow is co[n]stant. If, however, the ring be moved, still parallel to itself, [to] the edge of the field where it is no longer uniform, then the flow [of] force through the ring will vary with its position, and a deflecti[on] will be obtained. (b) Place the ring vertical or horizontal, but w[ith] its axis perpendicular to the lines of force; no effect results whe[n it] is moved either in its own plane or parallel to this plane, since th[ere] is no flow of force through it in any of these positions. The sam[e is] true if it be moved from the center to the edge of the field. [...] Place the ring with its axis parallel to the lines of force and ro[...]

it in its own plane about this axis. No variation of the flow of force occurs, and therefore no deflection. (d) Place the ring with its plane parallel to the direction of the field and rotate it about a diameter parallel to the lines of force. No induced current results. (e) Rotate the ring about an axis perpendicular to the direction of the lines of force as a diameter. A strong deflection will be produced.

699. Magneto- or Dynamo-electric Induction.—As already defined, magneto- or dynamo-electric induction results from the mechanical displacement of a field with reference to a circuit or of a circuit with reference to a field; this field being produced either by a permanent magnet or by an electromagnet. Evidently in this case the energy of the current is derived from the mechanical energy expended in producing the motion. Two cases may be considered: first, where the field is constant and the position of the circuit is so changed as to vary the flow of force through it; and second, where the circuit remains invariable in position and the field varies in strength from one value to another. If the time of variation is very short, the quantity of electrification generated produces a momentary current which is measured on a ballistic galvanometer (678).

700. Earth Induction.—Inasmuch as the earth's magnetic field is nearly constant at any given place, the displacement of a closed circuit in it in such a way as to vary the flow of force through this circuit will generate an induced current in it. Since when a linear conductor moves so as to cut the lines of force perpendicularly, the induced potential-difference is proportional to the strength of the field H, to the length of the conductor l, and to the speed of the motion s, we have $E = Hls$; provided the wire be maintained vertical and be made to move horizontally across the lines of force or be kept horizontal and be made to move vertically across them, remaining always parallel to itself, H being now the horizontal component of the earth's force. The current induced $I = E/R = Hls/R$, and the quantity of electrification $Q = It = Hlst/R$. Since $st = d$ is the distance d moved in the time t, and since ld is the surface described

by the conductor in its motion, $Q = Hld/R = HS/R$ Φ/R. Hence the quantity of electrification transferred is equal to the ratio of the flow of force to the resistance of the circuit; the flow of force being always equal to the intensity of the field (i.e., to the flow through unit surface) multiplied by the number of units of surface. Hence if S be the surface of a plane circuit and the circuit be moved in the earth's field from a position in which no lines of force pass through it to a position in which H lines so pass (i.e., from a position in which its plane is parallel to the lines of force to one in which it is perpendicular to these lines), the total flow of force is HS, and the quantity of electrification developed is HS/R. If, for example, a circular coil of surface S (i.e., a coil of n turns, each of surface s) be rotated about a vertical axis through 180°, from a position where the lines of force are parallel to the axis of the coil in the positive direction, to one where they are parallel in the negative direction, the total flow of force is evidently $2\Phi = 2HS$; and the quantity of electrification developed $Q, = 2HS/R$. This induction has been utilized by Weber for determining the direction of the earth's total force; i.e., the inclination or dip. For by placing such a circular coil horizontally and rotating it about a horizontal axis, the total flow of force on rotating it 180° will be $2VS$, in which V is the vertical component of the earth's magnetism; whence $Q_1 = 2VS/R$. From these two equations $Q : Q_1 :: H : V = \tan\delta : 1$.

701. Magnetic Field Measurement.—The equation $Q = 2FS/R$ has been applied by Rowland to the determination of F, the strength of a magnetic field in absolute measure. For if the discharge-deflection be determined with a ballistic galvanometer, $2FS/R$ $k \sin \frac{1}{2}\theta$. Magnetic fields may be compared with that of the earth, therefore, by placing a smaller inductor in circuit with the earth-inductor and noting the deflections θ and θ' when these inductors are alternately rotated 180° in their respective fields; and then $F : H = Sd' : S$ d and d' being the scale readings. In the same way the

relative strength of the same field at different points may be measured; and if the constant of the coil be determined from the discharge given by the coil when rotated 180° in a field of known strength, the absolute strength of any other field may be ascertained. So if a small coil be placed on the surface of a magnet and rotated 180°, the discharge-deflection will be proportional to twice the flow of force through the coil, and the quotient of this by the surface will represent the normal magnetic component for that part of the magnet. If the magnet be cylindrical and the coil be moved from a position M to another M', the discharge-deflection will measure the variation of the flow of force during the displacement; and this divided by the surface of the coil will give the mean value of the normal component of the magnetic force between those points.

702. Self-induction.—Mutual Induction.—In the forms of induction now to be considered, however, the variation of the flow of force is produced by variations in the strength of an electrical current (695); and the energy of the induced or secondary current is derived from that of the inducing or primary current. Faraday in 1831 observed that on completing or breaking the current flowing in a helix an induced current is generated in a second wire wound outside the first. And in 1832, Henry proved that any variation in the strength of a current in a helix is also capable of inducing currents in the contiguous spires of the helix itself. The former is called **mutual induction**, since it takes place mutually between two adjacent coils. The latter is called **self-induction**, because it represents the action of a coil on itself. It is also called **inductance**. When the ends of a coil of wire are connected with an electric generator, only a slight spark will be observed on making the contact, while a brilliant one appears on breaking it; the result depending, of course, upon the length of the coil and upon the current-strength. Moreover, the effect is increased by placing iron in the core of the coil. If the hands be made to touch the terminals of the coil when

the circuit is broken, a distinct shock is perceived; a fine platinum wire may be fused by the extra-cur thus developed. Since the time of variation of the is shorter on opening the circuit, the induced curre opening is stronger than that on closing.

EXPERIMENTS.—1. Place a large electromagnet in circuit galvanometer and a secondary battery. On closing the circu current will be seen to rise gradually and to take its full str only after considerable time; in the case of very large ma several seconds.

2. Place across the terminals of the magnet an incand lamp, of such resistance that the battery current will bring i to dull redness. Break now the connection with the batter observe that the lamp will become vividly incandescent f instant, due to the extra-current.

703. **Coefficient of Self-induction.** — A cu started in a coil does not attain at once its perma value. Since during the variable state the rate o crease is not uniform, we may represent the cou electromotive force developed, as the product of a f depending upon the coil itself into the rate of cur increase during an element of time. Calling this f L and i_t the rate of current-increase during the ti $e_t = Li_t$. The current developed will be e_t/R and quantity of electrification $e_t t/R$ or $Li_t t/R$. Since rate of increase of the current multiplied by the tim increase represents the entire current generated in time, $i_t t$ is the total current generated in the inter If the whole time T be divided into n intervals, such $nt = T$, the total quantity generated in the time T be $\Sigma(e_t t/R)$ for the n intervals; or since $t = T/n$, wi $\Sigma\left(\frac{e_t T}{nR}\right)$, or $\frac{\Sigma e}{n}\cdot\frac{T}{R} = \frac{\Sigma i}{n}\cdot\frac{LT}{R}$. In this expression, Σ is the mean potential-difference during the time T $\Sigma(i/n)$ the mean rate of increase of the current. Ther $\Sigma(i/n)$ multiplied by T or $\Sigma(iT/n) = \Sigma(it)$, is the e current produced in the time T. If this quantit assumed to be unity, the expression $\frac{\Sigma i}{n}\cdot\frac{TL}{R}$ bec

L/R; and if the resistance of the circuit be also unity, the whole quantity of electrification induced in the time T will be L. This factor L is called the **inductance** of the coil or its **coefficient of self-induction**; and it is defined as the quantity of induced electrification which is developed in the coil assumed to be of unit resistance, when unit current through it is made or broken.

Since, other things being equal, the quantity of electrification produced is directly proportional to the flow of force, the inductance-coefficient is expressed frequently in terms of the flow of force. Since the strength of the current-field is dependent not alone on the strength of the current flowing in the circuit, but also on the shape and size of the circuit itself, the flow of force developed by a given current is proportional to the product of the current-strength I by the factor L depending on the circuit, the inductance-coefficient; i.e., $\Phi = IL$. Whence $L = \Phi/I$; and this coefficient may be defined as the ratio of a flow of force to a current-strength. So that if I be unity $L = \Phi$; or the coefficient of inductance or self-induction of a circuit is unity when unit current started or stopped in that circuit induces unit flow of force through it. For a long coil having a single layer of wire, $L = 4\pi N^2 S/l$; in which N is the number of turns, S its section, and l its length.

Since the dimensions of flow of force are $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$ while those of current are $M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$, the dimensions of inductance, being the quotient of these quantities, is simply a length L. Hence in the C. G. S. system the unit coefficient of inductance is a **centimeter**. For practical purposes a unit one thousand million times as large has been adopted and called a **quadrant**; since this is the number of centimeters in a quadrant of the earth. A quadrant therefore is 10^9 C. G. S. units of self-induction. Since an ohm is a quadrant per second, an ohm-second (or a **secohm** as it is called by Ayrton and Perry)

is the equivalent of a quadrant. In this country
unit coefficient of self-induction is called a **henry**.

In the same way, since $e_t = Li_t$, $E = L\Sigma i_t t = L\int$
unit coefficient of inductance may be defined a
value of the potential-difference developed
circuit when the rate of variation of the cur
strength is unity. And again, since $W = \frac{1}{2}LI$
the energy stored up in the circuit during the var
period, and which appears on breaking the circuit a
extra-current spark, the coefficient of inductance m
defined as twice the energy stored up in a cir
during the variable period, the final streng
the current reached being equal to unity.
these values are identical when the permeability o
medium is constant.

704. Coefficient of Mutual Induction.—When
rent of strength I traverses a circuit a, it produces a
of force Φ. If a second circuit b be near it, more o
of this flow of force traverses it also, depending
the distance separating these circuits, upon their
the number of turns in them, their sizes, and their rel
positions. Hence the flow of force produced in b
current I in a is proportional to the current I mult
by a factor M representing these conditions and
the **coefficient of mutual induction** between the two cir
It is defined as the ratio of the flow of force
secondary circuit to the strength of the cur
flowing in the primary or inducing circui
that two circuits have unit coefficient of mu
induction when unit current flowing in one
velops unit flow of force through the ot
This unit, in the C. G. S. electromagnetic system
the unit of self-induction, is a **centimeter**; and
practical unit is 10^9 absolute units, or a **quadrant**.

Inasmuch, however, as both the primary and se
ary circuits possess self-induction, the phenome
mutual induction cannot be obtained free from th
self-induction; and hence the former are the more

plex. Thus, for example, on closing the circuit through the primary coil a, a current will be developed in the secondary coil b tending to oppose a's flow of force through b, and the self-induction of b acts in its turn to oppose the development of a current in b. So the current induced in b will react on a, modifying its variable period by an amount depending on L_a, L_b, and M, on the resistances R_a and R_b, and on the potential-difference in the primary. For two concentric coils of considerable length, $M = 4\pi NN'S/l$; in which N and N' are the number of turns in the outer and the inner coils respectively, S the surface of the inner coil, and l its length.

(*b*) *Applications of Induction.*

705. Applications of the foregoing Principles.—Suppose a coil of wire (Fig. 353) to rotate uniformly in the earth's magnetic field about a vertical diameter as an axis. Evidently there is no flow of force through it when its plane is in the meridian, and the flow of force is a maximum when its plane is perpendicular to the meridian. But the rate of variation of the flow is a maximum in the former position and a minimum in the latter. Moreover, with reference to the coil, the direction of the flow changes sign at each half-revolution, this change taking place at the instant when the plane of the coil is perpendicular to the meridian. Inasmuch as the potential-difference developed by the induction is proportional to the rate of variation of the flow of force, it will reach a maximum when the plane of the coil is in the meridian and become zero when this plane is perpendicular to it; changing sign at this point and reaching a negative maximum when the

FIG. 353.

coil has revolved 180° and its plane is again parallel to the meridian. In other words, the difference of potential at the terminals of the coil varies with the angle made by the axis of the coil with the meridian, being proportional to the sine of that angle. If S be the area of the coil and H the horizontal component of the earth's force, SH will be the total flow of force through the coil when the axis of the coil is in the meridian. If the angular velocity of rotation be ω, then ωSH will be the maximum rate of variation of the flow of force and $\omega SH \sin \alpha$ the actual rate when the axis of the coil makes the angle α with the meridian. As the angular velocity ω is $2\pi/T$, where T is the time of a complete rotation, we may write for the maximum potential-difference developed $E_{max} = 2\pi SH/T$ and for the actual potential-difference when the coil has turned through an angle θ, $E_\theta = (2\pi SH \sin \theta)/T$; or in time, since t is the time required to rotate the coil through this angle, $(t : T :: \theta : 2\pi)$,

$$E_\theta = 2\pi \frac{SH}{T} \sin 2\pi \frac{t}{T} \qquad [83]$$

Hence $E_\theta = E_{max}$ when $t = \frac{1}{4}, \frac{3}{4}, \frac{5}{4}$, etc., of T; i.e., at every half-revolution, when $\theta = 90°, 270°, 450°$, etc. While when $t = 0, \frac{2}{4}, \frac{4}{4}, \frac{6}{4}$, of T, the value of E becomes zero. Since $E_\theta = E_{max} \sin \theta$, the current-strength in the circuit, if the resistance be R, is $I_\theta = (E_{max} \sin \theta)/R$; and the activity or $E_\theta I_\theta$ will be $(E^2_{max} \sin^2 \theta)/R$. Here the potential-difference and the current vary periodically, and are alternate in direction; constituting an **alternating current**. The law of variation is that of the sinusoid; the lower curve here given (Fig. 354) showing this law for the potential-difference E and the current-strength I; and the upper curve (which since squares are positive is all above the axis) representing the variation of the energy in the circuit (Hospitalier).

The mean value of the potential difference and of the current flowing through the circuit is represented of course by the mean value of the ordinates to these

curves. This mean value is obtained by dividing the area of the curve by its length. The area of the curve between 0° and π is twice the maximum ordinate; whence calling this ordinate unity we have for the mean

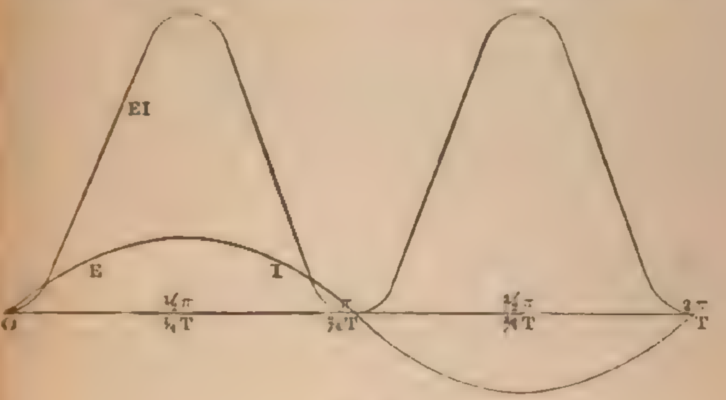


FIG. 354.

ordinate $2/\pi$ or 0·6366. Hence the mean potential difference of such an alternating current is 0·6366 E_{max} and the mean current is 0·6366 I_{max}.

Thus far we have neglected the self-induction of this coil. If L be its coefficient of self-induction, calculation shows that the resistance of the circuit will become, in-instead of R, the quantity $\sqrt{R^2 + 4\pi^2 L^2/T^2}$ or $\sqrt{R^2 + \omega^2 L^2}$. Calling this R_1, the current-strength $I_0 = E_{max} \sin(\theta - \phi)/R_1$; that is, $E_{max} \sin\left((2\pi t/T) - \phi\right)/R_1$. Hence while the new current has the same period as the old one, its phase is changed. It is retarded by the self-induction and reaches its maximum later than the potential-difference by the angular quantity ϕ, defined by the expression $\tan \phi = \omega L/R$. Since the maximum value of ϕ is $\frac{1}{2}\pi$, the difference of phase is $\frac{1}{4}$; and hence the maximum retardation is equal to one quarter of the entire period. In the figure (Fig. 355) the full line represents the current-variation and the dotted line the

variation of potential. Moreover, the effect of the self-induction in the circuit is to increase its apparent resistance. Since the tangent of ϕ is $\omega L/R$ it appears that the angle of retardation or lag is a function of the speed as well as of the ratio of the self-induction to the resistance of the circuit. Moreover, since the resistance is increased from R to $\sqrt{R^2 + \omega^2 L^2}$, the apparent increase due to the second term in this expression is a function also of the speed and of the self-induction; but is now a function of their product.

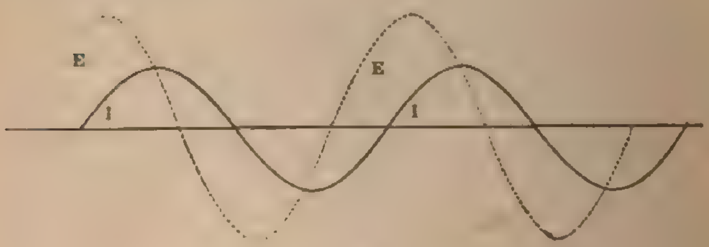


FIG. 355.

The alternating currents are led off from the rotating coil by means of two springs pressing upon two metallic rings on the axis of rotation, to which the ends of the coil are respectively connected. Since the current is reversed every half-revolution the number of alternations is twice the number of revolutions. This alternating current may be converted into a direct current by connecting the two ends of the coil to two semi-cylindrical metallic segments called a **commutator** fastened to the axis but insulated from it, upon which the springs press. The current in the external circuit is thus reversed every half-revolution and is thereby made uniform in direction.

706. Faraday's Disk.—In 1831, Faraday devised the first arrangement for producing continuous currents by means of electromagnetic induction. He rotated a copper disk in a magnetic field, the axis BO of the disk

being parallel with the lines of force H of the field (Fig. 356). By means of two contact springs A and B touching respectively the axis and the circumference of the disk the induced currents were carried to the external circuit. If we suppose the field uniform and of strength H, the angular velocity of the disk to be ω, the area of the sector de-

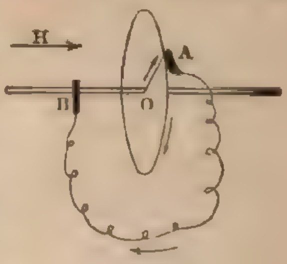


FIG. 356.

scribed in unit of time will be $\frac{1}{2}a^2\omega$, in which a is the radius of the disk; and the flow of force per second $\frac{1}{2}a^2\omega H$; which is the potential difference developed. Hence the current-strength will be $a^2\omega H/2R$, where R is the total resistance of the circuit. If T be the time of rotation and S the entire surface of the disk, $S = \pi a^2$ and $T\omega = 2\pi$. Whence $I = SH/TR$. Suppose the area of the disk to be a square meter and that it rotates ten times a second about a horizontal axis parallel to the magnetic meridian. Assuming the value of H, the earth's horizontal component, to be 0·2 we have for the potential difference developed $10^4 . 10 . 0{\cdot}2$ or $2 . 10^4$ C.G.S. electromagnetic units or $2 . 10^{-4}$ volts; so that if the resistance of the circuit be as low as 10^{-4} ohms, a current of two amperes will flow through it.

Barlow in 1823 had supported a copper disk capable of rotation about a horizontal axis, so that its lower edge just dipped into mercury contained in a hollow space in a board, between the poles of a U-shaped steel magnet, lying flat on the board. He found that rotation of the disk took place whenever a current was sent radially through the disk from the axis to the mercury. If we may suppose the field to be uniform, and to have a value H perpendicular to the plane of the disk, of surface S, then HS will be the flow of force through the disk during one rotation; and this therefore represents the fall of potential. Since the work done is EI, the expression HIS represents the entire work done in one

revolution of the disk under the action of the current. And if the speed is n rotations per second, the rate of work or work per second is n times this quantity or $nHIS$. Evidently the wheel of Barlow and the disk of Faraday are complements of each other. The former receives electrical energy and converts it into mechanical energy; thus acting as a motor. The latter receives mechanical energy and transforms it into electrical energy; thus acting as an electrical generator. Connecting a Faraday disk then with a Barlow wheel, the mechanical power applied to the generator could be transmitted electrically to the motor and there recovered more or less perfectly.

707. Induction in Irregular Masses. — Whenever variations take place in the strength of any magnetic field electrical currents are induced in all metallic masses within that field. These currents are commonly called **Foucault currents**, from their discoverer. Foucault was the first to observe that great mechanical resistance is experienced when an attempt is made to rotate a disk of copper in a powerful magnetic field. And he showed that, if the rotation is maintained the disk becomes heated. This is evidently only a simple consequence of Lenz's law. The currents induced by moving the conductor in the field are in such a direction as to oppose the motion. So that if energy be expended sufficient to maintain the motion, this energy appears as the electrical energy of induction currents; which currents circulating in the metal are converted into heat within it in consequence of its resistance.

EXPERIMENTS.—1. By means of multiplying gear, rotate a copper disk so that one edge passes between the poles of a powerful electromagnet. Notice that while the rotation is easy when the magnet is not charged, it becomes difficult when the current passes through it. Notice also that the work required increases with the speed of rotation. (Foucault.)

2. Suspend a copper cube between the poles of the magnet by a twisted string, and allow the cube to rotate. When it has attained a high speed, close the current through the magnet.

Notice that it will be brought at once to almost entire rest. (Faraday.)

3. Repeat Experiment 1 with a disk similar in all respects, except that radial slits have been cut in it at frequent intervals. Notice the much less effect when the magnet is charged, the circulation of the Foucault currents in the disk being partially prevented by the interruption of continuity.

708. Damping Effect of Induction.—In 1824, Gambey noticed that an oscillating magnetic needle comes to rest much sooner when a plate of copper is placed beneath it. Arago, in 1825, rotated a copper disk beneath a suspended magnetic needle, and observed that the needle is deflected always in the direction of rotation. Moreover, Babbage and Herschel found that the effect is proportional to the conductivity of the disk; being a maximum for silver and a minimum for bismuth. With glass no effect was observed. These phenomena were all explained by Faraday after his discovery of induction. The relative motion of magnet and metal induces currents in the metal tending to oppose the motion; and thus damps the oscillations of the needle or develops a mutual action between the two tending to move the needle in the opposite direction to that which would produce the currents. Thus in the figure (Fig. 357), suppose the needle *ab* to oscillate over the metal disk. As it moves in the direction of the arrows a current is induced on the *M* side tending to repel the needle, and on the *N* side tending to attract it; both of which currents oppose its motion and so damp its oscillations. Again, suppose the magnet fixed, and the disk to rotate from *N* toward *M*. Currents will be induced in that part of the disk which is approaching *a* in such direction as to repel the magnet; and in that part which is receding from *a* in such direction as to attract it; thus opposing the motion in both cases. Lastly, let the needle be at rest and the disk be moving

FIG. 357.

from N to M, as before. The currents induced in the approaching portion of the disk repel the needle, those in the receding portion attract it; hence the needle will tend to rotate with the disk. The damping effect of metallic masses by the induction currents developed in them is made frequent use of in practice to diminish the time required by an oscillating magnet to come to rest. The damping may be effected by the inductive action of a metal plate as in the Melloni galvanometer, or of a metal block as in the Wiedemann instrument; or by the currents induced by the needle in the coils of the galvanometer itself. Or it may be effected mechanically by means of a vane, whose motion is opposed by the resistance of the air. A galvanometer whose indications are given without oscillation is called **dead beat or aperiodic** (676).

709. Induction Screens.—In 1840, Henry showed that a plate of metal, when placed between two coils, acts like a screen, modifying more or less completely the inductive action of the one upon the other. Its effect is that of an equalizer, favoring the production of the current induced on closing the generator circuit and opposing that induced on opening it; so that the extra-current produced on breaking the inducing circuit is much reduced. Inasmuch as the effect is greater as the metal is a better conductor, it is evident that the effect is due to the production of induced currents in the metal plate itself. This device in the form of a tube placed between the helices is used in some forms of induction apparatus to vary the induction by sliding it in and out.

710. Induced Currents of Different Orders.—In 1838, Henry discovered the successive induction of currents. By placing a flat coil of copper ribbon, serving as a secondary, upon a similar coil transmitting an interrupted current and acting as a primary, he obtained the ordinary secondary induction current. But now if this current be sent through a third such coil upon which a fourth coil rests, a tertiary current is induced in this latter coil; and so on, the strength of these

induced currents gradually decreasing. These currents he distinguished as induced currents of the first, second, third, etc., orders; and he succeeded in producing these currents up to the ninth order. He observed that the successive currents alternate in direction, and he pointed out and utilized the distinction between long and short coil effects; or as he called them, between intensity and quantity induced currents. If the coil placed on the primary be made of fine wire and of many turns, the potential-difference at its ends is high; the current is an intensity current. While if the secondary coil be of coarse wire or ribbon of only a few turns, the induced current has a feeble potential-difference. Moreover, in the first case, the resistance of the secondary being high, the induced current through it will be feeble; while in the second case, the resistance being low, a stronger induced current will flow. Henry pointed out clearly the fact that either an intensity current or a quantity one can be induced from a quantity current at will, according to the character of the secondary coil; and an intensity primary can also induce a quantity or an intensity secondary.

711. Induction Balance.—In 1879, Hughes described a form of apparatus which he called an **induction balance**, designed for comparing differentially or by a zero method the effects of induction. The principle of its action, which had been already embodied in Dove's instrument, is simple: Suppose two equal primary coils to be connected with two equal secondaries, the two secondaries having a galvanometer in circuit, and so connected with it that their currents shall flow in opposite directions through it and thus antagonize each other. Evidently if an interrupted current is transmitted through the primaries connected in series, the induced secondary currents will have no effect on the needle. But if a bit of iron be placed in the core of one of the primaries, the stronger induction on this side will deflect the needle. In Hughes's induction balance, an auxiliary apparatus is introduced, called a sonometer or **audiome-**

ter, which consists simply of three coils all alike, one of them sliding on a horizontal graduated bar, and acting as a secondary. The other two coils act as primaries, being placed at opposite ends of the bar and reversed in position. By a switch these primary coils are placed in circuit with the interrupter and with the battery, while the secondary is put in circuit with a telephone, and is moved along the bar until no sound is heard; this point being in theory at the middle of the bar if the primary coils are equal. In fact, however, the construction is such that the zero-point is near one end of the bar. After the equilibrium has been disturbed in the induction balance proper, the auxiliary circuit is switched in and the secondary coil adjusted until the sound in the telephone is the same whichever of the circuits is in communication with the telephone. When this result is attained, the position of the secondary coil will measure roughly the effect. Since the ear cannot appreciate very sharply the equality in these sounds thus alternately heard, especially when they are very feeble, Hughes subsequently replaced the sonometer by an equilibrating device, consisting of a wedge-shaped strip of zinc, placed between the primary and secondary coils and on the opposite side to those containing the disturbing metal. By gradually moving the strip, the balance is restored and the sound ceases in the telephone. This instrument is of extraordinary sensitiveness. A fine iron wire or even a milligram of copper causes a loud sound, and two coins fresh from the mint show a distinct difference. Rubbing or breathing on them, after they have been balanced, disturbs again the equilibrium. Of course counterfeit coins are readily detected by this balance. As modified by Graham Bell, the induction balance has been applied to the location of bullets in the human body.

712. The Telephone.—The electric telephone is an instrument for reproducing sounds at a distance by electrical means. The apparatus required is (1) a device for converting the sound-energy into its equivalent electrical current-energy, and (2) another device for reconverting

this current-energy into sound-energy. The former is called the **transmitter**, the latter the **receiver**. Page in 1837 had observed that sounds were produced on magnetizing and demagnetizing iron by a current, Henry having already shown that a bar was elongated when magnetized; and Wertheim proved that the alternate elongations and contractions produced on magnetizing and demagnetizing an iron rod threw the rod into longitudinal vibration, producing the corresponding musical note. In 1861 Reis made use of a short iron rod surrounded by a wire coil of nearly the same length, mounted on a resonance-box as a receiver. As a transmitter he used a diaphragm of animal membrane stretched over the top of a cubical wooden chamber, having at its center a platinum disk in communication with one of its terminals. Upon this a platinum point, attached to a strip of metal bent at right angles, rested at the angular point; the strip being loosely connected through a mercury contact with the other terminal. If a musical note be sent into the box through an opening in its side, the vibration of the diaphragm will interrupt the contact, and will break the current into intermittances corresponding to the vibration-frequency of the note. On reaching the receiver, this note will be reproduced, and in this way a melody may be transmitted to a distance.

In 1873–74 Gray improved greatly the transmission of musical sounds, in connection with his harmonic telegraph; using a series of electromagnetically-vibrated steel reeds as transmitters, and as a receiver an electromagnet with an armature firmly fixed to one of its poles and extending over the other, but separated from it by from half to a third of a millimeter; the whole being mounted on a resonance-box. Electromagnetically-vibrated reeds were thus used both as transmitters and receivers. With this apparatus he discovered that composite tones could be transmitted on closed circuits; in other words, that the complex wave-form of harmony could be electrically reproduced. From this experimental

result it was but a step to conceive of the electrical reproduction of quality in sound in general; and this even to the electrical reproduction of speech. Hence in February, 1876, he filed his celebrated caveat for an apparatus capable of transmitting and receiving speech. The transmitter of this apparatus consists of a membrane or diaphragm, to the center of which a platinum wire is attached which enters a vessel of water, and the lower end of which approaches closely a second similar wire which comes up through the bottom. The receiver is made of an electromagnet, having its armature attached to a membrane forming the base of a cone-shaped chamber. The current sent through the lower wire of the transmitter passes through the small interval of water to the upper wire, and thence to the line and so to the receiver; going to ground after traversing the electromagnet. This current puts the membrane of the receiver under stress; so that on causing the membrane of the transmitter to vibrate, the water-space varies with the vibration, and so introduces a varying resistance into the circuit, causing an exactly corresponding variation of the current, and therefore of the receiving diaphragm. Words spoken into the transmitter cause diaphragm-vibrations in this transmitter, which translates these vibrations faithfully into electrical composite waves; and these in their turn are retranslated into sound-waves by the receiver; solving the electrical transmission of speech in virtue of the production of the proper electrical wave-form by varying the resistance of the circuit. During these investigations Gray discovered the great advantage of using an induction coil in connection with his telephone; placing the transmitter and battery in the primary circuit and connecting the secondary circuit to the line, at the end of which was the receiver provided with a high resistance electromagnet. The telephonic circuit was completed through the earth as usual.

At about the same time, Graham Bell was occupied with the question of harmonic electrical transmission. He devoted his attention especially to the production of

undulatory currents by variations of the electromotive force in the circuit; and this most readily by induction. He observed that if two electromagnets provided with vibrating iron armatures be placed in the same circuit with each other and with a battery, these armatures are magnetized by the current; so that if one of them is mechanically vibrated, it induces an undulatory current in the circuit which causes the second armature to vibrate. Hence it appeared that the electromagnetic device which had been used by Gray as a receiver was capable of use as a transmitter. On the same day that Gray filed his caveat, Graham Bell made application for a patent, which was issued in March, 1876, and in which is the remarkable claim so long hotly contested in the courts, and finally sustained by the Supreme Court of the United States: "The method of, and apparatus for, transmitting vocal or other sounds telegraphically as herein described by causing electrical undulations similar in form to the vibrations of the air accompanying the said vocal or other sounds." In its earliest form the Bell telephone was a practical facsimile of the Gray receiver; and in this form Bell exhibited it at the Centennial Exhibition. As first put into use commercially it consisted of a steel U-magnet attached to the poles of which were iron cores carrying spools of fine wire. Subsequently the magnet was made a straight bar and the Bell telephone took its present form, shown in Figure 358. On the pole of a compound bar magnet is a coil of wire. In front of this is a thin iron plate or disk acting as a diaphragm. When this is caused to vibrate by the impact of sound-waves, it induces currents in the coil. These, transmitted to the remote end, enter the receiver, which is a precisely

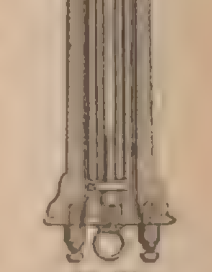


FIG. 358.

similar instrument, and reproduce vibrations of its diaphragm alike in character, though of course diminished in amplitude, to those of the transmitter; thus reproducing the air-waves and so the sound. These induced currents are extremely feeble, not exceeding those produced by one Daniell cell in a copper wire 4 mm. in diameter and long enough to go 290 times round the earth. To produce one cubic centimeter of mixed gases this current would have to pass through a voltameter for 19 years. Pellat states that the energy of one water-gram-degree would maintain a continuous sound in the telephone for 10,000 years.

Evidently since the energy of the current in this circuit is due to the energy of the sound-waves, it must be inferior to that in a circuit through which a battery current is flowing and which is only modified by the sound-waves. Hence a speedy return to the principle first recognized by Gray, i.e., of using a variable resistance telephone as transmitter and the Bell telephone only as receiver. In 1873, Edison had observed the peculiar property possessed by semi-conductors of varying their resistance with pressure; and early in 1877 he applied this property to the construction of a variable resistance transmitter. The material which he found best for this purpose was a lampblack taken from the chimney of a smoking petroleum lamp and compressed into a disk. This disk placed between two platinum surfaces, upon one of which the diaphragm rests while the other is adjustable, constitutes the carbon transmitter of Edison. The variations of pressure caused by the vibrating diaphragm, as Mendenhall has shown, vary the resistance of the carbon disk and so vary the current sent through it. Placing this transmitter in the primary circuit of a small induction coil, the secondary being in the line, the loudness and distinctness of the articulation leave nothing to be desired. This variation in the resistance of the soft carbon disk by pressure, Edison has also utilized in the construction

of his **tasimeter**, an instrument for detecting minute heat-changes.

In 1878, Hughes described certain phenomena due to loose or imperfect contact of conductors. If a piece of carbon be laid loosely across two others and a current be sent through the points of contact, a telephone being in the circuit, a loud noise is heard in the telephone whenever the carbon is jarred, even by sounds. Evidently the vibration of the carbon varies the extent of the surfaces of contact and so varies the resistance of the circuit. The **microphone** of Hughes consists of a thin board placed vertically, to which are fastened two pieces of electric-light-carbon, one above the other, having depressions near their outer ends, into which the pointed ends of a third carbon rest loosely. When a current passes through these loose contacts its strength varies as the surface of contact varies. So that if a receiving telephone be placed in the circuit and a watch be placed on the stand supporting the microphone, the ticking of the watch may be heard at many miles' distance. Since a part at least of the effect observed in the Edison carbon transmitter is due to surface contact, it is clear that the microphone embodies the principles first applied by Edison. The transmitting telephone now in general use is a modified microphone contrived by Blake. A platinum point abuts against a carbon disk, both being carried by springs through which the current flows to and from the contacts. Against the diaphragm this combination rests; and it is therefore jarred when the diaphragm vibrates; thus varying the resistance and the current-strength on the line in the same ratio. The long-distance transmitter of Edison is analogous except that the contact is multiple, being effected by coarsely pulverized and highly baked anthracite placed between the diaphragms.

The electrostatic telephone of Dolbear depends upon the alternate attraction and repulsion of two charged diaphragms acting as the armatures of an air condenser.

(c) *Mutual Induction Apparatus.*

713. Induction Coils.—An induction coil in the broad sense consists of a primary and secondary helix in connection, having a common core of iron (Fig. 359). Ordi-

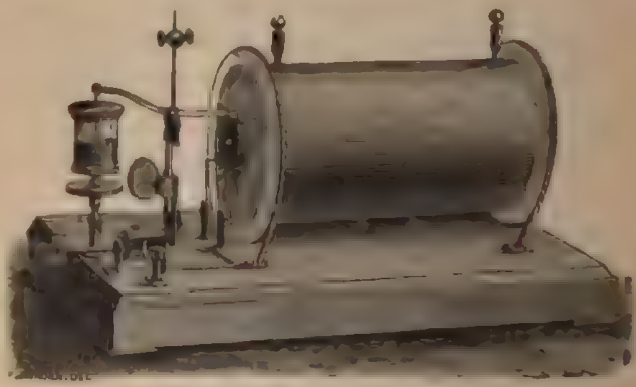


FIG. 359.

narily the primary helix is placed within the secondary and consists of a coarser and shorter wire; so that the mutual induction between the coils results in the production of a higher potential-difference at the terminals of the secondary. The construction of such coils for the purpose of obtaining a higher electromotive force, which followed very closely the experiments of Faraday and Henry, seems to have been effected in 1837–8 nearly simultaneously by Sturgeon, by Page, and by Callan. In Page's coils of 1842 sparks of ten or twelve millimeters were obtained from the secondary helix. And in 1851, by paying especial attention to insulation, Ruhmkorff constructed induction coils giving sparks equally long. In 1853, Fizeau proposed the use of a condenser in the primary circuit; and this addition increased the length of the spark to fifteen or twenty millimeters. In 1855, Poggendorff suggested winding the secondary coil in transverse sections; and in 1857, Ritchie, by increasing the number of these sections, produced coils giving

sparks thirty centimeters long. The final result in this direction has been attained by Apps, who in 1877 made for Spottiswoode an induction coil giving sparks more than a meter in length. The primary wire of this coil is 2·4 mm. in diameter and 603·5 meters long. It is wound in six layers, and has a resistance of 2·3 ohms. The secondary wire has a minimum diameter of 0·24 mm. and a length of about 450 kilometers, wound in four sections, and having a resistance of 110 200 ohms. The sections are separated by plates of ebonite, and the wire in the external sections is somewhat thicker than in the central ones. The core consists of a bundle of iron wires 110 cm. long and 8·9 cm. in diameter, the mass being 30 kilograms. A supplementary primary and core is provided, having its wire wound in a double strand and its core of nearly one half greater mass. The mass of the entire coil is about 700 kilograms.

Since the potential-difference induced at the terminals of the secondary coil is a direct function of the coefficient of mutual induction between this coil and the primary and also of the rate of variation of the current in the primary, and since this coefficient for unit current is proportional to the product of the number of turns in the two coils, it is clear that this potential-difference will be increased by increasing the number of turns in both primary and secondary circuits, by increasing the current in the primary and decreasing the time of its fall to zero. But an increase of length in the primary is inconsistent with an increase of current therein. And hence some constructors increase the length, others increase the size, of the primary wire; the former increasing the induction by the number of turns, the latter by the increased current. Hence the two primaries provided with the Apps coil above mentioned; the one of lower resistance, having a much increased iron core, giving a shorter secondary spark, but of increased quantity.

The function of the condenser in the primary circuit is simply to provide for the current of self-induction

developed in that circuit. The current induced in the secondary on opening the primary circuit is in the same direction as that flowing in the primary; as is also the self-induced current in the primary itself. When the primary is broken the self-induced current continues to flow across the interval in the form of a spark, thus prolonging the time of fall to zero of the primary current, diminishing the rate of variation of this current; and so the potential-difference in the secondary. By putting a condenser as a shunt to the primary wire, this current of self-induction passes into and charges the condenser; thus destroying the spark at the break-piece, and rendering the fall to zero of the current in the primary more abrupt; the condenser discharging itself immediately thereafter through the primary, producing an inverse current by which the demagnetization of the core is made more rapid. The greater effect produced by using a bundle of iron wires for the magnetic core instead of an iron cylinder is due to the fact that by this subdivision not only is a stronger magnetization produced, but the induction of Foucault currents which would retard the current-variation in the primary is prevented.

Inasmuch as secondary currents are induced only during the variable state of the current in the primary, some device is attached to an induction coil for varying the current between zero and its maximum; i.e., for interrupting the current; and this as abruptly as possible. The earlier forms of interrupter were mechanical and consisted in drawing the conducting wire along a rasp in circuit, or in rotating a toothed wheel so as to break contact as it turned. Subsequently, automatic interrupters were devised. That of Page consists of an electromagnet having its armature fixed upon the end of a spring on the back of which is a contact. When the current passes through this contact and through the electromagnet, the armature is drawn toward the magnet and so interrupts the current by opening the circuit at the point of contact. The elasticity of the spring restores this contact and the

armature continues to vibrate. In some of Page's coils the core of the coil itself acted as the electromagnet. Foucault's interrupter (Fig. 359) consists of an arm supported on a vertical spring, carrying an armature at one end and a vertical platinum contact-point at the other. The armature is supported above one end of the iron core and the point dips in mercury in a glass cup. The current flows through the primary coil, the arm, and the mercury; the armature is attracted to the magnetized core and its point is thereby raised out of the mercury, interrupting the circuit. As the contact is re-established when the arm vibrates back again, the attraction is repeated; thus making the oscillation continuous. In some cases the arm carries a second contact-point, dipping into mercury contained in a second glass cup; and through the circuit thus formed and the primary wire of the coil, the primary current flows; being interrupted continually by the vibrations of the arm. By placing alcohol above the mercury the break is sharper and the secondary spark is longer.

In 1879, Spottiswoode substituted the alternating current of a De Meritens dynamo for the intermitted direct current previously used; the reversals of the current being at the rate of 20000 per minute. Since the rate of variation of the alternating current is less than that produced with sudden interruptions of a direct current, the electromotive force developed is less and the spark shorter; a 17-centimeter spark only being obtained from a 50-centimeter coil. But owing to the strength of the primary current, the volume of the secondary is much increased; the spark being of the thickness of a lead-pencil.

714. Phenomena of the Induction Spark.—The phenomena of disruptive discharge can be studied much more satisfactorily with the spark of the induction coil than with that of the electrostatic machine. When the pointed terminals of the secondary of a large coil are separated by two or three centimeters, the spark is observed to consist of two distinct portions: one a fine

line of light more or less purple in color, constituting the spark proper; and the other a sort of flame or aureole of a yellowish color, surrounding the first. By blowing upon the spark through a glass tube this portion of the spark, the flame portion, is readily blown to one side; and by having a pair of points on this side, the blast may be so regulated that the flame is directed to one of them and the spark proper to the other. It is this portion of the spark which is affected in the magnetic field. As the distance between the points is increased the aureole disappears and the spark proper becomes more or less irregular in direction; due of course to its following the line of least resistance. By placing a glass condenser, or better a number of such condensers arranged in series, as a shunt in the circuit, the spark becomes shorter but also thicker and brighter; the quantity of the discharge being increased. If a jar-condenser be placed in series in the secondary circuit, its outer coating connected with one terminal of the secondary coil and its inner coating with one of the discharging points, the other discharging point being connected as usual to the other terminal of the secondary and the points not too widely separated, the jar may be charged by the coil precisely as with an electrostatic machine. This result proves that since the induction currents on opening and closing are opposite in direction, they must be unequal. This is always the case when the secondary circuit is open, the direct current produced on opening being always stronger than the inverse current developed on closing the primary circuit. Indeed with a sufficient break at the points, the latter current has not sufficient electromotive force to pass at all.

The length of the spark depends upon the electromotive force of the induced secondary current. According to Lord Kelvin's measurements, the potential-difference required to produce a spark in air, between parallel plates, and of a given length, diminishes rapidly as the distance increases, approaching a limiting value of 30000 volts per centimeter; which may be assumed

constant for distances greater than one centimeter. This potential-difference produces a spark of greater length in rarefied air, and of less length in condensed air. Gordon has confirmed Harris's law, that for rarefactions up to 28 centimeters the length of the spark is inversely as the barometric pressure. At less pressures than 28 cm. the potential-difference increases more rapidly than the law requires.

EXPERIMENTS.—1. To show the brief duration of the spark, place a condenser in the secondary circuit, and illuminate with the sparks a rapidly revolving disk divided into alternate black and white sectors. If there are n sectors and the interval between the sparks is $1/n$ or a multiple of $1/n$th of that of the time of rotation, the disk will appear to be at rest, its sectors being sharply defined.

2. Provide a disk having three rows of dots upon its face; the inner row having 8, the middle 9, and the outer 10. Adjust the interrupter so as to produce as many sparks during one rotation of the disk as there are dots in one of the rows. This row will then appear at rest, while the others will appear to rotate. Thus if there be 9 breaks during one rotation, the middle row will appear at rest, the others in motion: the inner row appearing to move forward and the outer row backward.

3. Provide a thin plate of vulcanite about a meter long and thirty centimeters wide, varnish one of its surfaces and scatter over it filings of iron, copper, and zinc. Place two narrow thin strips of metal across the ends and connect these to the terminals of the induction coil. The sparks of a 20-cm. coil will traverse the entire length of the plate, often dividing into two or three branches, the color depending on the metals between which it passes.

4. Connect one terminal of the coil to the metallic coating on the back of a mirror and place the other terminal near the center of the opposite face. The sparks will branch from this point in all directions, the ramifications being duplicated by reflection.

5. Exhaust a glass tube a meter or more in length while the metallic terminals with which it is provided are connected with the coil in action. Observe that at a certain exhaustion, the spark traverses the entire length of the tube, in nearly a straight line; the thickness of the spark increasing as the exhaustion proceeds until it fills the tube. In air, its color is reddish purple.

6. Exhaust an ovoid or egg-shaped glass vessel provided with electrodes, having previously placed within it a drop or two of alcohol. When the exhaustion is complete, it will be noticed that on passing the spark, the discharge within the vessel is broken up

into bands or strata transverse to the direction of its length; i.e., the discharge shows stratification.

7. Transmit the spark through rarefied air contained in tubes provided with annular spaces in which fluorescent solutions can be placed. The discharge being rich in ultra-violet waves, causes solutions of quinine or of æsculin to fluoresce blue; of uranine, green; and of naphthalene-red, crimson. These tubes already exhausted, made of fluorescent glass and containing these various solutions, are called **Geissler tubes,** having been made first by Geissler of Bonn. Some of them are very elaborate in design.

8. Vacuum tubes, consisting of cylindrical or spherical bulbs at the ends connected by a capillary section, and containing various gases or vapors, are called **Plücker tubes** (416). Place one of these tubes in the circuit of the secondary and notice the greater intensity of the light in the capillary portion. Notice also the variation of the color according to the substance in the tube. By placing the tube before the slit of a spectroscope, the spectrum of the gas or vapor may be examined.

9. Examine the spectrum of the spark in air taken between terminals of different metals, both with and without a condenser in the secondary circuit. Notice the lines of the oxygen and nitrogen of the air, mixed with those of the metal used. By making the terminals of different metals and by so adjusting the spark that the metallic lines will not go entirely across the field, these metals may in this way be identified.

10. Cement a piece of plate-glass a centimeter or more thick upon the end of a thick glass or vulcanite tube. Pass a wire connected with one terminal down the tube to touch the plate on one side while a wire connected with the other terminal touches the other side opposite it. When the spark is sent through the apparatus, the glass is perforated, if the coil be sufficiently powerful. To prevent injury to the coil, its points should be set at a safe sparking distance.

715. Crookes Tubes.—The phenomena of the electric discharge in high vacua have been studied by Crookes (199). As the exhaustion increases, the potential-difference required to effect the discharge decreases, but only up to a certain point, and then increases again. The best effects in ordinary "vacuum tubes" are obtained when the residual pressure is from two to four millimeters of mercury. When the exhaustion reaches one millionth of an atmosphere, no electric discharge in the proper sense takes place within the tube, the nega-

tive terminal alone now becoming important. If this terminal be a disk of aluminum of some size, placed in the middle of the tube and perpendicular to its axis, the purple glow ordinarily surrounding this terminal will be seen to recede from the disk as the exhaustion proceeds, until it reaches the walls of the tube, leaving a dark space surrounding the electrode. This purple glow represents the portion of space where the molecules driven from the negative plate collide with other molecules. As exhaustion proceeds, the mean length of the free path increases and the line of collision is driven farther and farther away from the terminal; thus increasing the dark space until finally it fills the entire tube. When the exhaustion reaches a millionth, the molecules impinge upon the glass of the tube and cause it to phosphoresce with a yellow or greenish-yellow color depending upon its character. Crookes by a series of most ingenious experiments has proved that these molecules, thus shot out from the negative terminal, move in straight lines and exert mechanical force and produce heat when they strike, and that their trajectory is altered by a magnet. But the most interesting result is their phosphorescent effect. A diamond or ruby subjected to this discharge glows strongly, the former giving a bright green, the latter a rich red light. Phenakite under the same circumstances phosphoresces blue, spodumene golden yellow, calcite orange, smithsonite a brilliant green, and emerald, red. By studying the spectra of this phosphorescent glow, Crookes has created a new branch of spectrum analysis; and he has succeeded, by fractional precipitation, in separating the earth yttria into five or more constituents, the phosphorescent spectra of which taken together make up the spectrum of the yttria itself.

Since the phenomena exhibited by these high vacua are as different from those of low vacua, or even from those of the gaseous state in general, as are these latter from those of the liquid state, Crookes has called this ultra-gaseous condition a "fourth state" of matter, the

matter itself being **radiant**. He has succeeded in carrying the exhaustion up to a twenty-millionth of an atmosphere.

716. Transformers.—The name **transformer** or **converter** is given to a mutual induction apparatus using alternating currents, whose function it is to transform currents of considerable potential and low quantity into induced currents of low potential and larger quantity. Hence, like the induction coil, the transformer consists of a primary coil, a secondary coil, and an iron core. But the primary coil has the higher resistance, and the iron core forms a complete magnetic circuit. One form consists of a ring of iron wire, upon which the primary and secondary coils are wound, either in alternate sections or the one superposed on the other. In other cases the primary and secondary coils are inside of the iron wire, which is wound upon the ring formed by these coils. Faraday's induction ring is therefore a complete transformer, one of the coils having several feet more of wire than the other. The object in construction is to have the inducing coils so arranged with reference to each other that the coefficient of mutual induction between them may be a maximum; the iron being as closely associated with the coils as possible. Evidently since the energy in the primary circuit is E_1I_1 watts, it follows that if the efficiency of the transformer is unity, the energy in the secondary circuit, E_2I_2 watts, must be equal to it; whence $E_1I_1 = E_2I_2$, or $E_1 : E_2 :: I_2 : I_1$; i.e., the electromotive forces in the two circuits are inversely proportional to the current-strengths. The ratio of the electromotive forces in the two coils is called the **coefficient of transformation**; and since each electromotive force is proportional to the number of turns in its coil, the ratio of the turns represents the same coefficient. Thus, for example, suppose it is desired to transform a current of 10 amperes at a potential of 1000 volts into a current of 200 amperes at a potential of 50 volts; the coefficient of transformation is one-twentieth; or the primary circuit must have twenty times as many turns as the

secondary. In practice the mass of copper in the two is made equal; so that the primary wire is finer than the secondary. The quantity of iron is determined by the condition that it must never attain more than half saturation as a maximum.

The use of alternating-current transformers is to secure economy in electrical distribution upon the theory that smaller conductors are required to send a given quantity of energy to a given distance, in proportion as the potential-difference is greater. From the equations $W = EI$ and $I = E/R$ we have $W = E^2/R$; so that by doubling the potential-difference, the resistance remaining the same, four times the energy can be transmitted to the same distance with the same loss. An example will make this clearer. It is required to transmit 50000 watts to a distance of 500 meters with a loss of ten per cent on the mains. By direct transmission the potential-difference being 100 volts and the current 500 amperes, the conductor, 1000 meters long, will have a resistance of 0·2 ohm; its cross-section therefore will be 81·7 square millimeters and its mass 731 kilograms. By the alternate-current transformer system, with a potential-difference of 1000 volts and a current of 50 amperes, transformed where utilized into a current of 500 amperes and a potential-difference of 100 volts, the wire resistance for the 1000 meters will be 20 ohms, its cross-section will be 0·817 square millimeter, and its mass 7·31 kilograms. Thus the transmission under the higher potential-difference requires a conductor of only one one-hundredth of the mass.

Self-induction also plays an important part in the operation of transformers. If in a coil through which an alternate current is flowing an iron core be inserted, the counter-electromotive force of self-induction may be increased so as to reduce almost to zero the generator current. Hopkinson suggested such a device under the name of "choke-magnet" for reducing the current in an alternating circuit, without increasing its resistance. Evidently when the secondary circuit of a transformer

is open, and it is doing no work, the maximum of counter-electromotive force is developed in the primary itself owing to its self-induction; and as this may be made nearly to equal the direct electromotive force of the generator, the current in the primary is very small. But as more and more work is thrown into the secondary circuit, the demagnetizing effect of its current upon the iron core diminishes the self-induction of the primary and so regulates automatically the current admitted to the transformer. Finally, when the converter is doing full work the self-induction becomes a minimum and the mutual induction a maximum.

(*d*) *Magneto-electric Induction Apparatus.*

717. Magneto-electric Generators.— A magneto-electric generator is a machine for converting mechanical energy into the energy of an electric current by the agency of magnets; this conversion being usually effected by rotating a coil of wire in a magnetic field. If the field is produced by an electromagnet, the machine is ordinarily called a dynamo-electric machine; or, in brief, a dynamo. The two essential parts of a dynamo are the revolving part, called the **armature**, and the fixed or magnetic part, called the **field**. Two fundamental forms of armature are in use, known as the Gramme armature and the Siemens armature, respectively. The former, invented by Gramme in 1871, consists of a ring of iron wire, upon which a number of coils of copper wire are wound; the whole rotating upon an axis perpendicular to the ring. The latter, proposed by Siemens in 1873, consists of a cylinder made up of thin iron disks, upon which the copper conducting wire is wound lengthwise. The former arrangement, in which the conductor is wound through a ring, is called a **ring armature**; the latter, in which it is wound upon the outside of a solid cylinder, is called a **drum armature**. Upon the axis of the armature and insulated from it, strips of copper equal in number to the

coils and parallel to the axis are placed edgewise, carefully insulated from each other; one of these strips being connected to the last end of one coil and to the first end of the coil immediately following. This arrangement is called the **collector.** The field consists of one or more electromagnets, which in series dynamos are magnetized by the passage of the entire current, and in shunt dynamos by the passage of a fraction of this current through it. In some cases the field is charged by the current of a separate machine called an **exciter.**

718. Mode of Operation of Dynamo-machines.—The operation of a dynamo is easily understood. In the figure (Fig. 360), which shows a ring armature revolving

FIG. 360.

between the magnetic poles *N* and *S*, it is evident that while the flow of magnetic force through any given coil is a maximum when this coil is at the top or the bottom of its revolution, the rate of variation of this force is a maximum when the coil is passing a line joining the poles; i.e., a horizontal line. For a coil on the right side, the potential-difference developed is equal but opposite in direction to that in a coil on the left side. Moreover, no potential-difference is developed in the coils at top and at bottom of the ring. As a given coil passes from the top toward the *N* pole a difference of potential is developed in it in a certain direction which increases steadily in amount until it reaches a maximum in the horizontal position and then decreases again until it is 180° from its first position. This potential-differ-

ence, however, while varying in its value according to the position of the coil, remains constant in its direction; a decreasing negative flow of force through the coil taking the place of an increasing positive one. On the left side of the ring a similar potential-difference will be developed, but opposite to the former in direction. Since the potentials of all the coils on the same side will be added together, the coils being in series, it follows that the maximum of negative potential will be found at the top of the ring and of positive at the bottom; or *vice versa*, according to the construction. So that on placing wire brushes at these upper and lower points, as is shown in the figure, the maximum potential-difference is developed between them. The relation between the coils on the two sides of the ring is analogous to that between a pair of batteries connected in multiple (Fig. 361); except that the potential-difference of the middle coils is greatest, and diminishes toward the ends.

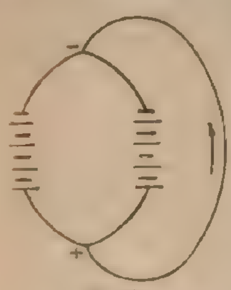


Fig. 361.

The operation of the drum armature is quite similar. In the figure (Fig. 362) suppose the armature to consist

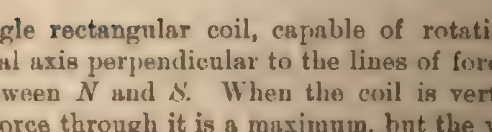


Fig. 362.

of a single rectangular coil, capable of rotating on a horizontal axis perpendicular to the lines of force of the field between N and S. When the coil is vertical the flow of force through it is a maximum, but the variation of the flow is a minimum; and *vice versa*. Hence the

electromotive force developed in the coil is proportional to the sine of the angle which the plane of the coil makes with the vertical; or in general, to the sine of the angle which a normal to this plane makes with the direction of the field. In the position of the coil shown in the dotted lines, this angle is 90° and its sine unity, and hence the potential-difference is a maximum. As the coil moves in the direction of the arrow, from the vertical position shown to a second position 180° from this, the electromotive force developed in it rises to a maximum at 90°, and then falls to zero again; this electromotive force being highest at the back of the descending side, and *vice versa*. Then as the coil continues to rotate from 180° to 360° to assume its initial position, the flow of force through it is in the opposite direction; and hence the electromotive force developed, while the same in amount, is opposite in direction, i.e., is highest at the front end of the now ascending wire. The two ends of this coil are connected with sliding contacts on the axis by which the machine is connected to the external circuit. If two rings of metal form this contact-piece, then as the electromotive force developed on these rings is reversed every revolution, the current will be an alternating one, as already stated. But if the contact-piece consists of two cylindrical segments of metal (Fig. 363)

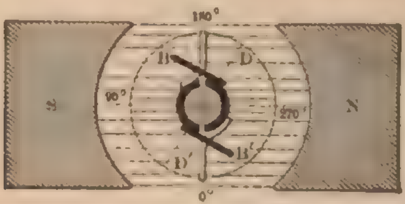


FIG. 363.

clasping, but insulated from, the axis, their dividing line being vertical, then evidently at the instant at which the current is reversed in direction by the motion of the coil, it is re-reversed by the change in the segment on

which press the brushes BB' leading to the external circuit. The function of this form of contact-piece is to reverse or commute the currents; and hence it is called a commutator. The form of the current produced by these two forms of contacts is shown in the diagrams; in the former (Fig. 364, curve A) the curve goes below

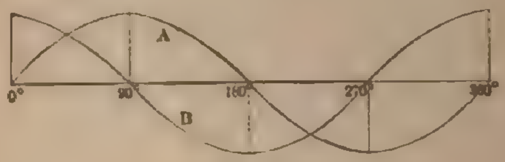


FIG. 364.

the zero line and the current is alternately positive and negative. In the latter (Fig. 365, curve A) it remains positive in direction, but varies in intensity according to the same law.

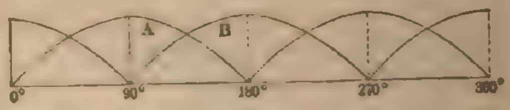


FIG. 365.

Suppose now that two coils be used in place of one, either on the ring or on the drum armature, these coils being connected to the commutator in multiple. If they are so placed as to conspire in their effect, the potential-difference will be the same as with a single coil, while the joint resistance of the two will be half that of either coil; and hence the current will be doubled. If two pairs of coils be used, however, and be so arranged that the second pair is at right angles to the first (Fig. 366), the potential-difference of the one pair will be a maximum when that of the other has a zero value; and if there be no commutator, the rise and fall of potential in the two pairs of coils will be that shown in curves A and B of Figure 364, one sine curve being 90° behind the

other. If separately commuted, however, the rise and fall of potential will be that shown in curves *A* and *B* of Figure 365. If there are four segments in the commutator, each segment being connected with one coil of each pair, then there will be two reversals and two commutations each revolution; but the brushes will be simultaneously in communication with both pairs of coils at a time. Consequently, since, through the commutator, all the coils are united with one another in series, the electromotive forces developed in them will unite to produce an algebraic sum, which constitutes the total potential-difference of the machine and the variation of which is shown in Figure 367. By increasing the number of

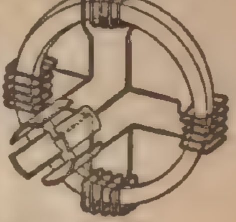


FIG. 366.

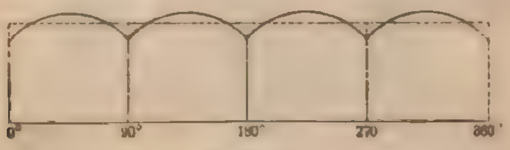


FIG. 367.

the coils thus connected the continuity of the direct current may be continually increased, becoming finally practically constant. The contact-piece in machines with many coils is generally called the **collector**.

When the machine is in operation, the magnetic lines of force due to the current in the armature compound with those due to the current in the magnets, and produce a resultant oblique to the direction of the field known as the **line of commutation**; as is seen at *DD'* in Figure 363. Hence the brushes must be advanced somewhat in the direction of rotation or given "a lead;" the amount of which varies with the load upon the machine.

719. Typical Forms of Dynamo-machine.—In practice a great variety of forms has been assumed by dynamos, both with reference to the armature and to the field-magnets. The typical form of the Gramme

machine used for lighting is shown in the figure (Fig. 368). The field consists of two ⊃-shaped electro-magnets placed horizontally with their planes vertical and their similar poles united by a "pole-piece" having curved sides. Within the cylindrical space thus formed the ring armature is made to rotate, its axis being parallel to the field coils. The potential-difference being

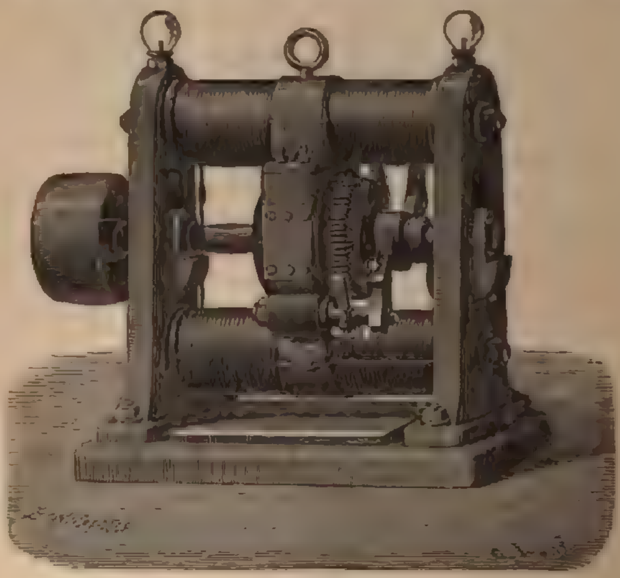


FIG. 368.

proportional to the rate of variation of the flow of force, is evidently, for a given field, dependent upon the number of turns in the coils and the speed of rotation. Because of the difficulties of construction, the drum armature has been more generally employed. The figure (Fig. 369) gives the Edison machine, having this form of armature, and one of the most efficient machines in use. The field-magnets are very massive and the field produced proportionately intense. The armature-core consists of thin disks of sheet-iron, insulated from each other, to prevent Foucault currents. Upon this core are

wound the coils, which are connected continuously to each other and are also connected between each pair to a collector having as many segments as there are coils.

In the Brush machine and the Thomson-Houston machine, the coils upon the armature are not connected together in series. Hence during some part of the rota-

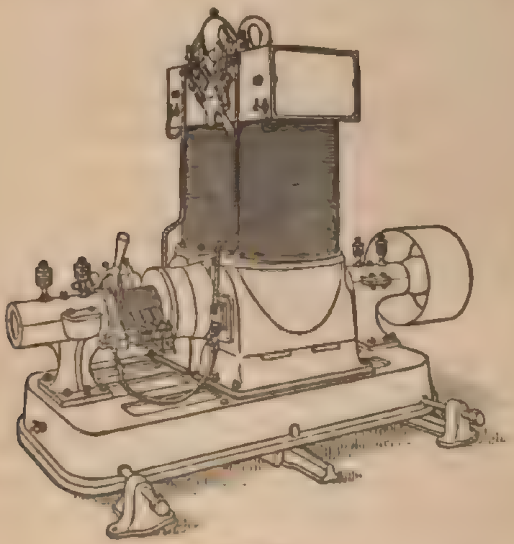


Fig. 369.

tion, certain coils are connected with collector segments not in contact with the brushes, and so are cut out of the circuit; thus diminishing the resistance of the machine. In the Brush machine the armature is of the ring form, having four or sometimes six pairs of coils upon it. In the Thomson-Houston machine, the armature is of the drum type though spherical in shape. But only three coils of wire are wound upon it, the inner ends of which are connected together, and the outer ends connected to a collector of three segments.

Of alternating machines, that devised by Stanley and known as the Westinghouse machine is one of the best

known. This machine has a cylindrical armature and a multipolar field. The field consists of sixteen radial electromagnets pointing inwards, their bases attached to an outer cylindrical frame. Their poles are alternately positive and negative. Within the cylindrical space thus formed revolves an armature-core made of thin iron disks, upon the circumference of which are laid flatwise sixteen rectangular coils of wire, their longer sides being parallel to the axis of rotation. The several coils are connected in series, the ends being joined to two collecting rings on the axis, upon which the brushes press. Evidently, since in alternate coils the electromotive force is opposite in direction, these coils must be wound alternately right-handed and left-handed; or must be connected alternately so that the electromotive force is in the same direction in all. Since there are sixteen alternations in each revolution, the total number of alternations per minute at a speed of 1000 revolutions will be 16000.

720. Series, Shunt, and Compound-wound Machines.—In 1867, Siemens, Wheatstone, and Farmer showed, almost simultaneously, that on rotating an armature connected with the field-magnets, the slight residual magnetism of the iron cores develops a weak current, which on being sent through the field coils, increases the magnetism of the field. This reacting on the armature increases the current in it and this current reacts to increase the field; until finally the full power of the machine is reached. In the earlier forms of dynamo-machine, the armature coils, the field coils, and the external circuit were all in series with one another and the machine was called a "series" machine. Of course as the external circuit increases in resistance the current round the field-magnets decreases and so the electromotive force diminishes. Hence such machines are best adapted to work on a circuit of constant resistance; such, for example, as an arc-light circuit.

In the plan suggested by Wheatstone, the current from the armature was divided; a portion being sent

through the field-magnets and the rest being sent through the external circuit. Since the field coils in this generator of Wheatstone are placed as a shunt to the main circuit, this form of machine is called a "shunt" machine. It is obvious that in this type of dynamo the current through the field-magnets is increased as the resistance of the external circuit becomes greater; and therefore that the electromotive force of the machine rises as the resistance to be overcome increases. Hence shunt machines are best adapted for circuits in which the electromotive force is to be maintained constant, the current being variable; as is the case, for example, in incandescent light circuits.

Since in "series" machines the electromotive force falls with increase of resistance, while in "shunt" machines it rises with such increase, it is plain that by the use of both devices, compensation may be more or less completely effected. If therefore the field-magnets be wound with two coils, one of low resistance in series with the external circuit, the other of high resistance in derived circuit with it, then, provided the speed be uniform, it becomes possible, by suitably proportioning these coils, to render the electromotive force of the machine practically independent of the resistance of the external circuit so that the machine regulates itself automatically. Such a machine is called a "compound-wound" machine.

721. Calculation of the Electromotive Force of a Dynamo-machine. — The calculation of the electromotive force of a dynamo may be easily effected. Suppose a drum armature having a coils of wire, each composed of b turns; then ab will be the number of turns of wire on the circumference; and since there are two circuits in multiple between the brushes, $\frac{1}{2}ab$ will be the number of wires in series on each half of the armature. Suppose the flow of force across the field to be N and the number of rotations per second n. Then since each line of force is cut twice in each revolution by each wire, the number of lines so cut in one revolution of the

armature will be $2N$, the number cut per second will be $2nN$, and the number cut by the wires on one half of the armature $\frac{1}{2}ab$ or $\frac{1}{2}C$ will be $\frac{1}{2}C \times 2nN$; or nCN. Hence the average difference of potential developed, being the rate of variation of the flow of force, will also be nCN absolute units; or $nCN/10^8$ volts. Thus the flow of force in a certain dynamo-field being 4 000 000, and the number of wires on its armature being 160, it appears that the electromotive force developed at a speed of 17·5 rotations per second will be $\frac{17{\cdot}5 \times 160 \times 4\,000\,000}{100\,000\,000} = 112$ volts. The current-strength is of course obtained by dividing this electromotive force by the resistance of the circuit. (Thompson.)

722. Efficiency of Dynamos.—Dynamos have two efficiencies: first, the ratio of the total energy developed in the **entire** circuit to the mechanical energy expended upon the machine; and second, the ratio of the energy expended in the **external** circuit to the mechanical energy absorbed. The former is generally called the **total efficiency**, the latter the **useful efficiency** of the machine. Since $W = EI$, it is evident that EI, measured with a wattmeter, for example, must approach W, measured mechanically by a transmission dynamometer, the more closely in proportion as the total efficiency is higher. In every case there are two circuits through which the current flows; i.e., the internal circuit or that through the dynamo itself; and the external or working circuit. If the resistances of these circuits be respectively r_i and r_e, then $I = E/(r_i + r_e)$. When the circuit is closed the potential-difference at the terminals of the machine is less than on open circuit; it falls from E to e. The current in the external circuit then is $i = e/r_e$, while that in the internal circuit is $i = (E - e)/r_i$; the fall of potential in the two circuits being directly as the resistances of these circuits. Moreover, since the current is the same in all parts of a circuit, the energy expended in the two portions of the above circuit is also directly

proportional to the resistances of these portions. If, for example, the potential-difference of a machine on open circuit be E volts and the current on closed circuit be I amperes, its rate of work or "activity" will be EI watts. If e be the potential-difference on closed circuit, then eI watts will be the rate of work in the external circuit. Since EI/W is the total efficiency and eI/W is the useful efficiency, the ratio of these quantities eI/EI or e/E represents evidently that fraction of the whole work done by the machine which is utilized in the external circuit; a value called by Thompson the **"economic coefficient"** of the machine. The "economic coefficient" of a machine then is simply the ratio of its electromotive force on closed circuit to the electromotive force on open circuit; or since $e/E = r_e/(r_i + r_e)$, it is also the ratio of the external to the total resistance. Evidently in order that a dynamo may have a large economic coefficient, and therefore a high useful efficiency, approximating closely to the total efficiency, it is necessary that r should be as small as possible compared with r_e. This law of maximum efficiency was first practically realized by Edison in 1879.

723. Electric Motors.—A dynamo is a completely reversible machine, and may therefore act as an electric motor and convert electric energy into mechanical energy with the same efficiency. The Sprague motor (Fig. 370) is one of the most efficient types of electric motor. A coil through which a current is passing and which is capable of rotation about a diameter as an axis, experiences when placed in a magnetic field a force tending to rotate it so as to make the flow of magnetic force through it a maximum. The work done per second in thus rotating it is the product of the moment of the rotating couple by the speed: i.e., to ωT, if ω be the angular velocity and T the twisting moment or torque. This mechanical "activity" is derived from the electrical energy supplied to the motor; so that this mechanical activity is the more nearly equal to the electrical work expended in the same time, as the motor is more efficient. Hence $\omega T =$

kEI; in which k is the efficiency. Two laws of efficiency are here to be considered (617). The first is the law of maximum rate of work, known as the law of Jacobi. If E be the counter-electromotive force developed by the motor and E the direct electromotive force supplied to it, the current through the motor being I, the rate W at which electric energy is expended upon the motor is $\mathsf{E}I$ watts, the motor being at rest. Suppose now it is allowed

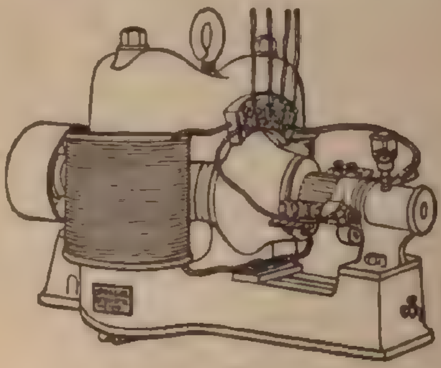


FIG. 370.

to run and so to do mechanical work. A counter-electromotive force E will be developed in it, which reduces the current to i, since by Ohm's law $i = (\mathsf{E} - E)/R$; the energy now expended per second being $W = \mathsf{E}i = \mathsf{E}(\mathsf{E} - E)/R$ watts. Besides doing mechanical work w, some of the current-energy is lost as heat H; and the energy equation is $W - w = JH = i^2R$, if R be the resistance of the circuit. Hence $w = W - i^2R$; or since $W = \mathsf{E}i$,

$$w = \mathsf{E}i - i^2R. \qquad [84]$$

The derivative of this equation with respect to i is $\mathsf{E} - 2iR$; and equating it to zero, we have $2iR = \mathsf{E}$ and $i = \frac{1}{2}\mathsf{E}/R$ as the value of i giving the maximum rate of work of the motor. But when the motor is at rest, the current has the value E/R. For the maximum rate of work, therefore, the motor must run at such a

speed as to reduce the current to one half that passing when the motor is at rest.

If, in the above equation for w, the value $(\mathsf{E} - E)/R$ be substituted for i, we get as the value of $w = E(\mathsf{E} - E)/R$. Since $W = \mathsf{E}(\mathsf{E} - E)/R$, we have $w : W :: E : \mathsf{E}$. As the current is $(\mathsf{E} - E)/R$ and also $\frac{1}{2}\mathsf{E}/R$, evidently $\mathsf{E} - E = \frac{1}{2}\mathsf{E}$, and therefore $E/\mathsf{E} = \frac{1}{2}$ and $w/W = \frac{1}{2}$. Or the efficiency is only fifty per cent when the motor does work at the maximum rate. Siemens, however, pointed out theoretically that a higher efficiency may be obtained from a motor if it be made to work at less than its maximum rate. True it does less work, but the amount of work done upon it is also lessened and to a greater extent; so that the efficiency is higher. The expression $w/W = E/\mathsf{E}$ means evidently that the ratio of the work done by the motor to the work done upon it is simply the ratio of the counter-electromotive force developed by the motor itself to that supplied to it by the generator. So that by increasing this counter-electromotive force, either by increasing the strength of the field or the speed of rotation, the efficiency may be made very nearly unity; exactly as is the case with the generating dynamo.

724. Graphic Representation of Efficiency.—These laws have been graphically represented by Thompson. If AB (Fig. 371) represent the potential-difference E and FB the counter-electromotive force E, $\mathsf{E} - E$ will be represented by AF; and since the diagram is a square, the total energy expended $\mathsf{E}(\mathsf{E} - E)$ will be given by the area $AFDH$, and $GHCL$ will represent the useful energy $E(\mathsf{E} - E)$. As R is constant, the ratios of $E(\mathsf{E} - E)$ to $\mathsf{E}(\mathsf{E} - E)$ or those of the corresponding areas $GLCH$ to $AFDH$ will represent the efficiency of the motor; i.e., the ratio of the useful energy done by the machine to the electrical energy expended upon it. The area $GLCH$ is a maximum when it

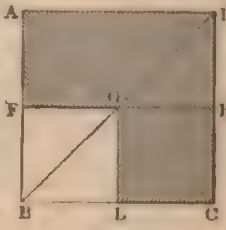


FIG. 371

is a square ; i.e., when its sides are equal. But then this area, which represents the useful work, will be one half the area $ADFH$ which represents the energy expended. The efficiency will then be 50 per cent and the ratio of E'/E will be one half. Suppose now the point G moves toward D (Fig. 372). As the two rectangles $AFGK$ and $GHCL$ remain constantly equal, the square $GHDK$, which represents the difference between the energy expended $AFHD$ and the useful energy $GHCL$, represents the energy wasted as heat. Clearly the less this heat-loss the greater the useful energy and the higher the efficiency. In the first diagram

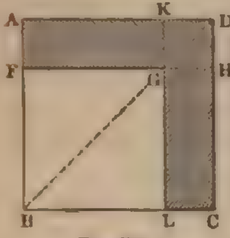


FIG. 372.

this heat-loss is enormous, being equal to the energy utilized. In the second it may be made as small as we please by making FB or E' very large compared with AB or E. If it be nine tenths of it, then the current will be reduced to one tenth of that flowing when the motor is at rest ; and the efficiency of the motor will be 90 per cent. The first diagram, therefore, illustrates the law of Jacobi ; the latter that of Siemens.

CHAPTER V.

ELECTROMAGNETIC CHARACTER OF RADIATION.

SECTION I.—RELATIONS BETWEEN LIGHT AND ELECTRICITY.

725. Electromagnetic Rotation.—In 1845, Faraday showed that the plane of polarization of a beam of light can be rotated by a magnetic field. The light in his experiment issued from an Argand lamp and was polarized in a horizontal plane by reflection from a glass surface. It then traversed first a piece of heavy glass (lead boro-silicate) placed on the poles of a large electromagnet, parallel to the direction of the lines of force; and second a Nicol prism. By suitably rotating this prism the field was made dark and the image of the lamp-flame was extinguished. On closing now the current through the electromagnet, the image of the flame became again visible; and on rotating the Nicol prism, the image could be again extinguished; thus showing that the plane of polarization was no longer horizontal as before, but had been rotated through a certain angle by the magnetic action. If the marked pole of the magnet is the one nearer to the eye, the rotation is left-handed; so that the prism has to be turned to the left to extinguish the image. In general, as Faraday proved, the plane of polarization is rotated in the same direction as that in which a current would have to circulate round the beam to produce the existing magnetism. The same property was observed in other solids and also in liquids; but no effect was detected in gases. This magnetic rotation differs from that pro-

duced by rotatory substances (460) in the fact that while the magnetic effect is a function of the direction of transmission, the latter is the same whatever the direction of transmission. It follows therefore that the magnetic rotation will be doubled on reflecting the beam back again, so as to traverse the medium twice in opposite directions.

In 1852, Verdet investigated this subject, using several powerful coils with hollow cores, their axes being in the same line. He employed the Faraday heavy glass, and also ordinary flint glass and carbon disulphide. By means of an induction method, i.e., rotating a coil 180° in the magnetic field and noting the current, the intensity of this field was measured and his experiments made quantitative. The results showed that this magnetic rotation is dependent: 1st, upon the nature of the medium employed; 2d, upon the intensity of the resolved part of the magnetic force in the direction of the light; and 3d, upon the distance traversed. So that for a given medium and a given wave-length the rotation of the plane of polarization between two points taken in the path of the beam is proportional to the difference of magnetic potential at those points. This is known as **Verdet's law.** For different media the angular rotation θ is numerically equal to the increase of magnetic potential in the medium $V_A - V_B$, multiplied by a coefficient ω which is generally positive in diamagnetic media and which is called **Verdet's constant.** It is defined to be the amount of rotation of the polarized beam, expressed in circular measure, which takes place between two points of its path one centimeter apart, within the medium, when the magnetic potential-difference between these points is unity; i.e., it is $\omega = \theta/(V_A - V_B)$. The value of this constant in absolute measure has been determined by Gordon (1877) and by Rayleigh (1885). For carbon disulphide, using thallium light of wave-length $5{\cdot}349 \times 10^{-5}$ cm., the former finds $\omega = 1{\cdot}52381 \times 10^{-5}$ radian. Rayleigh, using sodium light with CS_2 at 18°, obtained $1{\cdot}22231 \times 10^{-5}$ radian.

Hence a canal in the magnetic meridian 1·61 kilometers long and filled with this liquid, would rotate the plane of polarization of light of this wave-length, traversing it under the action of the earth's magnetism, about 50°. If the canal contained distilled water, then, since according to Gordon $\omega = 2{\cdot}248 \times 10^{-4}$ for white light, the rotation would be only about 7·5°. H. Becquerel (1877) continued these experiments and showed not only a relation between the magnetic rotatory power and the refractive index, the value $R/\overline{\mu^2\ (\mu^2 - 1)}$ being constant, but also a relation between the rotation and the wave-length, the expression just given when multiplied by the square of the wave-length being also constant. In 1879, he succeeded in detecting rotation in gases and even in measuring its amount. He proved that in coal-gas contained in a tube three meters long the beam being caused to traverse it nine times, the double rotation for sodium light was $+ 6{\cdot}8'$; that in liquid carbon disulphide under the same conditions being 513° or 30780′. This gives the approximate value 3×10^{-6} as Verdet's constant in coal-gas. In the same year, Kundt and Röntgen measured the rotation in compressed gases at a pressure of 250 atm. Their results show (1) that air, oxygen, nitrogen, carbon monoxide, carbon dioxide, coal-gas, ethyl, and marsh gas all rotate the plane in the direction of the magnetizing current; (2) that the amount of rotation is different for different gases; and (3) that for a given gas it is proportional to the density. The value of Verdet's constant for oxygen is $3{\cdot}322 \times 10^{-6}$; for nitrogen $3{\cdot}870 \times 10^{-6}$; for hydrogen $4{\cdot}023 \times 10^{-7}$; and for carbon monoxide $7{\cdot}060 \times 10^{-6}$. Hence light traveling in the meridian would have to traverse 253 kilometers of air, in order to be rotated 1° by the earth's magnetism. Becquerel has succeeded in detecting the rotation in the earth's atmosphere which is due to its magnetism.

In 1877, Kerr noticed that the plane of polarization of a beam of light is rotated when the polarized beam is reflected from the polished pole of an electromagnet;

and further that the rotation is contrary in direction to that of the magnetizing current; so that the rotation is right-handed when the reflection takes place from the north-seeking pole. In the diagram (Fig. 373) the beam

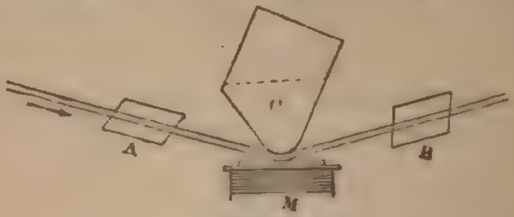


FIG. 373.

of light polarized by the Nicol prism A is reflected from the pole of the magnet M, and traverses the analyzing prism B. A soft iron wedge or sub-magnet C placed close to the pole is used to concentrate the magnetic field. To eliminate the elliptic polarization due to the metallic reflection alone, it is necessary to make the incidence normal. A plane-polarized beam from the Nicol prism A (Fig. 374) is reflected vertically by the unsil-

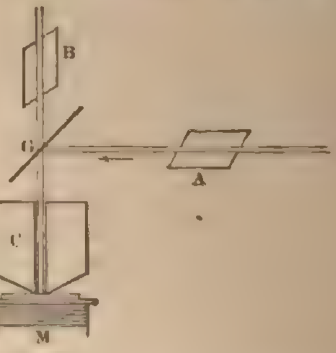


FIG. 374.

vered mirror G on to the pole of the magnet M, through the sub-magnet C, and after reflection is analyzed by the second Nicol prism B. The Nicol prisms being crossed, the field is dark; and then on energizing the magnet, the light reappears, and may be extinguished again by rotat-

ing the analyzer. Kerr has also obtained rotation by reflection from the side of a magnet; and he finds that when the plane of polarization is parallel to the plane of incidence, the rotation is always in the same direction as the magnetizing current.

726. Electrostatic Stress.—In 1875, Kerr observed that a dielectric under the influence of electrostatic stress becomes doubly refracting and hence converts plane into elliptically polarized light. The experiment is conveniently made with carbon disulphide, by sending a beam of plane-polarized light reflected from the mirror *M* (Fig. 375) through a glass tank containing the liquid, be-

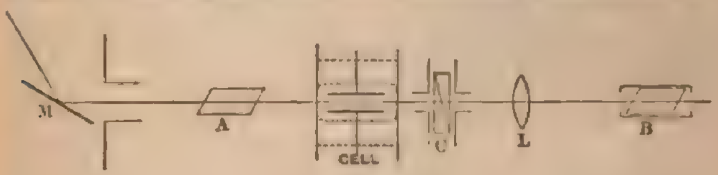


FIG. 375.

tween two plates of metal, the plane of polarization being at 45° to the vertical. The Nicol prisms *A* and *B* being crossed, no light passes; but on connecting the two plates with the two sides of a condenser, the light reappears as the condenser is charged; the phenomenon being at first white, then brilliantly colored. The magnitude of the optical effect is measured by a compensator *C* placed close behind the cell; and the potential-difference by the Thomson electrometer. The law of the action is thus stated by Kerr: "The intensity of electro-optic action of a given dielectric—that is, the quantity of optical effect (or the difference of retardations of the ordinary and extraordinary rays) per unit of thickness of the dielectric—varies directly as the square of the resultant electric force." Or, since these expressions are equivalent, the action may be said to vary directly either (1) as the energy of the electric field per unit of volume, (2) as the mutual attraction of the two conductors limiting the field, or (3) as the electric tension of

the dielectric. The optical effect in liquids varies not only in degree but also in character; those liquids being called positive which, like carbon disulphide, act like glass which is extended in the direction of the lines of force; and those being called negative which, like the fixed oils in general and especially colza oil, act like glass which is compressed in the direction of the lines of force.

Potier has suggested an optical galvanometer based on the magnetic rotatory power of liquids, since $\theta = \omega(4\pi NI)$. The liquid proposed is mercuric-potassium iodide, which has a magnetic rotation 2·8 times that of carbon disulphide. So that a tube one meter long, filled with it, surrounded with a coil 50 centimeters in length and having 5300 turns, would give with one ampere of current a rotation of 115°.

The electromagnetic rotations observed by Faraday and by Kerr are essentially similar, the Faraday effect being so large in iron that a distinct rotation is produced when the beam traverses the extremely thin film of this metal necessary to produce reflection. Suppose PQ (Fig. 376) to be the plane of polarization of the beam, and let it be resolved into the two circular components PNQ and PMQ. Evidently, since these are progressing perpendicularly to the plane of the paper, it is necessary only to suppose one of these components to be retarded or accelerated, in order to produce when recompounded a plane-polarized beam as before, but rotated through a certain angle.

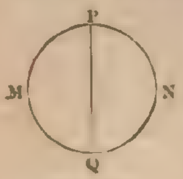


FIG. 376.

The speed of propagation of radiation is proportional directly to the square root of the elasticity and inversely to the square root of the density of the æther. Electrically, however, the relative elasticity of the æther in dielectrics is the reciprocal of the dielectric constant; and the density is the magnetic permeability. So that calling k the reciprocal of K, the dielectric constant, and μ the permeability, we have $s = \sqrt{k/\mu}$. Hence the speed of radiation is increased by whatever agency decreases either the mag-

netic or the electric permeability of the medium. In many media, such as iron for example, the permeability is not constant; and if the medium be already strongly magnetized, its permeability will be different for an increase of magnetizing force in the same direction, and for an equal increase in the opposite direction (Lodge). Hence one of the circular components being in the same direction as the existing magnetizing force while the other component is in the opposite direction, the change of permeability will not be the same for both and they will traverse the medium with different speeds and so when compounded will produce rotation of the polarized beam. The direction of the rotation will depend upon whether μ increases with small intensifications of the field or decreases with them; a substance in which μ is constant showing no Faraday effect. In iron μ is greatest for an increasing magnetizing force, and hence that component which agrees in direction with the magnetizing current will have the less speed; producing rotation in a direction opposite to that of the magnetizing current.

With reference to the electrostatic effect observed by Kerr, we may suppose that a beam polarized in the plane PQ (Fig. 377) is resolved into two perpendicular components AB and CD; and further, that one of these components traverses the medium with a different speed from the other. Evidently, when again compounded, the resulting vibration will be circular if the retardation be a quarter period, or elliptical if it be less than this. Evidently if a dielectric be subjected to a severe electrical stress, the value of its electric permeability K will be different along the lines of force and perpendicular to those lines. Consequently, the component in the direction of the lines of force will move faster than that at right angles to them and the medium will become doubly refractive and exhibit the effect described by Kerr.

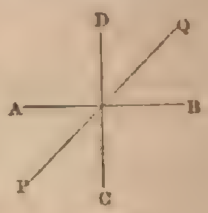


Fig. 377

Evidently a substance whose electric permeability is absolutely constant, such as free space for example, cannot show this effect.

727. The Hall Effect.—In 1880, Hall observed that when a current is made to traverse a thin sheet of metal, such for instance as gold-leaf, placed in a magnetic field with its plane perpendicular to the lines of force, a potential-difference is developed in the film, perpendicular at the same time to the lines of force and to the direction of the current. Thus if the current flows between A and B (Fig. 378), two points a and b can be

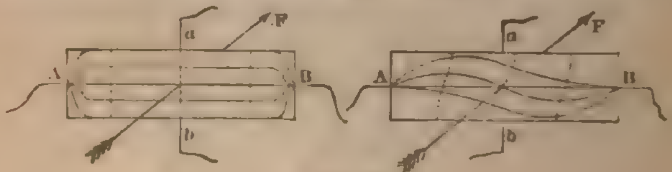


FIG. 378.

found having the same potential, the current-lines and equipotential-lines being as in the left-hand figure. If now the magnet be excited and F be the direction of its lines of force, a transverse electromotive force is developed and both the current-lines and the equipotential-lines in the conductor are displaced: (1) the potential of a being higher than b and the current through the galvanometer flowing from a toward b, in the case of iron, cobalt, and zinc, this direction being the same as that in which a conducting wire would be urged; and (2) from b toward a in the case of nickel, gold, silver, and bismuth; no effect being observable with platinum and lead. This displacement in the case of bismuth is shown in the right-hand figure. Experiment shows the electromotive force developed $E = CIF/e$; in which I is the strength of the primary current, F the strength of the field, e the thickness of the film, and C a constant, representing the electromotive force, when I, F, and e are each unity. For bismuth this constant has the value $-$ 8580·0; for nickel $-$ 14·0; and for gold $-$ 0·66.

Antimony gives + 14·0 as the value of C, iron + 7·85, cobalt + 2·40, and zinc + 0·83. The Hall effect is complicated by the action of the magnetic field upon the conductivity and thermo-electric properties of the metals used; these actions being especially large in bismuth.

The intimate relation between this transverse electromotive force observed by Hall and the Faraday effect has been pointed out by Rowland. If an electric displacement be produced in an insulator, this displacement ought to suffer a slight rotation when the dielectric is placed in a strong magnetic field so that the lines of electric and of magnetic force are perpendicular to each other, provided that such a transverse electromotive force exists as is indicated in the Hall effect; this rotation taking place during the variable state only.

728. Electrical Radiation.—The electrostatic experiments of Kerr prove clearly the displacements which are effected in dielectrics, and the consequent strains developed whenever these dielectrics are electrically charged. Whenever the distorting force is removed, however, the stress is relieved and the medium is restored to its primitive state; but then, being elastic, it passes its point of equilibrium and thus is thrown into oscillations which gradually die out. Precisely as a tuning-fork in vibration expends its energy in producing air-waves, so a condenser in its discharge throws the æther into vibration. In both cases the number of vibrations per second is determined by the vibrating body; but the speed of propagation is a function of the transmitting medium only. The time of a complete vibration of an elastic body is given by the simple harmonic equation $T = 2\pi \sqrt{m/k}$; i.e., is proportional to the square root of the ratio of the inertia and the elasticity-coefficients. In the case of a condenser-circuit, the reciprocal of the capacity of the condenser is the elasticity-coefficient, while the self-induction of the circuit represents the inertia-coefficient. So that if L be

the self-induction and C the capacity, the time of a complete oscillation will be $T = 2\pi\sqrt{LC}$, and the vibration-frequency the reciprocal of this. Again, the speed of propagation is $S = \sqrt{k/\mu}$, in which k is the reciprocal of the dielectric constant, and μ is the permeability of the medium. The discharge of a condenser-circuit therefore produces a succession of waves in the æther whose frequency is calculable from the first of the above formulas and whose speed of propagation is calculable from the second.

729. Hertz's Experiments.—Since neither of the constants k nor μ is known in absolute value for the æther, the speed of wave-propagation in this medium cannot be calculated. But evidently it may be determined experimentally. In theory a condenser-circuit may be made to face, at a given distance, a nearly closed circuit, and the time noted after the primary spark, before the secondary or induced spark makes its appearance. Practically, however, the time is too brief. Hertz

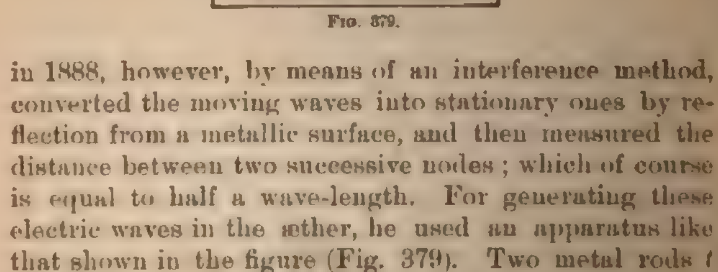


FIG. 379.

in 1888, however, by means of an interference method, converted the moving waves into stationary ones by reflection from a metallic surface, and then measured the distance between two successive nodes; which of course is equal to half a wave-length. For generating these electric waves in the æther, he used an apparatus like that shown in the figure (Fig. 379). Two metal rods t and t', terminating in small metal balls C, D, are provided with two metal spheres A, B sliding upon them

and by means of which their electrostatic capacities can be varied. The system is charged by an induction coil I, a spark passing between C and D when the potential reaches a value corresponding to the distance separating them. From the diameter of the spheres A and B, the capacity C of the system in microfarads can be approximately calculated, and from the dimensions of the rods t and t' its self-induction L in quadrants can be determined. These values known, the time of a complete oscillation, being $T = 2\pi \sqrt{LC}$, can be obtained. By varying the position of the spheres, on the rods, the value of L may be adjusted within certain limits. In order to produce waves of convenient length, such as a meter or two, the conditions must be such as to secure proportionally rapid oscillations; i.e., since $s = n\lambda$, both L and C must be small.

On putting the apparatus in action, two sets of waves of the same period are generated, the one electrostatic, the other electromagnetic. At a given point, therefore, two corresponding effects are produced, which by appropriate means may be separately studied. The electrostatic lines of force lie in planes passing through the centers of the spheres A and B, and go from one of these spheres to the other. The electromagnetic lines of force on the contrary are in planes perpendicular to those just mentioned, and are concentric about the rods t, t'. In order to explore the field, Hertz used an open wire ring having its ends provided with small balls adjustable micrometrically, so that their distance apart can be accurately measured. When this circuit is made to enclose the electromagnetic lines of force, induction sparks are produced between its terminals, which reach a maximum value when the plane of the coil passes through the axis of the vibrating generator. The best effects are obtained when the receiving coil is so adjusted that its oscillations are synchronous with those of the generating circuit; and hence this coil is called a **resonator**. Moreover throughout the field, sparks may be obtained between any two metallic ob-

jects such as keys or coins; and in fact two rods such as t, t' (Fig. 380) placed in the same line, with their ends close together, and supported in the box F by the glass tubes g, g', were actually used as a receiver. By attaching sheets of tinfoil A, B to these rods, the capacity may be increased so that sparks can be obtained at a distance of 20 or 30 meters from the oscillator. The wall opposite the oscillator was covered with sheet zinc, which being a conductor acted as a reflector for the electrical waves. By adjustment of the distance, and by

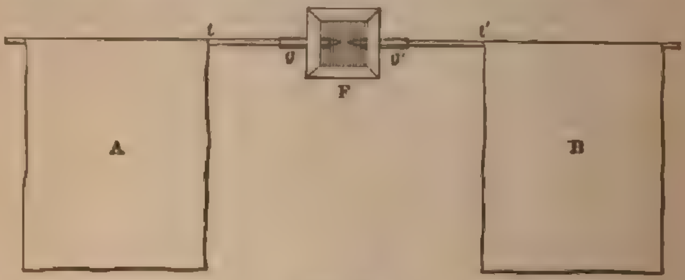


FIG. 380.

moving the resonator about, points of maximum and minimum disturbance were observed corresponding to the loops and nodes of a vibrating string. The product of the wave-length thus determined, by the vibration-frequency calculated from the constants of the oscillator, gave of course the speed of propagation; and this was found to be $3{\cdot}02 \times 10^{10}$ centimeters per second, or practically the speed of light. Moreover, Hertz found that this electric radiation can be reflected by plane conducting surfaces, while non-conductors transmit it; so that by metallic parabolic mirrors, it can be concentrated to a focus. Apertures in a series of screens show the rectilinear propagation of the waves, and a gridiron of metallic wires reflects them when its length is parallel to the direction of oscillation, and is transparent to them when the oscillations take place in a direction perpendicular to the wires. By means of a prism of pitch, with faces about a meter square and

having a refracting angle of 30°, Hertz showed the refraction of these radiations; and since the deviation produced by it was 22°, it follows that for the 60 cm. waves used in the experiment, the refractive index of the pitch was about 1·7.

SECTION II.—ELECTROMAGNETIC THEORY OF LIGHT.

730. Maxwell's Theory of Radiation.—In 1865, Maxwell propounded the theory that light is an electromagnetic disturbance. Optical as well as electrical phenomena require the existence of a medium the properties of which seem to be identical for both. Moreover, the speed of propagation of a disturbance in the medium seems to be the same whether the disturbance be an electric one or an optical one. If the disturbance be electrical, we see that the repulsion between two similar charges Q is proportional to Q^2; and since the lines of force coincide with the lines of displacement, the repulsion is also proportional to the elasticity of the medium. So that to keep the repulsion the same when the elasticity of the medium increases, the charges must be diminished; the square of either charge remaining inversely proportional to this elasticity, or the charge itself varying inversely as the square root of the elasticity. But since the electrostatic unit of quantity is defined as the quantity which repels an equal quantity at unit distance with unit force, it follows that this unit will vary with the dielectric in which the displacement is effected, being inversely proportional to the square root of its elasticity. Again, suppose two parallel currents to flow in the same direction. They will attract each other, and their attraction will be proportional to the product of the current-strengths; i.e., to the square of either if they are equal. The æther vortices developed by these currents will move with a speed proportional to the current-strength; and the pressure exerted by them will vary as the density of the medium and as the square of their velocities. To maintain the attrac-

tion between the currents constant when the density of the medium is changed, the velocity of the vortices must vary inversely as the square root of the density; i.e., since the velocity of the vortices is proportional to the current-strengths, the strength of each current must vary inversely as the square root of the density of the medium. But the electromagnetic unit of quantity being defined as the quantity which flows per second in a wire and attracts an equal quantity flowing per second in a parallel wire at unit distance with unit force, it follows that this unit will vary inversely as the square root of the density of the medium. Evidently, therefore, the ratio of the electromagnetic unit of quantity to the electrostatic unit will be proportional to the ratio of the square root of the elasticity to the square root of the density of the medium. But this latter ratio represents the speed of propagation through an elastic medium; and hence the ratio of these units represents the speed with which an electromagnetic disturbance traverses the æther.

731. Comparison of Electrostatic and Electromagnetic Units.—In 1868, Maxwell devised an experiment for determining this speed by a comparison of the electrostatic and the electromagnetic units, the principle of which consists in balancing the attraction between two electrified disks having known charges, against the repulsion of two coils through which known currents are flowing in opposite directions. The result of this experiment gave for the value of this ratio $2{\cdot}88 \times 10^{10}$ centimeters per second. Already in 1856, by measuring the charge of a condenser, 1st, electrostatically by the product of its capacity and potential-difference; and 2d, electromagnetically by discharging the condenser through a galvanometer,—Weber and Kohlrausch had obtained the value $3{\cdot}1074 \times 10^{10}$ cm. Lord Kelvin, in 1869, by comparing the value of a given electromotive force, measured electrostatically with an electrometer and measured electromagnetically with an electrodynamometer, the resistance being known, obtained the

mean value $2{\cdot}825 \times 10^{10}$ cm. A repetition of these experiments by McKichan gave $2{\cdot}93 \times 10^{10}$ cm. as a mean. Ayrton & Perry, in 1878, determined the capacity of an air-condenser both electrostatically and electromagnetically and obtained $2{\cdot}98 \times 10^{10}$ as a mean of 98 experiments. Hockin, in 1879, by the same method obtained the value $2{\cdot}988 \times 10^{10}$ cm. Rowland, in 1875, had shown that a charged gilt ebonite disk 21 cm. in diameter, revolving 61 times a second, deflected a magnetic needle. Moreover, he measured the deflection and compared the results with those required by theory. From his data, it follows that a value of $3{\cdot}0448 \times 10^{10}$ cm. for the ratio of the units would have given the deflection he observed. In 1883, J. J. Thomson obtained the value $2{\cdot}963 \times 10^{10}$. As a final mean, the value 3×10^{10} cm. is now generally adopted as the ratio of the two units.

But this ratio is identical with the speed of light as determined by the most recent experiments. Cornu's result (1874) as discussed by Listing is $2{\cdot}9999 \times 10^{10}$, Michelson's (1882) is $2{\cdot}99853 \times 10^{10}$, and Newcomb's (1882) is $2{\cdot}9986 \times 10^{10}$ cm. It would seem therefore that experimentally the value 3×10^{10} centimeters per second represents equally the speed of propagation of light and the speed of propagation of electric disturbances, both taking place in the æther. And hence that the electromagnetic theory of light as proposed by Maxwell is based upon a firm foundation of experimental fact.

732. Relation between the Dielectric Constant and the Refractive Index.—Further considerations may be adduced in support of this theory. Maxwell himself pointed out that since the attraction or repulsion between two charges is proportional directly to the elasticity and inversely to the specific inductive capacity of the medium, the elasticity of any medium must be the reciprocal of its specific inductive capacity. In other words, if k be the rigidity (or elasticity of form) of the medium, $k = 1/K$, where K is its specific inductive capacity or dielectric constant. Moreover, the density of the medium corresponds to the permeability or the

specific magnetic capacity, μ. Hence the speed of transmission in such a medium, which varies as $\sqrt{E/\mu}$, varies also as $1/\sqrt{K\mu}$. And this must be the speed of light if light is an electromagnetic disturbance. Now as in ordinary dielectrics, such as glass, quartz, sulphur, etc., the permeability is sensibly the same as in a vacuum, it follows that in such substances the speed of light must be inversely proportional to the square root of the specific inductive capacity. But the refractive index of a substance is the ratio of the speed of light in a vacuum to its speed in the substance; and hence the specific inductive capacity must be directly proportional to the square of the refractive index; i.e., since $s = 1/\sqrt{K\mu}$, $s^2 = 1/K\mu$, and n^2, which $= 1/s^2$, is equal to $K\mu$; or is proportional to K. In general, the index of refraction of a substance is the geometric mean of its electrostatic and magnetic specific capacities. Inasmuch, however, as the electromotive forces with which dielectric constants are measured continue for a much longer time than the duration of a light-vibration, the agreement will be the closer as the wave-length is longer; i.e., the dielectric constant is equal to the square of the refractive index only for light of infinite wave-length. The following are values experimentally obtained:

Substance.	$\sqrt{K}$	n	Authority.
Heavy flint glass	1·747	1·620	Gordon
Crown glass	1·763	1·504	"
Paraffin	1·412	1·422	"
"	1·523	1·526	Boltzmann
Sulphur	1·975	2·015	"
Resin	1·575	1·543	"
Turpentine	1·490	1·459	Silow
Carbon disulphide	1·345	1·611	Gordon
Air	1·000295	1·000294	Boltzmann
Hydrogen	1·000132	1·000138	"
Carbon dioxide	1·000473	1·000449	"

These numbers although not strictly accordant are sufficiently so to warrant the conclusion that if the

square root of K is not the complete expression for the index of refraction, it is yet the most important term in it; thus connecting very closely the two fundamental constants of electricity and light.

733. Relation between Opacity and Conductivity.—Again, as Maxwell also pointed out, an electromagnetic disturbance if it take place in a perfectly insulating medium must be transmitted indefinitely, since there is no outlet for its energy and consequently no loss. In a conductor, however, there are permanent displacement and conduction currents attended with frictional resistance and a consequent dissipation of the energy of the original disturbance, resulting in the production of heat, the energy being absorbed by the medium. As the disturbance progresses it continually diminishes, therefore, and soon becomes insensible. Electromagnetic disturbances consequently cannot be propagated in bodies which are conductors of electricity. Hence if light is an electromagnetic disturbance, it follows that conducting substances must be opake. As a matter of fact, most transparent solid substances are good insulators and all good conductors are very opake. Silver and mercury, for example, are good reflectors because they are good conductors and bad dielectrics; and hence do not allow the electric disturbance to be propagated through them without dissipation. Perfect insulators transmit the electric waves without loss; perfect conductors do not propagate them at all and therefore totally reflect them at their surfaces. In proportion as the conduction is less perfect there is penetration of the wave and dissipation of energy within the medium.

734. Conclusions.—These facts, taken in connection with the remarkable experiments of Hertz, appear to prove beyond a question that light is itself an electrical phenomenon, and that optics is a department of electrics. To produce radiation, it is necessary only to produce electric oscillations of sufficiently short period. Lodge calculates that a condenser of one microfarad ca-

pacity discharging through a coil having a self-induction of one quadrant will vibrate 157 times per second, and will produce waves in the æther 1900 kilometers long. A pint Leyden jar discharging through a pair of tongs may produce oscillations at the rate of ten million per second and so produce æther-waves not longer than 15 or 20 meters. A tiny jar like a thimble may give 300 000 000 oscillations per second and generate æther waves a meter long. Continuing this process, we may ask what will be the size of a circuit which will give waves comparable to those of light; say 0·6 of a micron or 6000 tenth-meters long. Since $T = 2\pi\sqrt{LC}$, its reciprocal or the wave-frequency will be $1/2\pi\sqrt{LC}$. The wave-length is s/n; or the quotient of the speed by the wave-frequency. Since $s = 1/\sqrt{\mathrm{K}\mu}$, the wave-length $\lambda = 2\pi\sqrt{\frac{L}{\mu}\cdot\frac{C}{\mathrm{K}}} = \cdot 00006$. From which it appears that the required circuit must have a self-induction in electromagnetic units and a capacity in electrostatic units such that their geometric mean is one tenth of a micron or 10^{-5} cm. This suggests at once a circuit of atomic dimensions and indicates that those æthereal waves which affect the retina and which we call light may be in fact produced by the electric oscillations of atomic circuits. An atom of sodium vibrates 5×10^{14} times in a second, or five hundred million times in one millionth of a second; and the range of vision is comprised between 4×10^{14} and 7×10^{14} vibrations per second. Could we produce electric atomic oscillations at this rate and permanently maintain them we could produce light. But this at present cannot be done. We can on the one hand create momentarily, æther-waves of a frequency as high as a few millions per second; and on the other create permanent atomic waves of the desired frequency by the agency of heat. A cylinder of lime or a carbon filament intensely heated emits waves of sufficient length; but with these waves comes a vast array of other and useless ones. Ordinarily, however, light is produced by

combustion; and Langley has shown that less than one per cent of the energy emitted is visible. The problem of the age is how to convert some other form of energy entirely into the energy of light. That this is possible in theory, Rayleigh long ago showed. That it is actually accomplished in Nature, Langley's remarkable measurements upon the glowworm abundantly confirm. Now that the mechanism of the process is before us, it would not seem impossible eventually to create and to maintain electric oscillations of the frequency required for light alone. When this is done the problem of the economical production of artificial light will have been solved.

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